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"DID NAPOLEON HAVE TO LOSE THE WATERLOO BATTLE?" – SOME SENSITIVITY ANALYSIS AND OPTIMIZATION EXPERIMENTS USING SIMULATION BY VENSIM

Introduction

Analysis, modeling and simulation of complex, non-linear, dynamic and multi-level systems have a long history, especially in the area of famous System Dynamics method (see: [Coyl77, Coyl94, Coyl96, Coyl98, Coyl99, Forr61, Forr69, Forr71, Forr72, Forr75, Rado01, Ster00, Ster02]). The modern simulation languages, such as Vensim [Vens09] can connect the simulation with the optimization and in this way to estimate the sensitive parameters in the modeled objects and choose the optimal decisions. The range of the modeled objects is quite wide, from industrial models, to ecological or economics. The review of famous application of SD method the Reader can find in the monograph [Kasp09]. The application of SD method can answer such questions as: what if? (typical for simulation type prognostic), what structure conditions the observed behavior of the system over time? (type descriptive – explanations study), how optimal in the particulars conditions? (normative, optimization investigation). The latest are not so popular like, in our opinion, should be. The authors of this paper have some experience with so called embedding the simulation in the optimization and vice-versa, working with the family of the models named DYN-BALANCE (see: [Kasp02, Kasp05, Kasp09, KaMa05, KaMa06, KaMS00, KaMS01, KaMS03, KaMS06, KaSl05, KaSl05a, KaSl06]). In order to make popular the possibilities of language Vensim in the scope of sensitivity analysis and the optimization, we decide to choose the model of Waterloo Battle, created by prof. Coyle [Coyl96], with the few authors' modifications. The results are new and help to answer the theoretical question: whether Napoleon had to lose the Waterloo Battle.

Presentation of the object of experiments and the assumptions of the simulation

The object of experiments is described in the literature of the fields (see: [Coyl96]). Now we summarized the main assumptions of the structure presented in Figure 1.

- Two enemies, the coalition and Napoleon are on the battlefield.
- The scenario is that both the coalition and Napoleon have the forces on the frontier and others in the reserve, some distance away; for both sides, if reserves are ordered into battle, there is a delay before they can arrive and because of limitation in the road network, there is a limit to the rate at which reserves can be.
- Each side inflicts combat losses on the other depending on the rate at which each man fires, the number of men in action and the proportion of shot which hit a target.
- Napoleon wishes to achieve a speedy victory and therefore commits his reserves as fast as possible; the coalition commander hope to wait for Allied forces to arrive from the neighboring country and then to achieve an overwhelming victory. He therefore commits reserves only when he needs to. His criterion of need is that he does not wish to be outnumbered, by more than 2 to 1 at the point of combat.

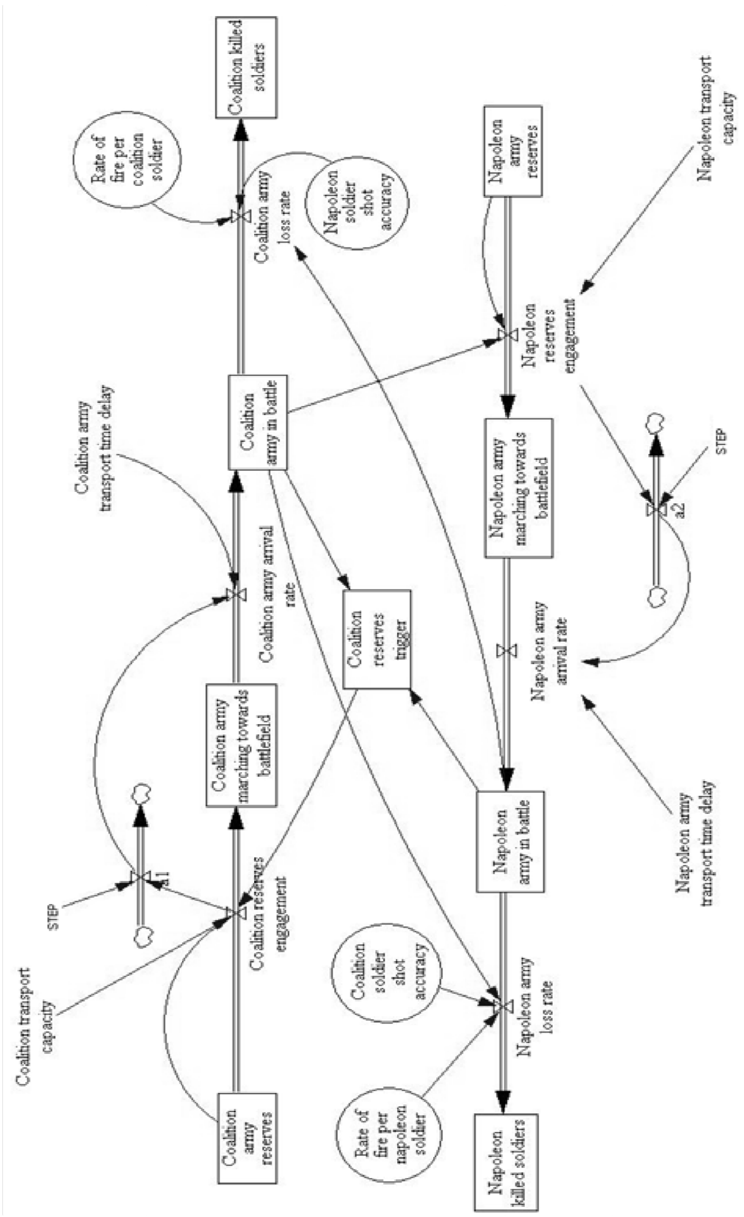

Fig. 1. The structure of model “Waterloo Battle” in Vensim convention

Source: Own idea.

Below we present the list of variables and parameters and the mathematical model of Waterloo Battle, interpreting the structure from Figure 1.

List of the variables and the parameters of this model:

Coalition army reserves – Car
Coalition reserves engagement – Cre
Coalition army marching toward battlefield – $Camtb$
Coalition army arrival rate – $Caar$
Coalition army in battle – $Caib$
Coalition army loss rate – $Calr$
Coalition killed soldiers – Cks
Coalition reserves trigger – Crt
Napoleon killed soldiers – Nks
Napoleon army loss rate – $Nalr$
Napoleon army in battle – $Naib$
Napoleon army arrival rate – $Naar$
Napoleon army marching towards battlefield – $Namtb$
Napoleon reserves engagement – Nre
Napoleon army reserves – Nar
Napoleon army transport time delay – $Nattd$
Napoleon transport capacity – Ntc
Coalition transport capacity – Ctc
Coalition army transport time delay – $Cattd$
Rate of fire per napoleon soldier – $Rofpns$
Rate of fire per coalition soldier – $Rofpcs$
Napoleon soldier shot accuracy – $Nssa$
Coalition soldier shot accuracy – $Cssa$
Auxiliary number one – $a1$
Auxiliary number two – $a2$
STEP start of the battle – $STEP$

The mathematical model of the Waterloo Battle:

$$Car(t+dt) = Car(t) - dt \cdot Cre(t) \tag{1}$$

$$Cre(t) = \begin{cases} MIN\left(Ctc, \frac{Car}{dt}\right), Crt \geq 1 \\ 0, Crt < 1 \end{cases} \tag{2}$$

$$Camtb(t+dt) = Camtb(t) + dt(Cre(t) - Caar(t)) \tag{3}$$

$$Caar(t) = DELAY3(a1(t), Cattd) \tag{4}$$

$$a1(t) = Cre(t) \cdot STEP(1,1) \tag{5}$$

$$Caib(t + dt) = Caib(t) + dt(Caar(t) - Calr(t)) \tag{6}$$

$$Cks(t + dt) = Cks(t) + dt(Calr(t)) \tag{7}$$

$$Calr(t) = \begin{cases} MAX(Naib(t) \cdot Rofpns \cdot Nssa, 0), Caib > 0 \\ 0, Caib \ \leq 0 \end{cases} \tag{8}$$

$$Nks(t + dt) = \ Nks(t) + dt \cdot Nalr(t) \tag{9}$$

$$Nail(t) = \begin{cases} MAX(Caib(t) \cdot Rofpcs \cdot Cssa, 0), Naib > 0 \\ 0, Naib \leq 0 \end{cases} \tag{10}$$

$$Naib(t + dt) = Naib(t) + dt\big(Naar(t) - Nalr(t)\big) \tag{11}$$

$$Naar(t) = DELAY3(a2(t), Nattd) \tag{12}$$

$$a2(t) = \ Nre(t) \cdot STEP(1,1) \tag{13}$$

$$Nar(t + dt) = Nar(t) - dt \cdot Nre(t) \tag{14}$$

$$Nre(t) = \begin{cases} MIN\left(\frac{Nar}{dt}, Ntc\right), Caib \geq 0 \\ 0, Caib < 0 \end{cases} \tag{15}$$

$$Crt(t) = \begin{cases} 1, -(2 \cdot Caib(t) - Naib(t)) \geq 0 \\ 0, -\big(2 \cdot Caib(t) - Naib(t)\big) < 0 \end{cases} \tag{16}$$

Now we present the assumptions of the basis simulation in the Table 1.

Table 1

Assumptions of the simulation

Levels or parameters	Initial value	Dimension
Car	30000	[Coalition Soldier]
Camtb	0	[Coalition Soldier]
Caib	15000	[Coalition Soldier]
Cks	0	[Coalition Soldier]
Nks	0	[Napoleon Soldier]
Naib	9050	[Napoleon Soldier]
Namtb	0	[Napoleon Soldier]
Nar	32000	[Napoleon Soldier]
Nssa	0.00083	[Coalition Soldier/Napoleon soldier shot]
Cssa	0.00081	[Napoleon Soldier/Coalition soldier shot]
Rofpns	150	[Napoleon soldier shot/hour/Napoleon soldier]
Rofpcs	150	[Coalition soldier shot/hour/Coalition soldier]
Cattd	6	[hour]
Nattd	6	[hour]
Ctc	4000	[Soldiers/hour]
Ntc	2500	[Soldiers/hour]

The results of basis simulation are presented in Figure 2 and Figure 3. We can observe that at the assumption for the model and at the equations 1-16 Napoleon could win the battle, what is easily notice in the values in Table 2, and Table 3. It will be interesting to suggest the changes in model, specially the strategy of coalition commander. If we change the value 2 for outnumbered coalition forces by Napoleon forces, the result of battle will be quite opposite. We can observe this in Figure 4 and Figure 5.

The authors have performed many experiments on the model and noticed that the model is very sensitive on values of parameters and the initial values of levels. And because of this we decide to execute the sensitivity analysis and the optimization on the model on the bases of possibilities of language Vensim [Vens09].

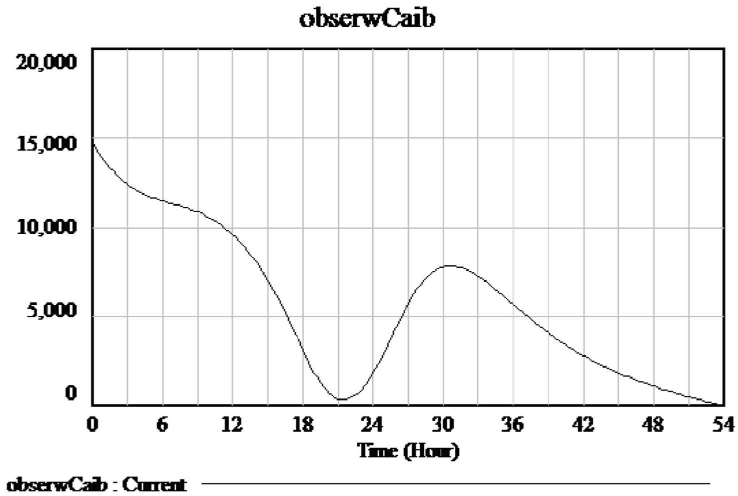

Fig. 2. The dynamics of the variable: Coalition army in the battle (Coalition lose the battle)

Source: Own results.

Table 2

End of the Waterloo Battle, values of variable *Caib* (observed)

Table Time Down

Time (Hour)	"obserwCaib"	obserwCaib
49.75	Runs:	716.282
50	Current	668.666
50.25		621.712
50.5		575.383
50.75		529.644
51		484.462
51.25		439.801
51.5		395.625
51.75		351.897
52		308.576
52.25		265.622
52.5		222.993
52.75		180.645
53		138.535
53.25		96.6196
53.5		54.8546
53.75		13.1961
54		0

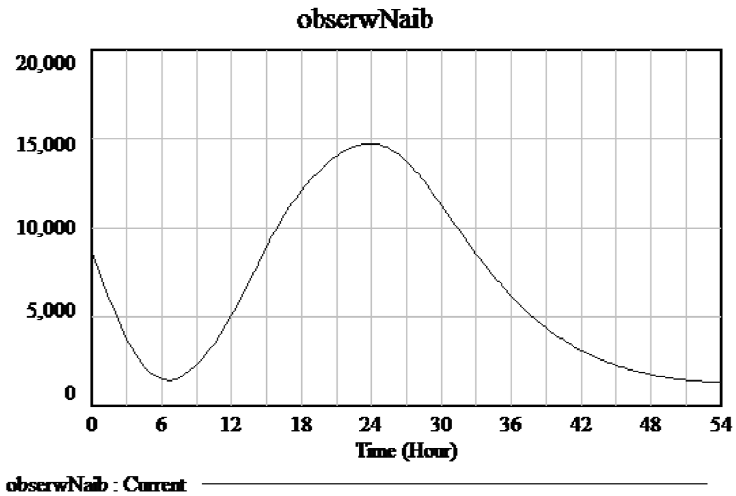

Fig. 3. The dynamics of variable: Napoleon army in the battle (Napoleon win the battle)

Source: Own results.

Table 3

End of the Waterloo Battle, values of variable *Naib* (observed)

Table Time Down

Time (Hour)	"obserwNaib"	obserwNaib
49.5	Runs:	1557.55
49.75	Current	1534.32
50		1512.57
50.25		1492.26
50.5		1473.37
50.75		1455.9
51		1439.81
51.25		1425.09
51.5		1411.73
51.75		1399.72
52		1389.03
52.25		1379.66
52.5		1371.59
52.75		1364.81
53		1359.33
53.25		1355.12
53.5		1352.19
53.75		1350.52
54		1350.12

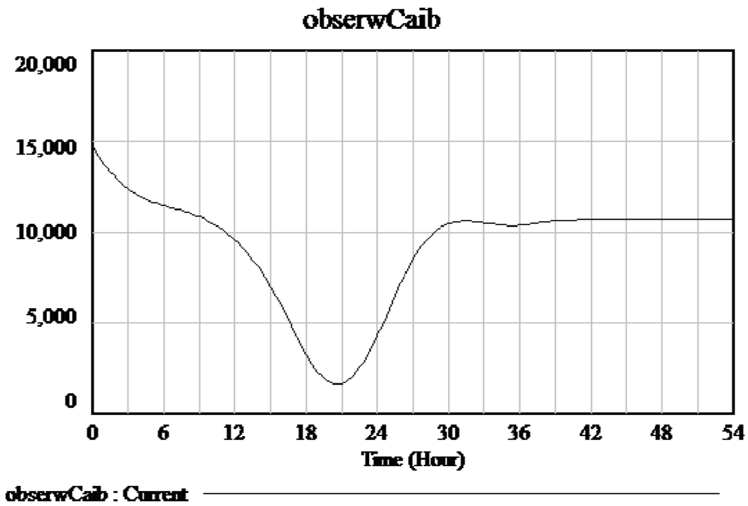

Fig. 4. The dynamics of the variable: Coalition army in the battle (Coalition win the battle)

Source: Own results.

Table 4

End of the Waterloo Battle, values of variable *Caib* (observed)

Table Time Down

Time (Hour)	"obserwCaib"	obserwCaib
32.25	Runs:	10589.9
32.5	Current	10571.8
32.75		10551.1
33		10528.6
33.25		10505.2
33.5		10481.8
33.75		10458.9
34		10437.4
34.25		10417.8
34.5		10400.8
34.75		10387
35		10376.8
35.25		10370.9
35.5		10369.5
35.75		10373.3
36		10382.5
36.25		10397.6
36.5		10418.9
36.75		10446.7

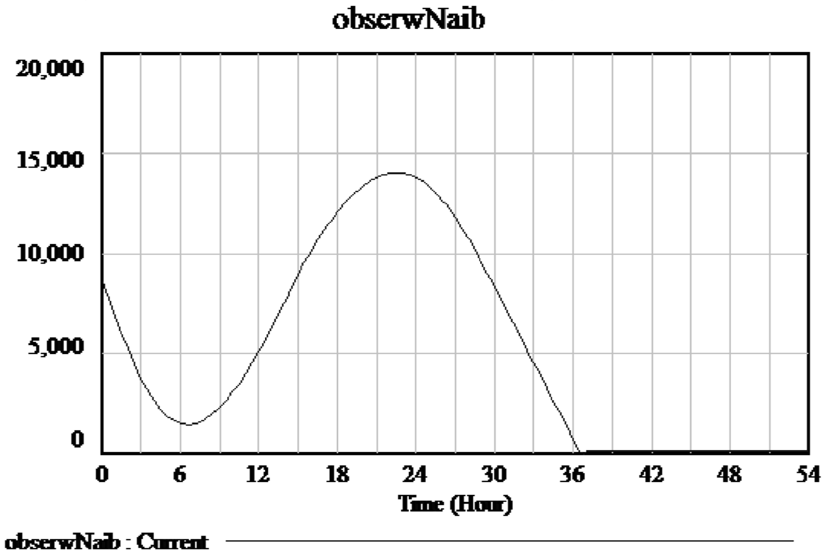

Fig. 5. The dynamics of the variable: Napoleon army in the battle (Napoleon lose the battle)

Source: Own results.

Table 5

End of the Waterloo Battle, values of variable *Naib* (observed)

Table Time Down

Time (Hour)	"obserwNaib"	obserwNaib
32.25	Runs:	5505.52
32.5	Current	5185.23
32.75		4865.35
33		4545.97
33.25		4227.15
33.5		3908.95
33.75		3591.36
34		3274.38
34.25		2957.98
34.5		2642.11
34.75		2326.7
35		2011.65
35.25		1696.86
35.5		1382.21
35.75		1067.56
36		752.758
36.25		437.649
36.5		122.053
36.75		0

Some sensitivity analysis and optimization experiments using simulation by Vensim

The SD models contain usually many parameters. It is interesting to examine the effect on their variation on simulation output. We select some parameters and assign maximum and minimum values along with a random distribution over which to vary them to see their impact on model behavior. Vensim has a method of setting up such sensitivity simulation. Monte Carlo multivariate sensitivity works by sampling a set of numbers from within bounded domains. To perform one multivariate test, the distribution for each parameter specified is sampled, and the resulting values used in a simulation. When the number of simulation is set, for example, at 200, this process will be repeated 200 times. In order to do sensitivity simulation you need to define what kind of probability distribution values for each parameter will be drawn from. The simplest distribution is the Random Uniform Distribution, in which any number between the minimum and maximum values is equally likely to occur. The Random Uniform Distribution is suitable for most sensitivity testing and is selected by default. Another commonly-used distribution is the Normal Distribution (or Bell Curve) in which that values far from the mean. Results of sensitivity testing can be displayed in different formats. Time graphs display behavior of a variable over a period of time. The variables spread of values, at any period in time, are displayed either in terms of confidence bounds, or a separate values which combine to form individual simulation traces.

Now let present the assumptions of the sensitivity experiments:

A) First experiment will be related to the initial values of Coalition army reserves and Napoleon army reserves. They will be chosen (by Monte Carlo method) from the intervals: (25000, 35000) for the *ICar* and (30000, 35000) for *INar*. The results of simulations (200 iterations) are presented in Figures 6, 7, 8.

We can notice in the form of confidence bounds, how the model is sensitive for choice of the initial value of variables: *ICar, INar.*

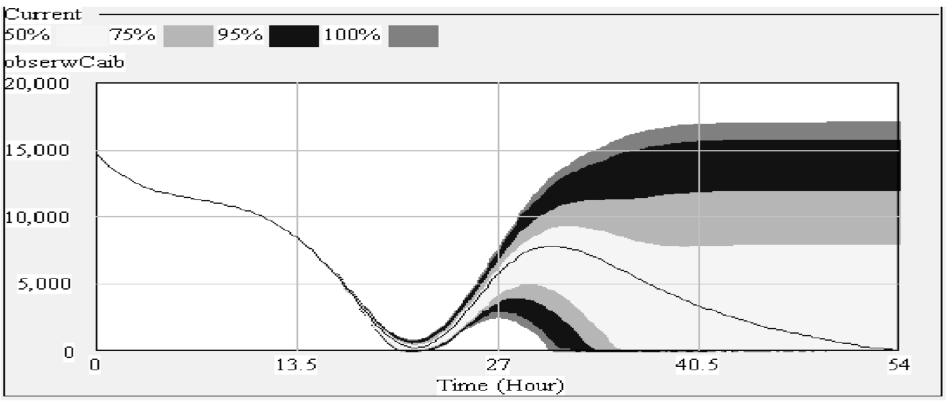

Fig. 6. Confidence bounds for variable *Caib* (at the multi-variants simulation for the parameters: $ICar \in (25000, 35000)$, $INar \in (30000, 35000)$)

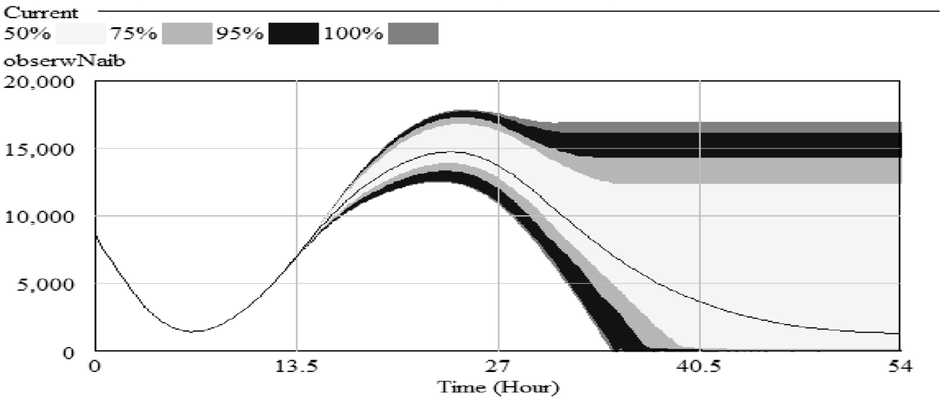

Fig. 7. Confidence bounds for variable *Naib* (at the multi-variants simulation for the parameters: $ICar \in (25000, 35000)$, $INar \in (30000, 35000)$)

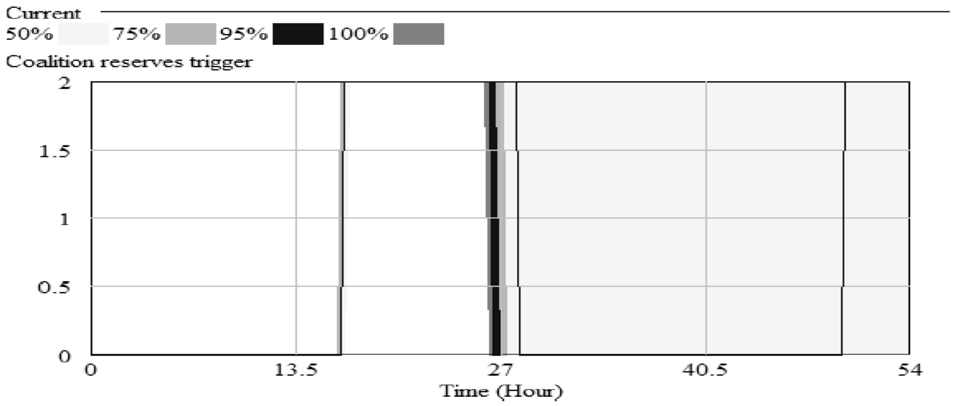

Fig. 8. Confidence bounds for variable Crt (at the multi-variants simulation for the parameters: $ICar \in (25000, 35000)$, $INar \in (30000, 35000)$)

B) Second experiment will be related to the sensitivities of the parameters Ctc and Ntc. They will be chosen (by the Monte Carlo method) from the intervals: (3000, 4000) and (2000, 2500). The results of the simulations are presented in Figures 9, 10 and 11.

C) Third experiment will be related to the sensitivities of the parameters: $Nattd$ and $Cattd$. They will be chosen (by the Monte Carlo method) from the intervals: (4,6) for $Nattd$, (4,6) for $Cattd$. The results of the simulation are presented in Figures 12, 13 and 14. We can see in the form of confidence bounds, how the model in sensitive for the choice of the values for the parameters: $Nattd$, $Cattd$.

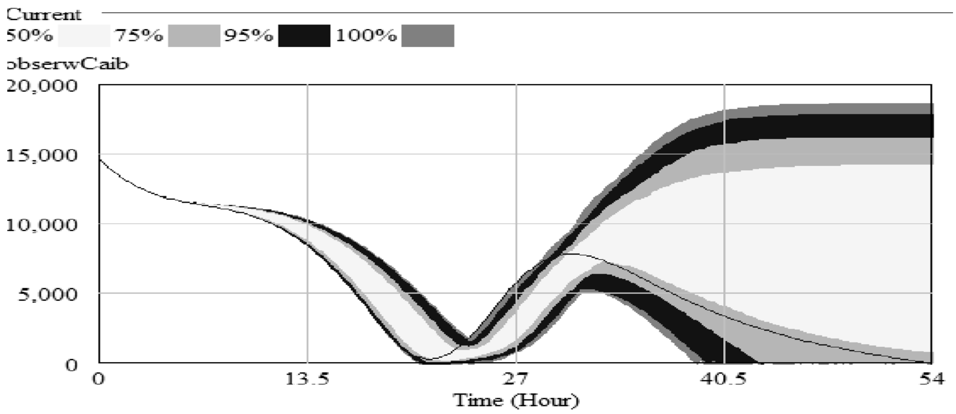

Fig. 9. Confidence bounds for variable $Caib$ (at the multi-variants simulation for the parameters: $Ctc \in (3000, 4000)$, $Ntc \in (2000, 2500)$)

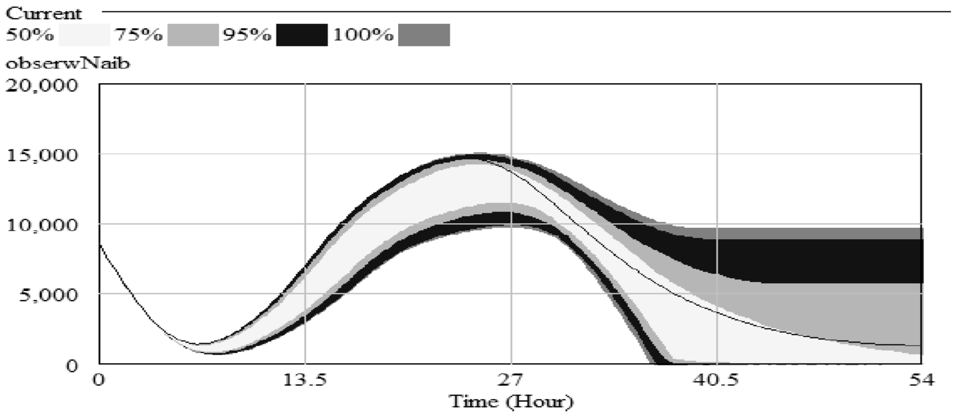

Fig. 10. Confidence bounds for variable $Naib$ (at the multi-variants simulation for the parameters: $Ctc \in (3000, 4000)$, $Ntc \in (2000, 2500)$)

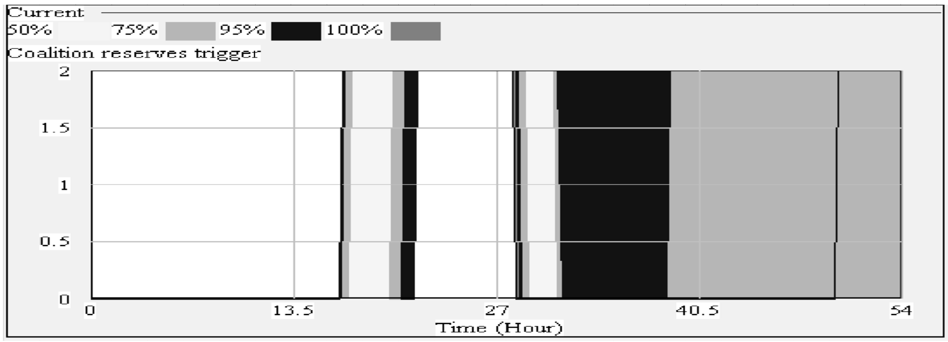

Fig. 11. Confidence bounds for variable Crt (at the multi-variants simulation for the parameters: $Ctc \in (3000, 4000)$, $Ntc \in (2000, 2500)$)

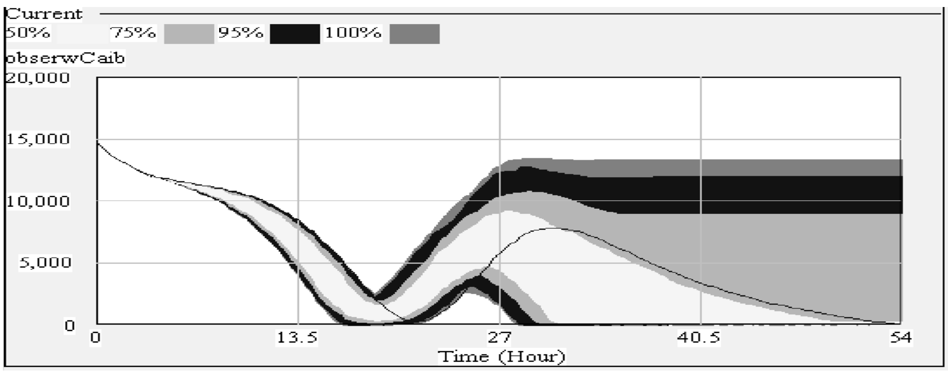

Fig. 12. Confidence bounds for variable $Caib$ (at the multi-variants simulation for the parameters: $Nattd \in (4, 6)$, and $Cattd \in (4, 6)$)

D) Fourth experiment is the most interesting because we decided in the multi-variants simulation investigate the results of the choices:

$$ICar \in (25000, 35000), INar \in (30000, 35000), Ctc \in (3000, 4000), Ntc \in (2000, 2500), Nattd \in (5.5,6), Cattd \in (5.5,6))$$

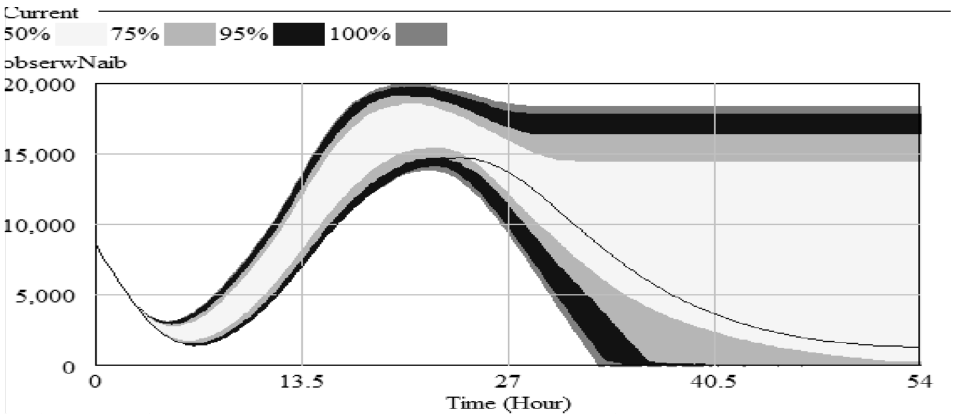

Fig. 13. Confidence bounds for variable *Naib* (at the multi-variants simulation for the parameters: $Nattd \in (4, 6)$, $Cattd \in (4, 6)$)

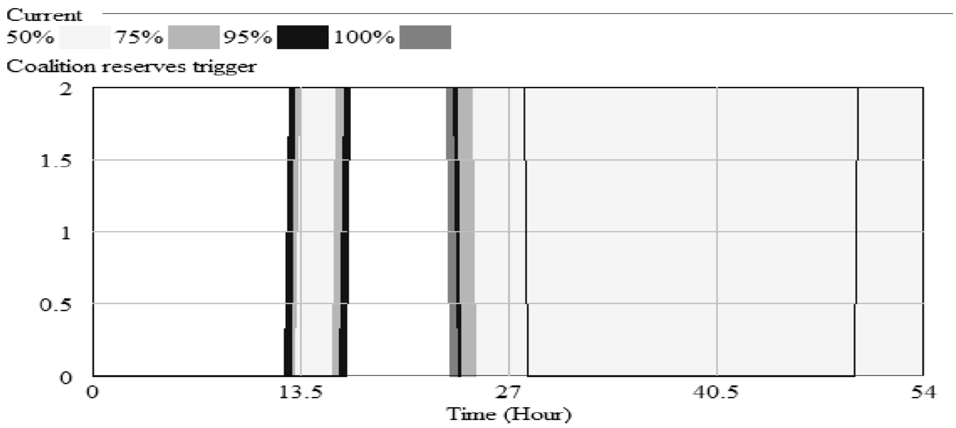

Fig. 14. Confidence bounds for variable *Crt* (at the multi-variants simulation for the parameters: $ICar \in (25000, 35000)$, $INar \in (30000, 35000)$, $Ctc \in (3000, 4000)$, $Ntc \in (2000, 2500)$, $Nattd \in (4,6)$ and $Cattd \in (4,6)$)

In this case the variations are "all together", and we can see, in Figures 15, 16 and 17 how the model in sensitive for such choices. This is the entrance for the optimization experiments.

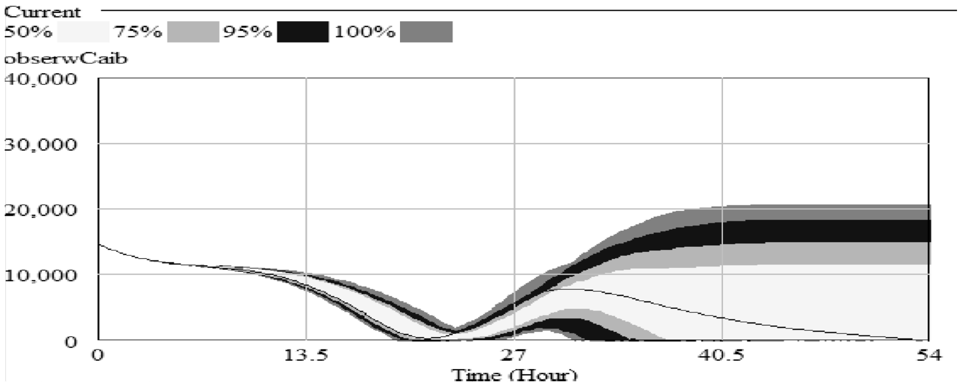

Fig. 15. Confidence bounds for variable *Caib* (at the multi-variants simulation for the parameters: *ICar* ∈ (25000, 35000), *INar* ∈ (30000, 35000), *Ctc* ∈ (3000, 4000), *Ntc* ∈ (2000, 2500), *Nattd* ∈ (5.5,6) and *Cattd* ∈ (5.5,6))

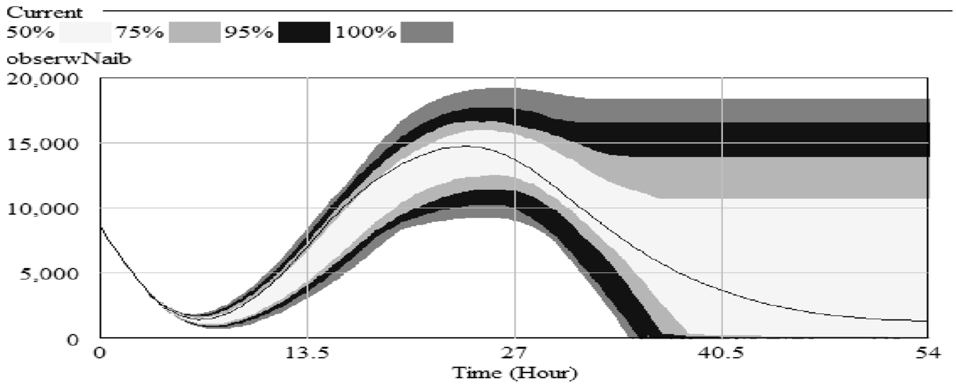

Fig. 16. Confidence bounds for variable *Naib* (at the multi-variants simulation for the parameters: *ICar* ∈ (25000, 35000), *INar* ∈ (30000, 35000), *Ctc* ∈ (3000, 4000), *Ntc* ∈ (2000, 2500), *Nattd* ∈ (5.5,6) and *Cattd* ∈ (5.5,6))

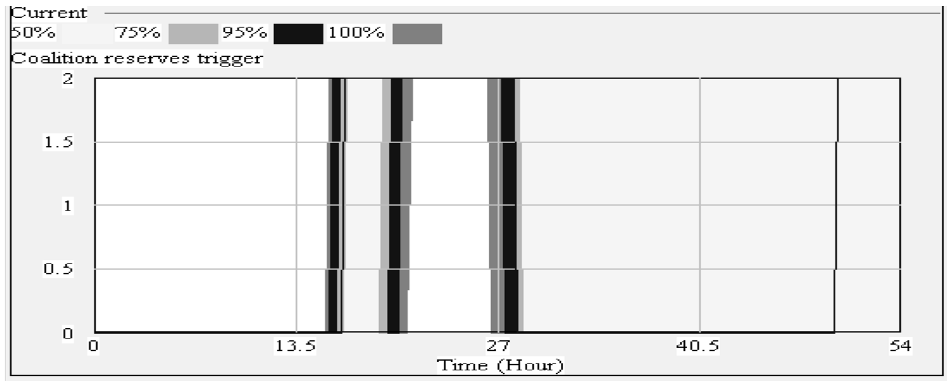

Fig. 17. Confidence bounds for variable *Crt* (at the multi-variants simulation for the parameters: *ICar* ∈ (25000, 35000), *INar* ∈ (30000, 35000), *Ctc* ∈ (3000, 4000), *Ntc* ∈ (2000, 2500), *Nattd* ∈ (5.5,6) and *Cattd* ∈ (5.5,6))

We can do the optimization considering the different point of view, for example:

- Coalition commander wants to achieve the maximum loses of Napoleon soldiers.
- Coalition commander wants to achieve the minimum loses of own soldiers.
- Napoleon wish to maximize the loses of Coalition soldiers.
- Napoleon wish to minimize the own loses.

Possibilities of the language Vensim, allow to perform such experiments, using different so called "payoff" functions and choosing optimized parameters from given intervals. The intervals will be the same like at sensitivity analysis, or sometimes modificated, consider the real world circumstances. The results of experiments type optimization are presents in Table 6.

On the base of results of the optimization, performed in Table 6, we can notice as follows:

- Napoleon reaches the minimum of own killed soldiers as a result of specific partition of the forces on the frontier and in the reserve at the specific partition forces of coalition (see: the experiment number two). On the contrary the maximum of coalition killed soldiers (what is the advantageous from Napoleon point of view) is reached in the experiment number 3.
- At the assumption that only forces in the reserve (both for Napoleon and coalition) can be distributed from the given intervals, we reached the value of minimum Napoleon killed soldiers different than that in the experiment 2, which is performed on the position number 6. Similarly the maximum of coalition killed soldiers is reach in the experiment 7 (different than value in the experiment 3).
- At the assumption that it is possible to change the transport capacity (both for Napoleon's army and army of coalition), Napoleon can win in the experiment 10 and 11, which is in that case quite obvious.
- At the assumption that it is possible to change the transport time delay (both for Napoleon's army and army of coalition), Napoleon reaches the minimum of own killed soldiers in the experiment 14, and the maximum of coalition killed soldiers of forces in the reserves in the experiment 15.
- In most complex version of the experiments, we assume the distribution of the transport capacity and the transport delays (both for Napoleon's army and army of coalition). Napoleon reaches the minimum of own killed soldiers in experiment 18, and the maximum of coalition killed soldiers in the experiment 19.
- The interesting experiments were performed, using the "force ratio" (recall the value of criterion: 2 to 1 for ratio of the forces Napoleon to coalition at the point of combat). The bases simulation performed in chapter 2 of this paper, shown us that Napoleon can win the battle when the "force ratio" has value 2, and lose the battle at the value 1,5 (the method we use then was "trial and error"). Now, after optimization experiments it is reached that results are quite good, both in the experiments with individual variable force ratio as in with the initial values of Napoleon and coalition reserves (see: experiments 21-28).

Table 6

The results of the optimization on the model Waterloo Battle

Number of experiment	Type of optimization	Point of view (whose)	Objective Function / Payoff Function	Intervals for optimized parameters	Optimized value of objective function	Optimal value of parameters
1.	MAX	Coalition Commander	Napoleon killed soldiers Napoleon army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$ $Icuib \in (10000,20000)$ $Inaib \in (6000,11000)$	41385.6	33952.9 35000 18034.5 11000
2.	MIN	Napoleon	Napoleon killed soldiers Napoleon army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$ $Icuib \in (10000,20000)$ $Inaib \in (6000,11000)$	6022.08	25000 35000 10000 10930.7
3.	MAX	Napoleon	Coalition killed soldiers Coalition army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$ $Icaib \in (10000,20000)$ $Inaib \in (6000,11000)$	52078	35000 35000 16720.4 10860.7
4.	MIN	Coalition Commander	Coalition killed soldiers Coalition army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$ $Icaib \in (10000,20000)$ $Inaib \in (6000,11000)$	1267.44	35000 30000 20000 6099.11
5.	MAX	Coalition Commander	Napoleon killed soldiers Napoleon army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$	40084.7	35000 33473
6.	MIN	Napoleon	Napoleon killed soldiers Napoleon army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$	24399.1	25000 35000
7.	MAX	Napoleon	Coalition killed soldiers Coalition army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$	46892.1	31605.7 35000
8.	MIN	Coalition Commander	Coalition killed soldiers Coalition army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$	30313.5	35000 30000
9.	MAX	Coalition Commander	Napoleon killed soldiers Napoleon army loss rate	$Ctc \in (3000,4000)$ $Ntc \in (2000,2500)$	39438.3	4000 2072.66
10.	MIN	Napoleon	Napoleon killed soldiers Napoleon army loss rate	$Ctc \in (3000,4000)$ $Ntc \in (2000,2500)$	28403.8	3000 2500
11.	MAX	Napoleon	Coalition killed soldiers Coalition army loss rate	$Ctc \in (3000,4000)$ $Ntc \in (2000,2500)$	45190.3	3000 2500
12.	MIN	Coalition Commander	Coalition killed soldiers Coalition army loss rate	$Ctc \in (3000,4000)$ $Ntc \in (2000,2500)$	24902.8	4000 2000

Table 6 cont.

Number of experiment	Type of optimization	Point of view (whose)	Objective Function / Payoff Function	Intervals for optimized parameters	Optimized value of objective function	Optimal value of parameters
13.	MAX	Coalition Commander	Napoleon killed soldiers Napoleon army loss rate	$Nattd \in (4,6)$ $Cattd \in (4,6)$	38814.7	5.35145 4
14.	MIN	Napoleon	Napoleon killed soldiers Napoleon army loss rate	$Nattd \in (4,6)$ $Cattd \in (4,6)$	19554.2	4 6
15.	MAX	Napoleon	Coalition killed soldiers Coalition army loss rate	$Nattd \in (4,6)$ $Cattd \in (4,6)$	45553.2	4 6
16	MIN	Coalition Commander	Coalition killed soldiers Coalition army loss rate	$Nattd \in (4,6)$ $Cattd \in (4,6)$	29792.9	6 4
17.	MAX	Coalition Commander	Napoleon killed soldiers Napoleon army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$ $Ctc \in (3000,4000)$ $Ntc \in (2000,2500)$ $Nattd \in (4,6)$ $Cattd \in (4,6)$	42473.6	32869.6 34977.2 4000 2000 5.9622 4
18	MIN	Napoleon	Napoleon killed soldiers Napoleon army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$ $Ctc \in (3000,4000)$ $Ntc \in (2000,2500)$ $Nattd \in (4,6)$ $Cattd \in (4,6)$	18241.1	25000 35000 3000 2500 4 6
19.	MAX	Napoleon	Coalition killed soldiers Coalition army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$ $Ctc \in (3000,4000)$ $Ntc \in (2000,2500)$ $Nattd \in (4,6)$ $Cattd \in (4,6)$	50465	35000 32501.1 3018.39 2500 5.1578 6
20.	MIN	Coalition Commander	Coalition killed soldiers Coalition army loss rate	$Icar \in (25000,35000)$ $Inar \in (30000,35000)$ $Ctc \in (3000,4000)$ $Ntc \in (2000,2500)$ $Nattd \in (4,6)$ $Cattd \in (4,6)$	19166.7	35000 30000 4000 2000 6 4
21.	MIN	Coalition Commander	Coalition killed soldiers Coalition army loss rate	$force\ ratio \in (1.5,2)$ $Icar \in (25000,35000)$ $Inar \in (30000,35000)$	26587.2	1.5 35000 30000

Table 6 cont.

Number of experiment	Type of optimization	Point of view (whose)	Objective Function / Payoff Function	Intervals for optimized parameters	Optimized value of objective function	Optimal value of parameters
22.	MAX	Napoleon	Coalition killed soldiers Coalition army loss rate	$force\ ratio \in (1.5,2)$ $Icar \in (25000,35000)$ $Inar \in (30000,35000)$	50220.1	1.98773 35000 34427.7
23.	MIN	Napoleon	Napoleon killed soldiers Napoleon army loss rate	$force\ ratio \in (1.5,2)$ $Icar \in (25000,35000)$ $Inar \in (30000,35000)$	24399.1	2 25000 35000
24.	MAX	Coalition Commander	Napoleon killed soldiers Napoleon army loss rate	$force\ ratio \in (1.5,2)$ $Icar \in (25000,35000)$ $Inar \in (30000,35000)$	41512.1	1.5 34809.5 34889.8
25.	MIN	Coalition Commander	Coalition killed soldiers Coalition army loss rate	$force\ ratio \in (1.5,2)$	34240	1.51917
26.	MAX	Napoleon	Coalition killed soldiers Coalition army loss rate	$force\ ratio \in (1.5,2)$	45018.8	1.9874
27.	MIN	Napoleon	Napoleon killed soldiers Napoleon army loss rat	$force\ ratio \in (1.5,2)$	37199.9	1.9874
28.	MAX	Coalition Commander	Napoleon killed soldiers Napoleon army loss rat	$force\ ratio \in (1.5,2)$	38742.5	1.50945

So in many circumstances Napoleon can win the Waterloo Battle. But the question is did that circumstances were possible and by what cost?

The next question is that we used the specific objective function, and by the different criterion, for example the survivors on the battlefield, the results could be quite different. The dimension of paper doesn't allow to develop that aspect of the research. Moreover it will be interesting to use more complex multivalued objective functions and functions with the cost factors.

Final conclusions

The aim of this paper was the presentation of some new results authors' investigation in the area of simulation and optimization with use the model of Waterloo Battle. We applied simulation language Vensim which is very effective tool for the analysis, modelling and simulation on the models System Dynamics type. Many experiments we performed can answer the theoretical question: did Napoleon have to lose the Waterloo Battle?

Interesting visualization by so called confidence bounds help to estimate the sensitivities of parameters, which are used in the optimization experiments. It should be stress that possibilities of the experimentation are practically unlimited. We can use many of the objective functions, which the cost elements and complex form (with weight factors and scaling). The dimension of that paper not allowed to develop many aspect of the research. We hope that the object of the experiments help to cause the concern with the subject of sensitivity analysis and optimization, what is very interesting from the methodical point of view.

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"CZY NAPOLEON MUSIAŁ PRZEGRAĆ BITWĘ POD WATERLOO?" – ANALIZA WRAŻLIWOŚCI I OPTYMALIZACJA Z UŻYCIEM VENSIMA

Streszczenie

Problem analizy wrażliwości i optymalizacji nieliniowych, dynamicznych i wielopoziomowych systemów jest bardzo interesujący zarówno z punktu widzenia metodologii, jak i zastosowań praktycznych. Celem tego artykułu jest zaprezentowanie nowych wyników autorów w dziedzinie symulacji i optymalizacji z użyciem modelu typu SD (Dynamiki Systemowej). Modelowany system jest interesujący i prezentowany w prostszej wersji jako „Bitwa pod Waterloo" autorstwa profesora Coyla. Wykorzystywane przez autorów oprogramowanie Vensim umożliwia analizę wrażliwości (metodą Monte Carlo) i optymalizację z różnymi typami funkcji celu. Z kolei graficzna wizualizacja wyników zwana „confidence bounds" jest pomocna przy szacowaniu „wrażliwych parametrów" modelu i w ten sposób przy odpowiedzi na pytanie zawarte w tytule artykułu „Czy Napoleon musiał przegrać bitwę pod Waterloo?".