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ENGINEERING ASPECTS OF MATHEMATICAL MODEL OF DEEP-PROFILED SANDWICH PANEL

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Abstract. The paper concerns the problem of static analysis of a deep-profiled sandwich panel. Equilibrium conditions in the form of differential equations are discussed. The solutions to example problems are given taking into account boundary conditions. The effects of transversal load and thermal excitations are presented and discussed.

Introduction

Sandwich panels are commonly used in many branches of industry. This type of structures is made of three layers: two external, thin and relatively rigid steel facings and a thick, but light and flexible core. The facings can be flat or deep-profiled. Sandwich structures are very attractive because of their high load-bearing capacity at low self-weight and excellent thermal insulation. This type of structures requires taking into account various aspects of structural behaviour, like the shear flexibility of the core, influence of thermal actions and diverse failure mechanisms.

A wide variety of problems concerning sandwich panels was presented by Zenkert [1] and Davies [2]. The papers also present the problem of the mechanical behaviour of flat and deep-profiled sandwich panels. The engineering aspects of the structural response of flat panels were presented in [3]. The paper analyses the mathematical relations of the solution of differential equations. The modern point of view on the process of the design and analysis of the structures was presented in [4]. The mathematical optimization of sandwich structures taking into account various boundary conditions was discussed in [5].

Despite the great importance of sandwich structures, ignorance of the principles of mechanics leads to many misunderstandings or even mistakes in design and usage. The authors present the problem of the mechanical analysis of deep-profiled sandwich panels. The issue is described by using two differential equations. The class of equations and form of the solution influence the internal force and stress distribution.

1. Sandwich panel theory

This paper discusses single-span and multi-span panels with parallel, deep-profiled facings and a soft core. The model of the two-span panel is shown in Figure 1.

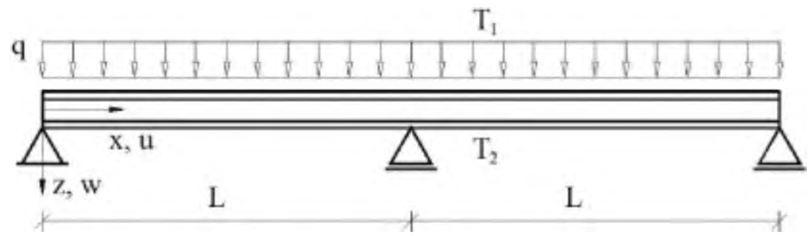


Fig. 1. Multi-span sandwich panel loaded mechanically (q) and thermally ($\Delta T = T_2 - T_1$)

The sandwich panel theory was originated by Hoff [6], Reissner [7] and the people working for Forest Product Laboratories [8] and NACA (NASA at present) [9]. The theory was popularized by Allen [10] and Plantema [11].

In the theory, it assumes that the strains are small and the materials are isotropic, homogeneous and linearly elastic. The normal stress in the foam core is negligible ($\sigma_{xC} = \sigma_{yC} = 0$) because the modulus of elasticity of the core is much lower than the steel faces (about 50 000 times). Therefore, the shear stresses in the core are constant along transverse axis z ($\tau_{xzC} = \tau_{yzC} = \text{const.}$).

The cross-sectional equilibrium condition for a panel with thick or deep-profiled faces can be written in the form of two independent differential equations [12]:

$$\frac{B_{F1} + B_{F2}}{G_C A_C} \cdot w^{VI} + \frac{B}{B_S} \cdot w^{IV} = \frac{q}{B_S} - \frac{q''}{G_C A_C} - \theta'' \tag{1}$$

$$-\frac{B_{F1} + B_{F2}}{G_C A_C} \cdot \gamma^{IV} + \frac{B}{B_S} \cdot \gamma'' = -\frac{q'}{G_C A_C} - \frac{B_{F1} + B_{F2}}{G_C A_C} \cdot \theta''' \tag{2}$$

where vertical displacement w and shear strain γ are the functions of position coordinate x. The horizontal displacement u of the mid-plane of the panel is equal to zero. The superscripts of w and γ denote the order of the derivative with respect to x. G_C and A_C denote the shear modulus and cross-sectional area of the core, q is the distributed transverse load and θ is the initial curvature induced by temperature difference $\Delta T = T_2 - T_1$. In Equations (1) and (2) the normal force is omitted.

Because the bending stiffness of the core is negligible, the total bending stiffness of panel B consists of three parts:

$$B = B_S + B_{F1} + B_{F2} \tag{3}$$

Term B_S represents the bending stiffness of the facings with respect to the global centre line of the sandwich panel, whereas B_{F1} and B_{F2} are the bending stiffness of the upper and lower facings with respect to their own centre lines.

Integrating twice Eqs. (1)-(2) and using differential relations $M' = V$, $V' = -q$, the following constitutive equations are obtained:

$$-\frac{B_{F1}+B_{F2}}{G_C A_C}\cdot w^{IV}+\frac{B}{B_S}\cdot w''=-\frac{M}{B_S}-\frac{q}{G_C A_C}-\theta \tag{4}$$

$$-\frac{B_{F1}+B_{F2}}{G_C A_C}\cdot \gamma''+\frac{B}{B_S}\cdot \gamma=\frac{V}{G_C A_C}-\frac{B_{F1}+B_{F2}}{G_C A_C}\cdot \theta' \tag{5}$$

Terms M and V denote the bending moment and the shear force, respectively. It is worth noticing that Equations (4) and (5) are only useful in the case of statically determined systems because the moment and shear force functions should be known.

Taking into account the assumptions concerning sandwich element deformation [12, 13], the bending moment and shear force can be divided into three parts:

$$M = M_S + M_{F1} + M_{F2} = B_S\left(\gamma' - w''\right) - B_{F1}w'' - B_{F2}w'' \tag{6}$$

$$V = V_S + V_{F1} + V_{F2} = G_C A_C \gamma - B_{F1}w''' - B_{F2}w''' \tag{7}$$

where M_S and V_S correspond to the pure "sandwich" effect and the terms with subscript $F1$ and $F2$ denote the forces which are taken over by the external facings.

Introducing the span of beam L and non-dimensional quantities:

$$\alpha = \frac{B_{F1}+B_{F2}}{B_S} \tag{8}$$

$$\beta = \frac{B_S}{G_C A_C L^2} \tag{9}$$

$$\lambda^2 = \frac{1+\alpha}{\alpha\beta} \tag{10}$$

differential equations (4) and (5) can be written as:

$$w^{IV} - \left(\frac{\lambda}{L}\right)^2 w'' = \frac{1}{B}\left(\frac{\lambda}{L}\right)^2\left[M + \beta L^2 q + \frac{B}{1+\alpha}\theta\right] \tag{11}$$

$$\gamma'' - \left(\frac{\lambda}{L}\right)^2 \gamma = -\frac{\beta\lambda^2}{B}V + \theta' \tag{12}$$

According to [13], the bending stresses in the faces are calculated using:

$$\sigma_{F1} = \sigma_{F1}^{MS} + \sigma_{F1}^{MF1} = -\frac{M_S}{eA_{F1}} + \frac{M_{F1}}{I_{F1}} z_1 \tag{13}$$

$$\sigma_{F2} = \sigma_{F2}^{MS} + \sigma_{F2}^{MF2} = \frac{M_S}{eA_{F2}} + \frac{M_{F2}}{I_{F2}} z_2 \tag{14}$$

where e, A_{F1} and A_{F2} denote the distance between the centroids of the faces, the cross-sectional area of the external (upper) and internal (lower) face, respectively. Symbols I_{F1} and I_{F2} represent second moments of the area of the faces, whereas z_1 and z_2 are the coordinates of a point belonging to the respective facing (Fig. 2).

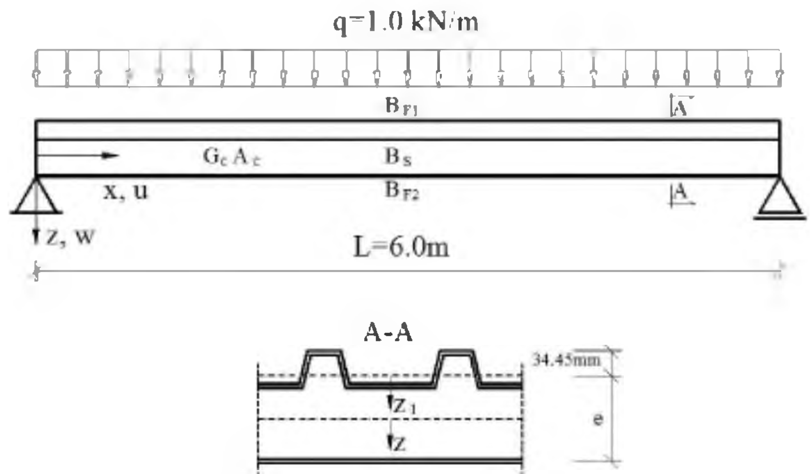


Fig. 2. Geometrical parameters and normal stress distribution within deep-profiled cross section of sandwich panel

The shear stresses in the core and faces are calculated according to:

$$\tau_C = \frac{V_S}{A_C} \tag{15}$$

$$\tau_{C1} = \frac{V_{F1}}{A_{w1}}, \qquad \tau_{C2} = \frac{V_{F2}}{A_{w2}} \tag{16}$$

where A_{w1} and A_{w2} represent the cross-section area of the webs of the respective facing.

2. Solution of differential equations

The general solution of differential equations (11) and (12) has the following form:

$$w(x) = C_1 \cosh\frac{\lambda x}{L} + C_2 \sinh\frac{\lambda x}{L} + C_3 + C_4 x + w_p(x) \tag{17}$$

$$\gamma(x) = D_1 \cosh\frac{\lambda x}{L} + D_2 \sinh\frac{\lambda x}{L} + \gamma_p(x) \tag{18}$$

where $w_p(x)$ and $\gamma_p(x)$ are the particular integrals of respective differential equations. It is worth noticing that separate analysis of the temperature influence represented in Eqs. (11)-(12) by θ and the influence of transverse load q is very comfortable. If the system is statically undetermined, the problem comes down to a few statically determined problems with additional kinematic conditions.

Particular integrals $w_p(x)$ and $\gamma_p(x)$ can be found assuming the polynomial form of the solution of (11) and (12). The power of the polynomial should correspond to the form of functions q and θ.

The constants of the general solutions are derived taking into account the prescribed boundary conditions. Let us note that even in the case of a simply supported sandwich panel with deep-profiled facings, six boundary conditions are demanded, whereas in the case of flat facings, only four boundary conditions are necessary. The examples of solutions are presented below.

3. Influence of tanverse load

Consider a simply supported sandwich panel with an upper deep-profiled facing and lower flat facing subjected to uniform transverse load q. The structure is presented in Figure 3 and the boundary conditions are as follows:

$$w(0) = 0, \qquad w''(0) = 0, \qquad \gamma'(0) = 0 \tag{19}$$

$$w(L) = 0, \qquad w''(L) = 0, \qquad \gamma'(L) = 0 \tag{20}$$

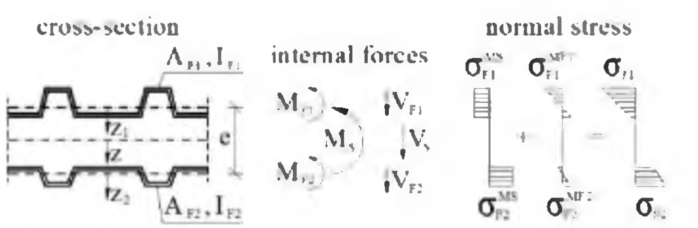


Fig. 3. Simply supported, deep-profiled sandwich panel subjected to uniform load

The solution of the problem has the form:

$$w(x)=\frac{qL^4}{B}\left[\frac{1}{24}\xi\left(1-2\xi^2+\xi^3\right)+\frac{1}{2\alpha\lambda^2}\xi(1-\xi)-\frac{1}{\alpha\lambda^4}\frac{\cosh(\lambda/2)-\cosh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)}\right] \tag{21}$$

$$\gamma(x)=\frac{qL^3}{B}\beta\left[\frac{1}{2}(1-2\xi)-\frac{1}{\lambda}\frac{\sinh\left(\lambda(1-2\xi)/2\right)}{\cosh(\lambda/2)}\right] \tag{22}$$

where $\xi = x/L$. Bending moment M and shear force V can be divided into:

$$M_S(x)=qL^2\frac{1}{1+\alpha}\left[\frac{1}{2}\xi(1-2\xi)-\frac{1}{\lambda^2}\frac{\cosh(\lambda/2)-\cosh\left(\lambda(1-2\xi)/2\right)}{\cosh(\lambda/2)}\right] \tag{23}$$

$$M_{F1}(x)=qL^2\frac{\alpha_{F1}}{1+\alpha}\left[\frac{1}{2}\xi(1-2\xi)+\frac{1}{\alpha\lambda^2}\frac{\cosh(\lambda/2)-\cosh\left(\lambda(1-2\xi)/2\right)}{\cosh(\lambda/2)}\right] \tag{24}$$

$$M_{F2}(x)=qL^2\frac{\alpha_{F2}}{1+\alpha}\left[\frac{1}{2}\xi(1-2\xi)+\frac{1}{\alpha\lambda^2}\frac{\cosh(\lambda/2)-\cosh\left(\lambda(1-2\xi)/2\right)}{\cosh(\lambda/2)}\right] \tag{25}$$

and

$$V_S(x)=qL\frac{1}{1+\alpha}\left[\frac{1}{2}(1-2\xi)-\frac{1}{\lambda}\frac{\sinh\left(\lambda(1-2\xi)/2\right)}{\cosh(\lambda/2)}\right] \tag{26}$$

$$V_{F1}(x)=qL\frac{\alpha_{F1}}{1+\alpha}\left[\frac{1}{2}(1-2\xi)+\frac{1}{\alpha\lambda}\frac{\sinh\left(\lambda(1-2\xi)/2\right)}{\cosh(\lambda/2)}\right] \tag{27}$$

$$V_{F2}(x)=qL\frac{\alpha_{F2}}{1+\alpha}\left[\frac{1}{2}(1-2\xi)+\frac{1}{\alpha\lambda}\frac{\sinh\left(\lambda(1-2\xi)/2\right)}{\cosh(\lambda/2)}\right] \tag{28}$$

with:

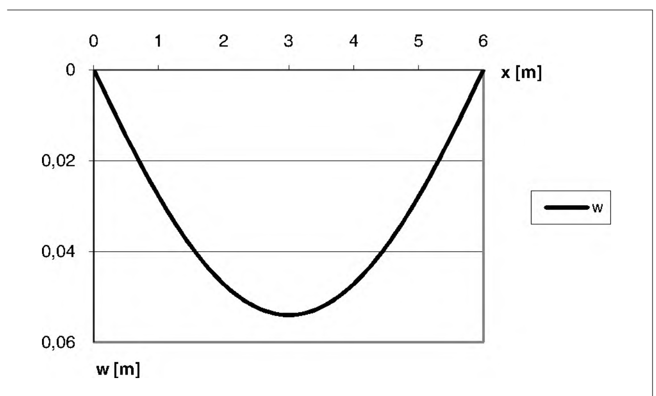
$$\alpha_{F1}=\frac{B_{F1}}{B_S},\qquad \alpha_{F2}=\frac{B_{F2}}{B_S},\qquad \alpha_{F1}+\alpha_{F2}=\alpha \tag{29}$$

The stresses are calculated using (13) and (14). The displacements, internal forces and extreme stresses are presented in Figures 4 and 5. The geometrical and mechanical parameters used in the example correspond to typical roof sandwich panels, namely: width of panel b = 1.0 m, span L = 6.0 m, e = 85.3 mm,

G_C = 3550 MPa, A_C = 85300 mm^2, A_{F1} = 550 mm^2, A_{F2} = 480 mm^2, B_S = 393.3 kNm^2, B_{F1} = 20.56 kNm^2, B_{F2} = 0.0018 kNm^2, I_{F1} = 97920 mm^4, A_{w1} = 62 mm^2. The structure is loaded uniformly by q = 1.0 kN/m. To find the extreme stresses in the deep-profiled face, coordinate $z_1 = -34.45$ mm, of the most distant point belonging to the upper face was used.

Figures 5a and 5b show that internal forces M_{F1} and V_{F1} corresponding to the bending stiffness of the upper face are relatively small comparing to the total values of M and V. In fact, moment M_{F1} gives a higher stress (–100.2 MPa) in the upper face than moment M_S (–89.8 MPa). This fact is presented on Figure 5c. The total extreme stress in the upper face is equal to –190.0 MPa, whereas in the lower flat face it is only 102.95 MPa. The shear forces cause at the support an extreme shear stress in the core τ_C = 30.6 kPa and in the upper face τ_{F1} = 6.30 MPa. The stress in the face is much higher than in the core, but it is still a very low value comparing to the shear strength of the steel. The shear strength of the core material is 100÷120 kPa.

a)



b)

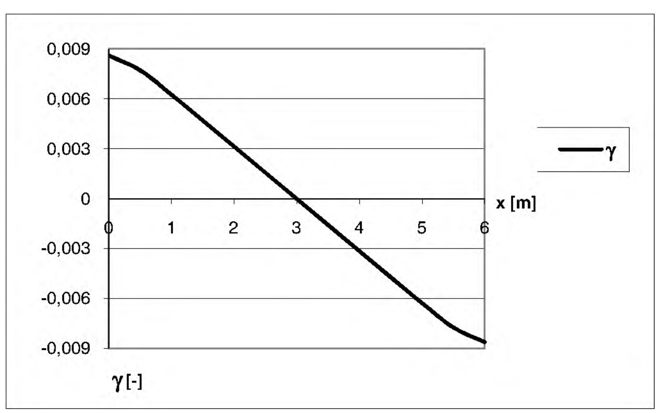
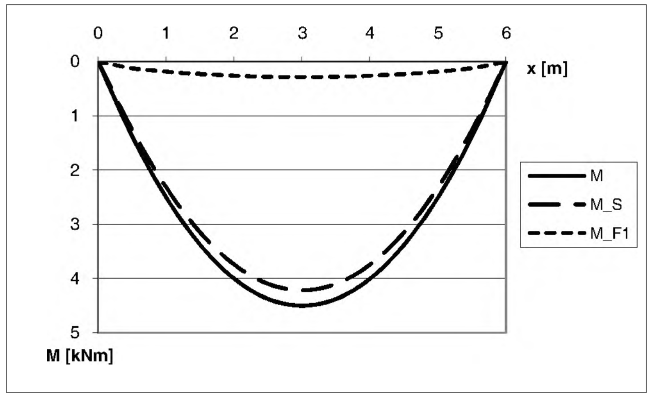


Fig. 4. Solution of simply supported, deep-profiled sandwich panel subjected to uniform transverse load q: a) displacement w, b) shear strain γ

a)



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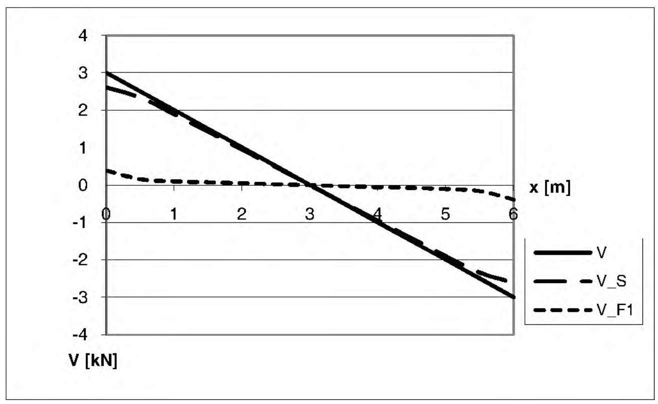


c)

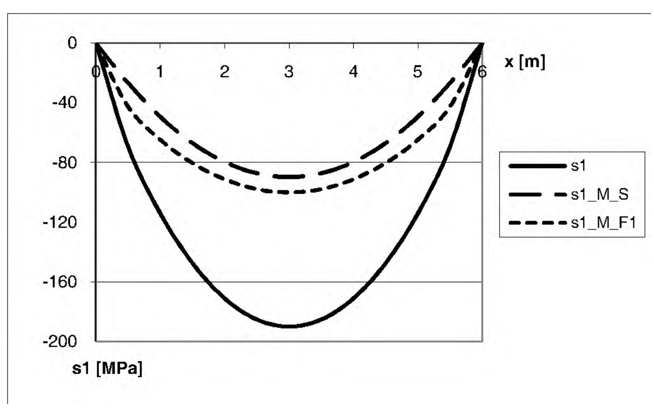


Fig. 5. Solution of simply supported, deep-profiled sandwich panel subjected to uniform transverse load q: a) bending moments M, M_S, M_{F1}, b) shear forces V, V_S, V_{F1} c) extreme normal stress in upper face: s1 - total stress

4. Influence of thermal action

Let us consider the same simply supported sandwich panel, subjected to thermal action. Temperature difference $\Delta T = T_2 - T_1$ between the internal and external faces triggers initial (constant) curvature θ:

$$\theta = \frac{\alpha_2 T_2 - \alpha_1 T_1}{e} \tag{30}$$

where e is the distance between the centroids of the faces and α_1, α_2 are the thermal expansion coefficients of the respective faces. In the case of panels with flat faces, this kind of excitation results only in displacements (maximum deflection $\theta L^2/8$) and does not cause internal forces. In the case of a panel with deep-profiled faces, bending moments and shear force appear within the structure. Using differential equations (11), (12), arbitrary solutions (17), (18) and boundary conditions (19), (20), knowing that $q = 0$, $V = 0$, $M = 0$, we obtain the displacement functions:

$$w(x) = \frac{\theta L^2}{1+\alpha}\left[\frac{1}{2}\xi(1-\xi) - \frac{1}{\lambda^2}\frac{\cosh(\lambda/2) - \cosh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)}\right] \tag{31}$$

$$\gamma(x) = \frac{\theta L}{\lambda}\frac{\sinh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)} \tag{32}$$

and internal forces:

$$M_S(x) = -\frac{\alpha\theta B_S}{1+\alpha}\frac{\cosh(\lambda/2) - \cosh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)} \tag{33}$$

$$M_{F1}(x) = \frac{\alpha_{F1}\theta B_S}{1+\alpha}\frac{\cosh(\lambda/2) - \cosh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)} \tag{34}$$

$$M_{F2}(x) = \frac{\alpha_{F2}\theta B_S}{1+\alpha}\frac{\cosh(\lambda/2) - \cosh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)} \tag{35}$$

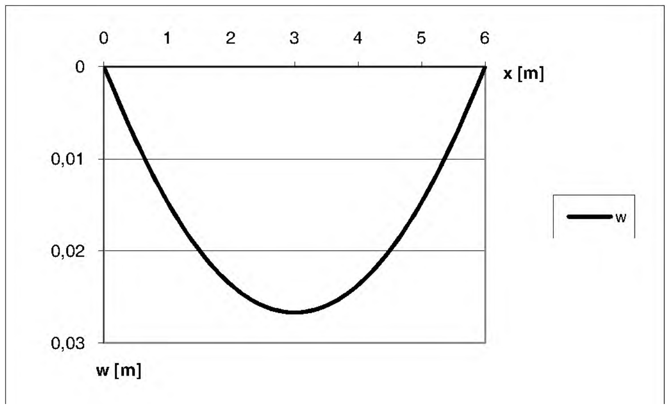
$$V_S(x) = -\frac{\theta B_S}{\beta\theta L}\frac{\sinh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)} \tag{36}$$

$$V_{F1}(x) = \frac{\alpha_{F1}\theta B_S}{\alpha\beta\theta L}\frac{\sinh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)} \tag{37}$$

$$V_{F2}(x) = \frac{\alpha_{F2}\theta B_S}{\alpha\beta\theta L}\frac{\sinh(\lambda(1-2\xi)/2)}{\cosh(\lambda/2)} \tag{38}$$

The displacements, internal forces and extreme stresses are presented in Figures 6 and 7.

a)



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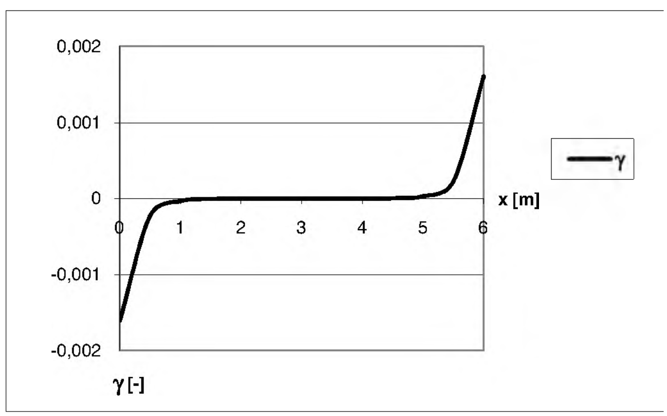

Fig. 6. Solution of simply supported, deep-profiled sandwich panel subjected to uniform curvature θ: a) displacement w, b) shear strain γ

Results w, γ presented in Figure 6 and M, V, σ 7 in Figure 7 should be compared to Figures 4 and 5, respectively. The deflection caused by temperature is comparable to the deflection of the structure loaded transversely. In the case of thermal action, the shear strains are essential only near the supports. The bending moments, shear forces and normal stresses have very low values, but indeed the existence of such quantities is interesting. This is the effect of the fundamental assumptions of the theory and type of differential equations.

a)

b)

c)

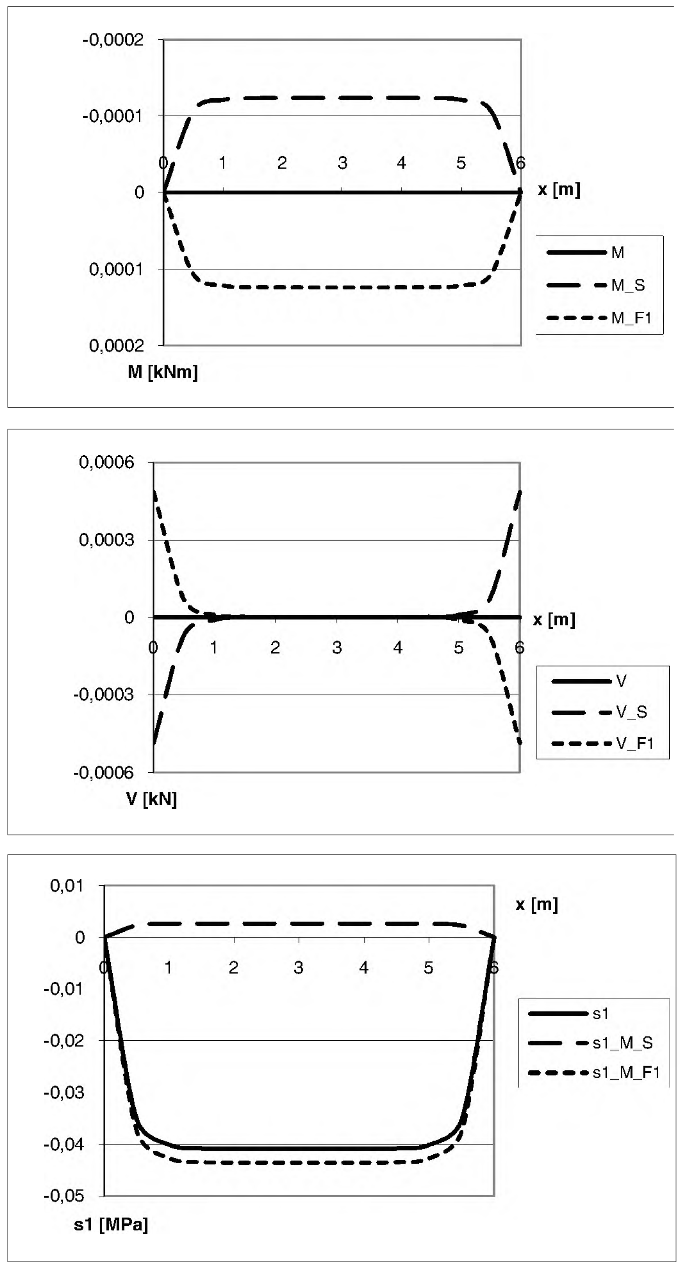

Fig. 7. Solution of simply supported, deep-profiled sandwich panel subjected to uniform curvature θ: a) bending moments M, M_S, M_{F1}, b) shear forces V, V_S, V_{F1} corresponding to sandwich effect, c) extreme normal stress in upper face: s1 - total stress

Conclusions

The presented analysis shows that the structural behaviour of deep-profiled sandwich panels is much more difficult than panels with flat facings. The total bending moment is divided into parts which correspond to the sandwich effect and bending stiffness of the faces. It results in very high stress in deep-profiled faces. This fact is even more important in multi-span panels where high stresses lead to yielding of the face at the internal supports.

In the case of a simply supported structure, thermal excitation results in relatively high displacements and very low forces and stresses. Nevertheless, the distribution of the quantities seems to be very interesting. The final stresses depend on the complicated relations between the bending and shear stiffnesses of the panel. In the end, very often engineering intuition fails. The proper analysis of deep-profiled sandwich panels is crucial to ensure serviceability of the structures.

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