Elementa Geometriae.

a
Professore Matheseos
Habichtio
elaborata.

C. R. Bibliothecae Scherschnikianae
Teschinii

Q. D. B. V.

Caput I^{mum}

De primis Triangulorum et Parallelogrāmorum adfectionibus.

§1. Definitio 1.

Geometria eſt ſcientia extenſorum quatenus terminata ſunt, h.e. Linearum, ſuperficierum et ſolidorum.

§2 Scholion.

Quia extenſio ex ſimultanea rei cujuſdam per locum diffuſione oritur, in mente nobis repræſentamus eam multa in uno continuo ſimul percipientes, inde quidem extenſionis notio, totius et partium notiones involvit, atq; eadem ipſa in aliarum rerum notiones irrepit, quæ ideo per lineas ſuperficies et ſolida repræſentari poſſunt. Inde patet Geometriæ uſum eſſe utique latiſſimum. Dicemus autem continuum si in composito partes eo ordine juxta se invicem collocentur ut alias inter ipsas alio ordine interponi posse prorsus sit impossibile.

§3. Definitio. II.

Terminus est, quod alicujus extremum est, seu ubi id, quod hactenus ponebatur, desinit et cessat, vel incipit, quod antea non erat.

§4. Definitio III

Congruere dicuntur, quorum iidem esse possunt Termini et Congruentia est Coincidentia Terminorum.

§5 Scholion

Quae dici poterant de Congruentia omnia vid: in Isaac. Barrow: Lectionum Mathemat. 3tia Antiblos.

§6 Definitio IV.

Punctum est, quod quaquaversum seipsum terminat seu quod non habet Terminos alios a se distinctos, ſeu cum Euclide: Punctum eſt, cujus pars nulla eſt.

§7 Corollarium

Ergo punctum omne alteri cuicunque congruit.

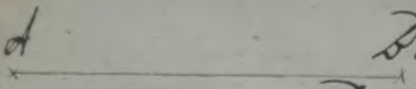
§8 Scholion 1.

Punctum Geometricum nec pingere poſsumus, nec imaginationi exhibere, ſed ſola mentis abſtractione concipitur. Opus tamen eſt hac definitione ne ſcilicet in Geometrica praxi punctum pars lineae exiſtat id quod ſummo ſtudio cavendum, uti quidem patebit inferius.

§9 Scholion 2.

De Definitionibus Puncti Lineae ſuperficiei atque ſolidi Geometricis noſtris, non noſtro demum, ſed jam antiquioribus ſeculis diſputatum eſt, agmen ducente Sexto Empirico circa ſeculi III. initium celebri, in notiſsimo, quod contra Mathematicos et Scepticos inſcripſit, opere. Calculos ipſi ſuos adjicientibus Hen. Cor. Agrippa de Vanit. Scient. c. 11 ſqq Fr. Verulamio de Dignit. et Augm. Scientiarum L. III. c. 6. Joh. Clerico in Logica P. III. c. 12 et Auctore Artis cogitandi P. IV. c. 4 ſqq Joh. Chr. Meingio in diſs. de Non-Ente Mathematicorum Puncto, ſuperficie et corpore Lipſ. 1710. Chr. Thomaſio in Cautelis circa Praecognita Juris prudentiae Cap XI. Enimvero, quemadmodum Sexti dubiis praeter Carolum Renaldinum in Arte Mathematum analytica, paulo infelicius refutationis onus ſuſcipientem Guil. Langius de Veritatibus Geometricis tractatu rariſſimo Hafn. 1656. 4 abunde ſatis fecit, ita hoc aevo idem ſaxum

Joh. Friedr. Weidlerus in Diss. Vindiciæ Mathematum contra quorundam Philosophorum maxime H. C. Agrippæ F. Verulamii Joh. Clerici et Auct. Artis cogitandi objectiones Witteb. 1713 et in Programmate inscripto Mathemata adversus celeberrimi D. C. Thomasii Objectiones vindicata ibid. 1715. Joh. Petr. Reuschius in Vindiciis certitudinis Mathematum adversus Illustr. Thomasii Cautelas Jen. 1718. 4. Joh. Mathias Hasius in Dissert. de Philo Mathematico Witteb. 1727 studio et accuratione maximis volventes, quæ objici poterant omnia felicissime sustulerunt. Add. Krügers Anmerkungen über des Herrn von Wolfs Geometrie. §§ et Bellnagels ausführliche Erläuterung über des Herrn von Wolfs Arithmetik Geometrie und Trigonometrie.



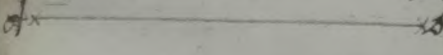
§10. Definitio V.

Linea deſcribitur, ſi Punctum A ad alterum B movetur.

§11 Corollarium.

Quia punctum nullas partes habet §6. linea nec lata eſſe poterit nec profunda in ſolam tantum longitudinem exporrecta ſ. extenſa.

§12. Definitio VI.



Linea recta A B eſt cujus pars quecunque toti ſimilis.

§13. Scholion.

Linea motu ſ. fluxione puncti unius ad alterum deſcribitur §10. quia itaq; pars quæcunq; rectæ lineæ ſimilis dicitur toti facile apparet motum puncti deſcribentis in omnibus lineæ partibus eundem eſſe debere, ſecus enim ex motus diverſitate agnoſcerentur i.g.c.h.s. definit. Quia vero motus differe nequit, niſi vel celeritate, vel directione celeritas autem ad deſcriptionem nihil con fert ſola directionis ratio habenda; hinc linea recta deſcribitur, ſi punctum A verſus alterum D eadem Directione fluat ſ. moveatur.

§14 Definitio VII.

Linea curva eſt, cujus partes diſſimiles Toti

§15 Scholion.

Notanda ſunt merito ad hanc definitionem § antec: monita. Quodſi vero utraque Definitio ob ſimilitudinis notionem obſcurior forte videatur Euclideam adferre lubet hujus tenoris: Recta Linea eſt, quæ ex æquo ſua interjacet puncta, h.e. in qua nullum punctum intermedium ab extremis ſurſum vel deorſum huc atque illac deflectendo ſubſultat in qua denique nihil flexuoſum reperitur. Repræſentat rectam ejusmodi optime filum ſuma vi extentum, in eo enim omnes mediæ partes cum extremis æqualem ſitum obtinent cf Clavius in Coment Eucl. p.m. 8. Hinc quid Curva ſit, facile colligitur. Eadem ſignificatione Plato, Rectam definit eſse illam cujus extrema obumbrant omnia intermedia, et Archimedes, eſse omnium linearum eosdem terminos habentium minimam ſ. breviſsimam quæ inter duo puncta duci poſsit.

§16. Definitio VIII.

Metiri idem eſt, ac Quantitatis pro Unitate aſsumta rationem ad aliam exprimere, indeque menſura dicitur Quantitas Unitatis loco aſsumta.

§17 Definitio IX.

Superficies eſt magnitudo duabus dimenſionibus prædita ſ. in Longitudinem et Latitudinem extenſa.

§18. Definitio X.

Superficies plana s. Planum est, quod ex æquo suas interjacet lineas vel si e quovis Perimetri puncto ad quodvis ejusdem rectam in eadem ducere licet.

§19 Definitio XI.

Est autem Perimeter continuum quo aliud continuum terminatur.

§20 Definitio XII.

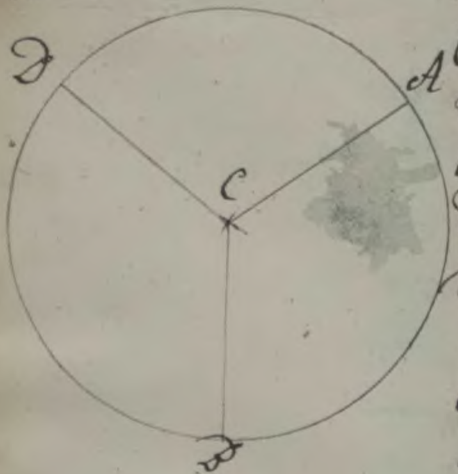
Figura est continuum Perimetro terminatum.

§21. Definitio XIII.

Figura rectilinea est, cujus Perimeter ex Lineis rectis. Curvilinea cujus Perimeter ex curvis; Mixtilinea cujus Perimeter ex Lineis partim rectis partim curvis constat.

§22. Definitio XIV.

Latus est linea, quæ est pars Perimetri, figuræ superficialis. ~~[illegible]~~

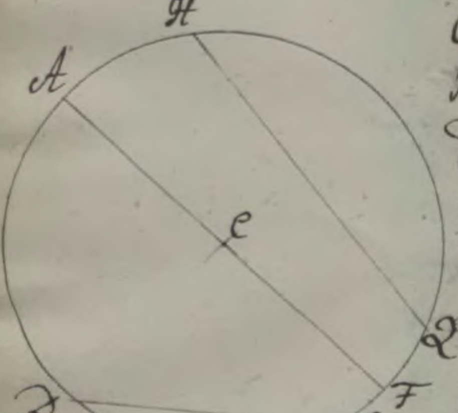


§.23. Definitio XV.

Circulus est figura plana, linea in se
redeunte terminata, ex cujus sing
lis punctis ductae rectae AC, DC, B
ad punctum intermedium C, Centrum sunt inter se
aequales
lineam in semet redeuntem Peripe
riam Vel
Circulus describitur ex Motu rectae
circa punctum fixum C.

§24 Definitio XVI.

Chorda s. subtensa AF, DG, HQ. est
Recta a Peripheria ad Peripheriam
ducta.



§25 Definitio XVII.

Diameter AF est Chorda per cen-
trum transiens, ejus dimidium
AC, CF, vel recta ex Centro ad Peri-
pheriam ducta, dicitur Radius

§26. Corollarium.

Hinc ejusdem vel aequalium Circu-
lorum Radii sunt inter se aequa
les. Dicimus autem Circulos

æquales, radiis æqualibus descriptos.
peri. Definitio XVIII.
Arcus et AD, DF, FA, est pars quanta-
libet Peripheriæ.
Gradus autem pars 360ma Peripheriæ. Mi-
nutum primum pars 60ma Gradus
Minutum secundum pars 60ma Minu-
ti primi et ita deinceps.

§28 Scholion

Facile apparet, cum cujusvis Cir-
culi Peripheria in 360 Gradus habe-
t, Gradus Circuli majoris majo-
res fore, Gradibus Circuli mino-
ris; Dicuntur autem Circuli ma-
jores minoresque radiis majoribus
itque minoribus descripti.

§29. Definitio XIX.

Linea AB secat aliam CD in E si
eam dirimat in partes cis et
ultra sitas

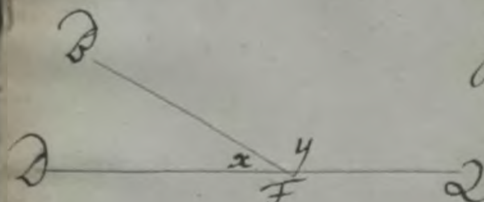
§30. Definitio XX.

Angulum dicimus duarum re=

ctarum et AB et BC in puncto B co
currentium mutuam inclination
Lineæ AB, BC, crura: B autem h.
Concursus Punctum Vertex Angu
audit.

§31 Hypothesis 1.

Angulus hic, vel una tantum liter
Vertici adscripta, vel minuscula
eidem inscripta, vel si plures An-
guli fuerint in eodem vertice conc-
currentes, evitandæ confusioni
causa tribus literis significat
ea tamen lege, ut Vertici adscrip-
ta medio semper loco ponatur. Sic
ut Angulus x in quæstionem venit
F. medium locum occupabit scri-
bendo: DFB vel BFD si Angulus y,
BFQ vel QFB.

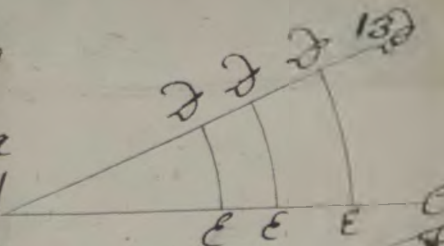


§32 Hypothesis 2.

Aliquando Anguli signum erit:
∠ v.c. de angulo x demonstraturi
quidpiam scribemus: ∠x aut ∠DFB.

§33 Definitio XXI.

Mensura Anguli DAE est Arcus DE, Radio prorsus arbitrario AE intra ura illius AE, et AD descriptus.

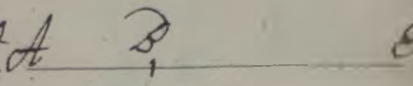


§34 Definitio XXII.

nguli contigui sunt FGH. et HGI quorum idem est Vertex G et unum rus G.H. commune.

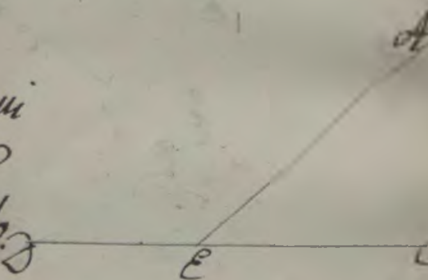
§35. Definitio XXIII.

Recta Linea AD, DE in directum sita dicuntur si eiusdem rectae AE partes xistant.



§36. Definitio XXIV.

ngulus deinceps positus AEC, est qui fetur Li DEA crure uno ED in C roducto s. g. i. e. in directum posito.

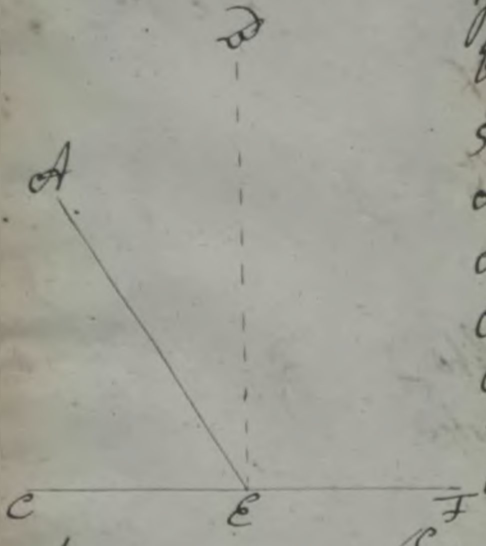


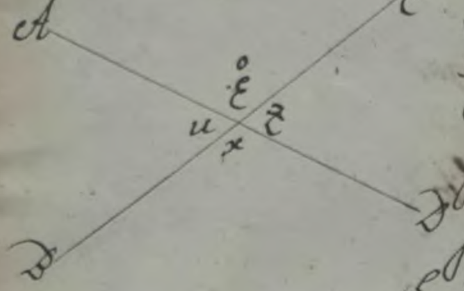
§37 Corollarium

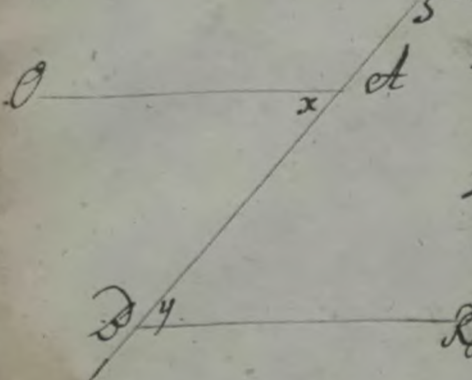
nguli deinceps positi sunt quidem ntigui §34 sed non contra.

§38. Definitio XXV.

ngulus rectus KLM. est cui Angulus deinceps positus KLN. est aequalis.







§.39. Hypotheſis 3.

Brevitatis cauſa in Demonſtra
nibus Anguli recti ſignum merito
ſpide quidem Angulum KLM ſo
bemus: $\angle KLM = R$.

§40 Definitio XXVI.

Angulus obliquus AEC et AEF. eſt
cui deinceps poſitus eſt inæqualis
Et in ſpecie:
Angulus acutus AEC. obliquus mi
nor Recto DEC.
Angulus obtuſus AEF obliquus ma
jor Recto DEF. dicitur.

§41 Definitio XXVII.

Anguli verticales o et x item u et z
ſunt, ſi crura unius in directum
ſita ſunt in directum, cruribus al
rius np. crura $\angle$li o AE, et EC crura
$\angle$li x np. DE, BE.

§42 Definitio XXVIII.

Si Lineæ S.T duæ aliæ OA et RB
a diverſis plagis O et R et in div
ſis punctis A et B occurrant, Angu

quos cum ea efficiunt x et y, dicuntur alterni.

§ 43 Definitio XXIX.

Si vero Linea ST duo alias AB et DR itidem in diversis quidem punctis A et D, sed ab eadem plaga B et R occurrant; Anguli, quos cum ea efficiunt z et y dicuntur oppositi; et in specie dicitur u oppositus externus, z vero oppositus internus ipsius y.

§ 44. Definitio XXX.

Linea KL dicitur normalis aut perpendicularis ad alteram si cum ea efficiat Angulum rectum.

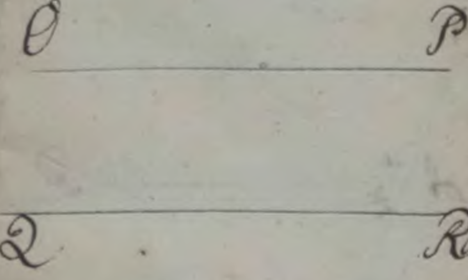
§ 45 Corollarium

Quodsi igitur KL ad ML fuerit normalis anguli ad L deinceps positi sunt aequales § 38.

§ 46 Hypothesis 4.

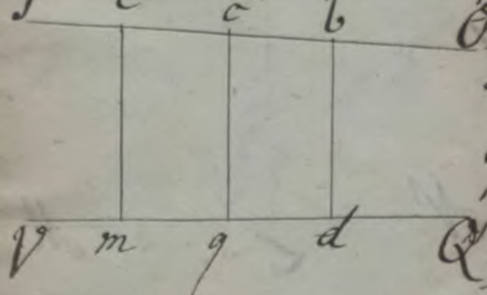
Normalem unam ad alteram significabimus: ⊥ vel ⅃ : v. c. LK ad LM normalem scribemus LK ⊥ ad LM.

§47 Definitio XXXI.
Distantia est linea brevissima inte
duo h.e. data puncta.



§48 Definitio XXXII.
Linea OP parallela dicitur alter
QR, si eandem ubiq; ad alteram d
stantiam servet.

§49 Hypothesis 5
Parallelismum rectarum signi
cabimus h.m. = &c. si OP parallel
fuerit ipsi QR. scribemus: OP = Q



§50 Definitio XXXIII.
Lineae convergentes TO et VQ s
quarum distantiae lm, cq, bd conti
fiunt minores.

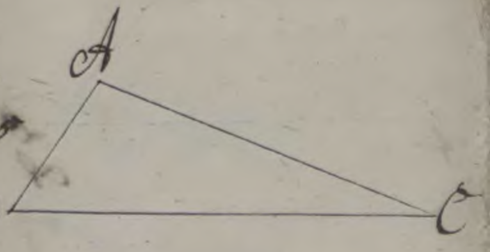
§51 Definitio XXXIV.
Lineae autem divergentes O
QV. sunt, quarum distantiae b
cq, lm, continuo fiunt majores.

§52. Scholion
Quae §§ antecedentibus de signis
allata fuerunt ex Th. Backeri cla
ve Geometrica Catholica Lond

b84. 4. Lat, et arg. s. edita maximam partem desumta sunt.

§53 Definitio XXXV.

Triangulum ABC est figura tribus lineis terminata.

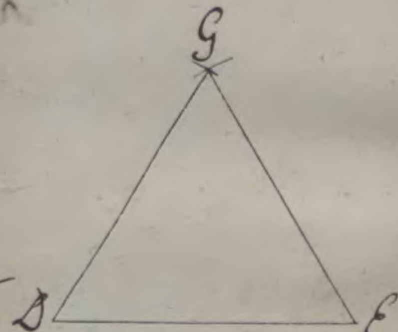


§54 Scholion.

Quoniam in Geometria elementari versamur, quæ nullam curvarum excepto circulo admittet, facile colligitur §53. Triangulum rectilineum intelligi, id quod et in reliquis tenendum, ubi figurarum Geometriæ elementaris definitiones tradentur.

§55 Definitio XXXVI.

Triangulum æquilaterum GDE est cujus singula latera æqualia.

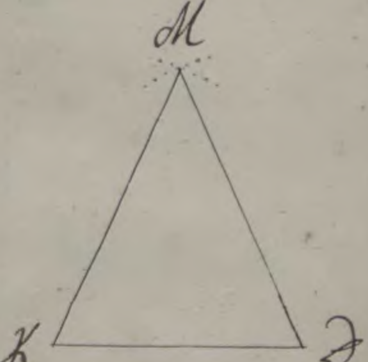


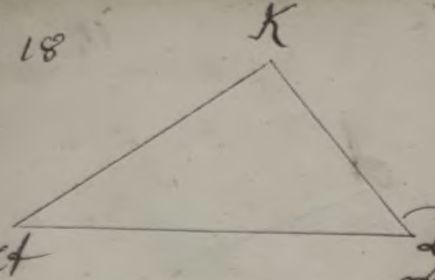
§56 Definitio XXXVII.

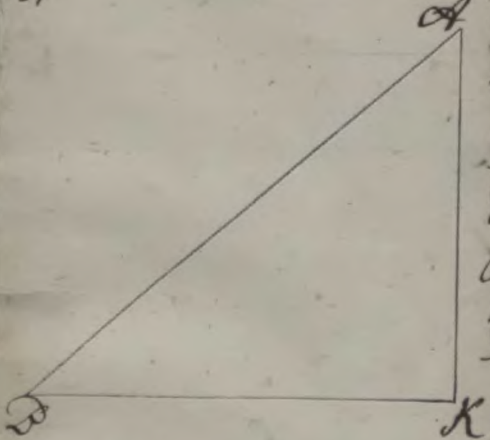
In genere figuram æquilateram dicimus, cujus omnia latera sunt inter se æqualia.

§57 Definitio XXXVIII.

Triangulum æquicrurum vel Isosceles est MDK, cujus duo latera MD et MK sunt æqualia.







§58 Definitio XXXIX.

Triangulum Scalenum est cujus
singula Latera sunt inaequalia.

§59 Definitio XL.

Triangulum Rectangulum KAD
est cujus ∠lus K Rectus est.

§60 Definitio XLI.

Hypothenusa AD est Latus ∠li Rec
oppositum.

§61 Definitio XLII.

Cathetus autem AK aut KD latus
cum latere KD aut AK rectum effi
ciens ∠lum dicitur.

§62 Definitio XLIII.

Reliquum tandem Latus KD aut
basis est.

§63 Definitio XLIV.

Est autem basis in genere pars
ima perimetri figurae cujuslib

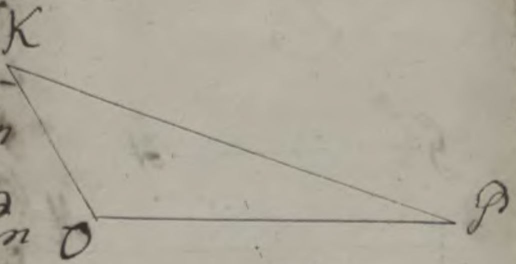
§64 Scholion.

Scilicet basis cum sit in genere
pars ima perimetri figurae §63
situs autem figurae ipsi non sit
essentialis, Cathetus et basis Tri

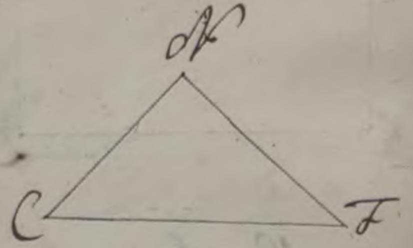
nguli rectanguli relative dicuntur.

§ 65 Definitio XLV.

Triangulum obliquiangulum est cujus singuli anguli sunt obliqui in specie autem.



Triangulum obtusangulum KOP in quo unus ∠lus O obtusus. Triangulum acutangulum NCF cujus singuli anguli sunt acuti.



§ 66 Scholion.

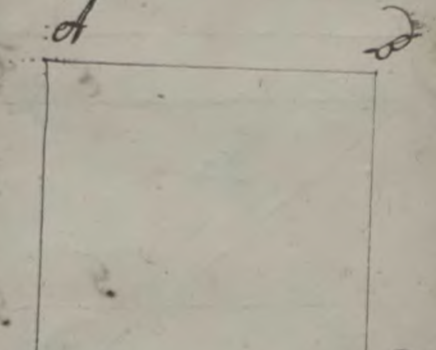
Cum in Triangulis et Laterum et Angulorum habenda sit Ratio definienda quoque erant α) respectu Laterum § 55. 57. 58. β) respectu Angulorum § 59. 65.

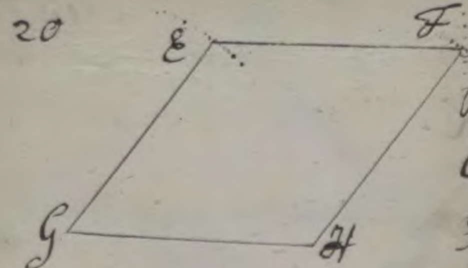
§ 67 Definitio XLVI.

Figura quadrilatera est, cujus Perimeter quatuor Lateribus absolvitur, et in specie rectangula dicitur, si singuli ∠li recti, obliquangula si singuli ∠li obliqui fuerint.

§ 68 Definitio XLVII.

Quadratum ABCD est Figura quadrilatera, æquilatera et rectangula.

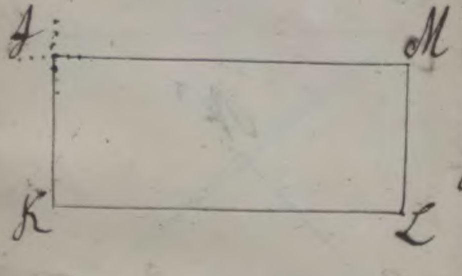




§ 69 Definitio XLVIII.

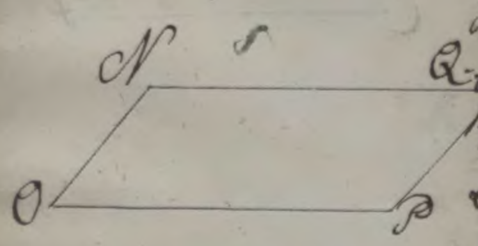
Rhombus EFGH est figura quadri latera æquilatera, obliquangu

§ 70 Definitio XLIX.



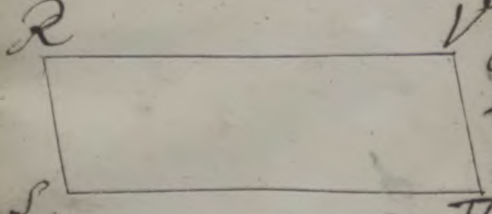
Rectangulum s. oblongum IKLM est figura quadrilatera, rectangula latera opposita IM et KL, ut et IK et LM æqualia habens.

§ 71 Definitio L.

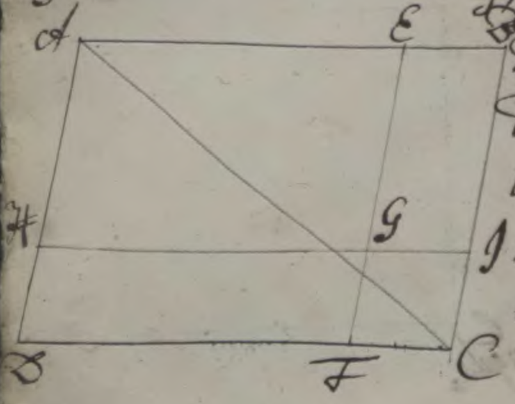


Rhomboides NOPQ est figura quadrilatera obliquangula latera opposita NO et PQ itemq. OP et QN habens æqualia.

§ 72 Definitio LI.



Parallelogrammum RSTV est figura, quadrilatera opposita habens parallela.

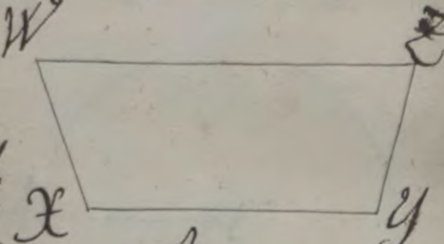


Cumq. in parallelogramo ABCD Diameter s. q. h. l. i.e. Diagonali ducta fuerit, duoq. lineæ EF et HI lateribus parallelæ secantes Diametrum in eodem puncto G ita ut Parallelogrammum ABCD

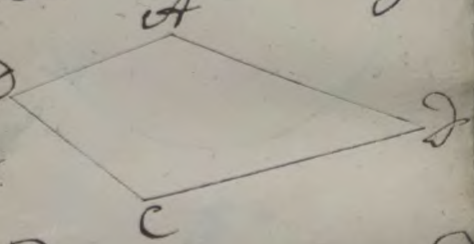
istas parallelas in quatuor distribu-
antur parallelogramma, appellantur
duo illa DG et GB per quae Diameter
non transit complementa, duo vero
reliqua HE et FI per quae Diameter
incedit circa Diametrum consiste-
re dicuntur.

§ 73. Definitio LII.

Trapezium WXYZ est figura qua-
drilatera cujus duo tantum late-
ra opposita sunt parallela WZ et XY.
Dicitur etiam Trapezium paralle-
larum basium.

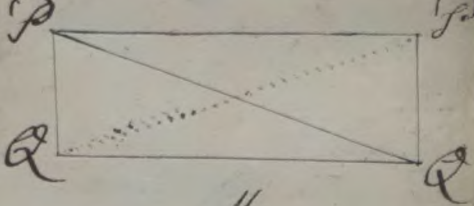


Trapezoides autem ABCD est figu-
ra quadrilatera non parallelogramma



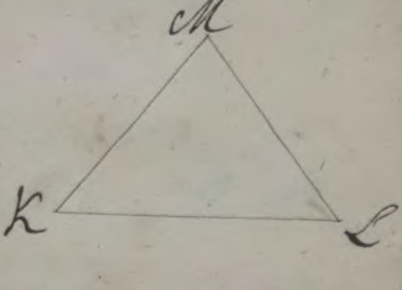
§ 74 Definitio LIII.

Diagonalis PQ est recta ex vertice
Anguli unius P in verticem alterius
Anguli Q ducta.



§ 75 Definitio LIV.

Vertex figurae M est vertex An-
guli Basi KL oppositus.



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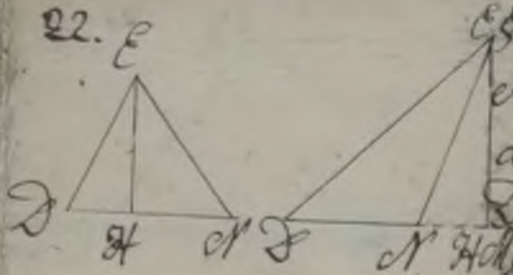


§76. Definitio LV.

Altitudo figurae EH est Distantia a Vertice E ad Basin vel ipsam DN vel Continuatam DM.

§77 Definitio LVI.

Figura aequiangula est cujus singuli ∠li sunt inter se aequale

§78 Definitio LVII.

Eodem modo determinari dicuntur si data per quae unum determinatur, fuerint similia datis per quae determinatur et alterum sibi utrobique ex datis similibus per easdem regulas reliqua determinantur.

§79 Corollarium.

Quae eodem modo determinantur in iis coincidunt ea, per quo discerni debent, adeoque characteres iidem sunt ergo sunt similia §80tr

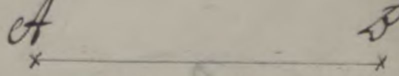
§80 Axioma.

Omnes Lineae rectae se tantum in unico puncto intersecare possunt Nec gaudent lineae segmento alio commu:

quam punctuali. Idem et de Peripheriis valet.

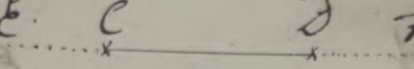
§ 81. Postulatum 1.

A dato puncto A ad alterum B duci possit recta linea AB.



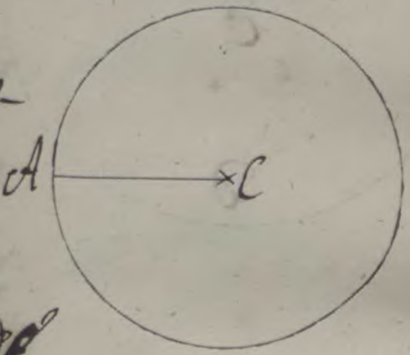
§ 82. Postulatum 2.

Linea recta terminata CD utrinque in E et F produci possit.



§ 83 Postulatum 3.

Dato quovis centro C et radio quovis CA circulum describere liceat, adeoque et Circuli arcum.

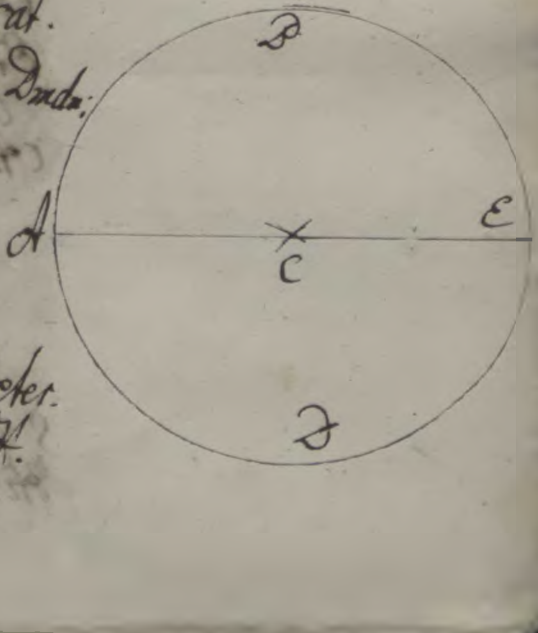


§ 84. Theorema 1.

Diameter AE et Circulum ABEDA et Peripheriam ABEDA bifariam secat. h.e. in duas partes aequales. h.e. Dmdr: si AE fuerit Diameter p. 74. fore 1) ABECA = ADECA 2) ABE = ADE.

Demonstratio

Mbr. 1. Pars circuli ABECA determinatur Diametro AE p. 74.



Pars circuli ADECA determina
eadem Diametro AE p. H.
Ergo Partes ADECA et ADECA eode
modo determinantur §78.
Ergo ADECA ∼ ADECA §79.

Ergo
ADECA : ADECA = ADEA :
ADEDC §148 Ar.

Ergo
ADECA = ADECA §151 a 152 Ar.
Q.E. I.

Mbr II.
Pars Pphiae ADE determinatur
Diametro AE p. H.
Pars Pphiae ADE determinatur
eadem Diametro AE p. H.
Ergo ADE ∼ ADE. §78. 79.

Ergo
ADE : ADE = ADECDA : ADED
§148 Ar

Ergo
ADE = ADE §150. 152 A
Q.E. II

§ 85 Corollarium.
Hinc patet ſuper quavis recta

deſcribi poſſe ſemi circulum.

§ 86 Theorema 2.

Quæ ſibi mutuo congruunt ea 1) æqualia et 2 ſimilia ſunt.

Demonſtratio.

Mbr. 1.

Quæ ſibi mutuo congruunt, eorum iidem termini eſſe poſſunt § 4. Termini autem iidem ſalva quantitate ſubſtitui poſſunt § 3 Arithm. Quæ vero ſalva quantitate ſubſtitui poſſunt æqualia ſunt § 10. Ar. Ergo Quæ ſibi mutuo congruunt, æqualia ſunt. Q.E.I.

Mbr. 2.

Quæ ſibi mutuo congruunt eorum termini æquales eſſe poſſunt § 4. Enimvero in quantitatibus continuis termini ſunt ea, per quæ a ſe invicem diſcerni debent, h.e. ſunt continuarum quantitatum characteres. Ergo, quæ ſibi mutuo congruunt, iis ea eadem ſunt per quæ a ſe invicem diſcerni debent.

In quibus autem ea eadem ſunt per quæ a ſe invicem diſcerni debent ea ſunt ſimilia. § 8 etr. Ergo quæ ſibi mutuo congruunt, ſimilia ſunt. Q.E.D.

§ 87 Corollarium.

Quod si ergo lineæ rectæ et ∠li congruant æquales ſunt. § 86.

§ 88 Theorema 3.

Quæ æqualia et ſimilia ſunt, ea ſibi mutuo congruunt.

Demonstratio

Quia ſimilia ſola quantitate differunt, ergo ſi ſimilia fuerint æqualia p. 11. prorſus amplius non differunt. Quæ autem prorſus non differunt, iisdem terminis ut comprehendantur opus eſt. Sed quæ iisdem terminis comprehenduntur congruunt § 4. Ergo quæ æqualia et ſimilia ſunt congruunt. Q.E.D.

§ 89 Corollarium 1.

Lineæ rectæ et ∠li ſi fuerint æquales congruunt § 88.

§90 Corollarium 2.

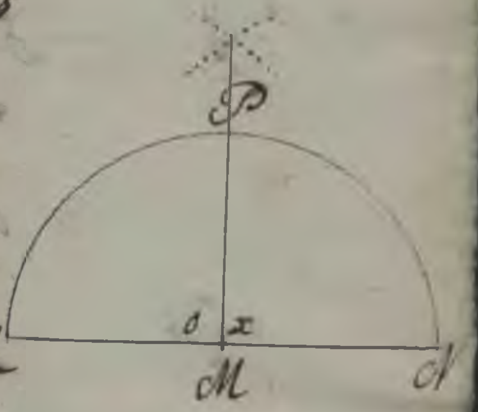
Ergo et inter duo puncta non nisi unica recta cadit, semper enim eadem recta prodit. §81.

§91 Theorema 4

Anguli Recti Mensura est Quadrans Circuli h. e.

Sit $\angle$lus o = R p H

Dico $\angle o = \frac{\text{Circulo}}{4} = \frac{LPNL}{2}$



Demonstratio

Produc LM §82 in N. Si radio quovis LM describas Semicirculum §85. 83 erit

LN = Diametro §25

et LPNL = $\frac{1}{2}$ Circulo §84 Ergo.

Mens. $\angle\angle$ o+x = $\frac{1}{2}$ Circulo §33.

Sed quia $\angle$o = R. p. H

erit $\angle o = \angle x$ §38

Mens. $\angle\angle$ o+x = Mens. $\angle\angle$ o+o §10 etc.

= Mens. 2×$\angle$o.

Mens. 2×$\angle$o = $\frac{1}{2}$ Circ. §41

Mens $\angle$ o = $\frac{1}{2}$ Circ. : 2. §43 et 38 etc.

☆ ☆

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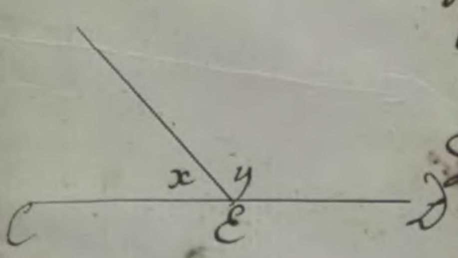
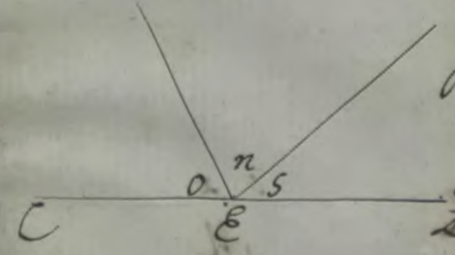
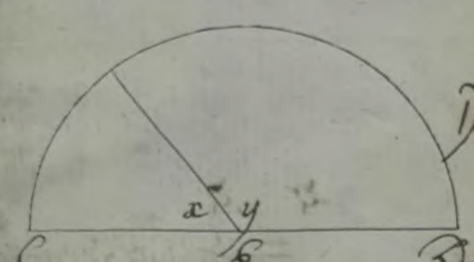
= $\frac{1}{4}$ Circ. §220 etc.

= $\frac{LPNL}{2}$

Q. E. D.







§92. Corollarium

Omnes ergo ∠li Recti sunt inter se aequales, et ∠lus rectilineus aequalis Recto est Rectus. Clavius ad XII. 1 Com. Euclid.

§93 Theorema 5.

Duo Anguli qui sunt deinceps x et y vel quotcunque alii super recta eadem constituti ad eundem punctum E sunt aequales duobus Rectis. Et contra: si $x+y$ sunt aequales duobus Rectis erit CE in directum sita ipsi ED.

Demonstratio:

Mbr 1. Quod ostendendum

1) $x+y = 2R$.

2) $o+n+s = 2R$.

1) Quia $x+y$ sunt ∠∠ d.p. ptt

Ergo CE et ED in directum sita sunt §30. 35.

Descripto itaque semicirculo ex centro E. §85.

Erit Mens. ∠∠ $x+y = \frac{1}{2}$ Circ. §33

sed $\frac{1}{2}$ Circ. $= 2R$. §91

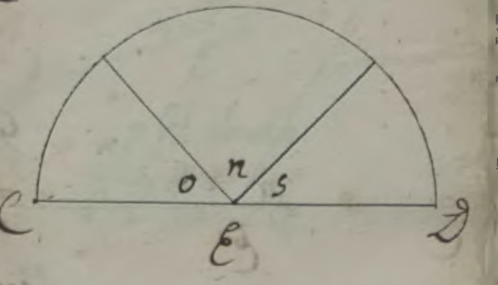
Mens ∠∠ $x+y = 2R$ §41 Ar.

Q.E.I.

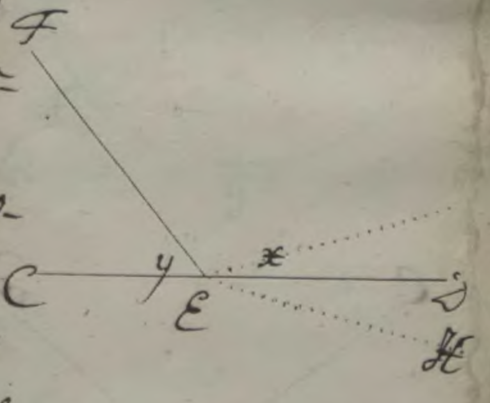
2) Quia CD = rectæ p. H.
Si ex communi omnium ∠∠orum vertice E p. H describatur semicirculus §88. erit ut ante:

Mens. ∠∠∠ $o+n+s = \frac{1}{2}$ Circ. §33
Sed $\frac{1}{2}$ Circ. $= 2R.$ §91

Mens ∠∠∠ $o+n+s = 2R.$ §41 Ar.
Q.E.D.



Menbrum 2dum. Quod mdm.
Si $x+y = 2R.$
fore CD Lineam rectam.
Nam cum $x+y = 2R.$ p. H.
aut 1) ED ipsi CE in directum iacet
aut 2) ED ipsi CE non iacet in directum
Ponamus non jacere, duci itaque poterit alia recta vel
1) supra ED cadens, qualis EG
2) infra ED cadens, qualis EH
quæ sit in directum posita ipsi CE §82. Hinc erit in
Casu 1mo



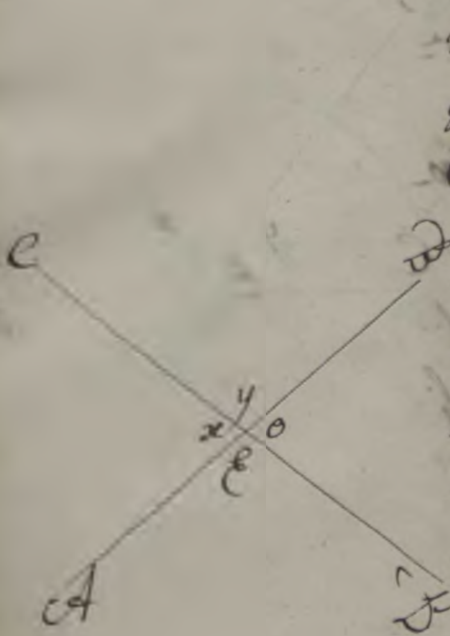
$y + FEG$ ∠∠ d. p. p. H aſſ.
Ergo $y + FEG = 2R$ p. Mbr 1.
ſed $\underline{y + x = 2R}$. p. H Theor.
$y + FEG = y + x$ §41 } Ar.
∠ $FEG = x$. §43 }
I. Q. E. A. §47 Ar.

Caſu 2do:
∠∠ $y + FEH$ ſunt d. p. p. H aſſ.
Ergo ∠ $y + FEH = 2R$ p. m. 1 §39 8.
ſed $\underline{\angle y + x = 2R}$ p. H. Th.
∠ $y + FEH$ = ∠ $y + x$. §41 Ar.
∠ $FEH = x$ §43 } Ar.
I. Q. E. A. §47 }

Quare cum ſub data Hypotheſi re
neq ſupra CD neq infra CD duci pu
rit in directum cadens ſ. ſita ipſi
CE plas et 2dum. Erit omnino
D ipſa in directum ſita ipſi CE. Q. E. D.

§94 Theorema 6.

Si recta AD alteram CB ſecet
in E, ∠li verticales x et o, item
y et E ſunt inter ſe aequales.



Demonſtratio.

$x+y$ ſunt ∠∠ d. p. §36. 41.

$x+y=2R$. §93

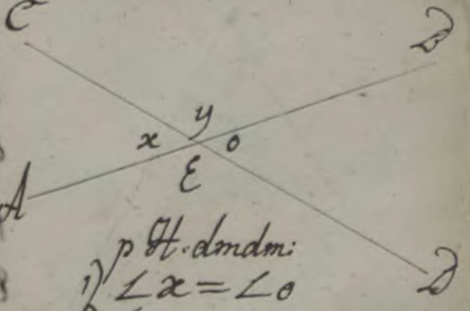
$o+y$ ſunt ∠∠ d. p. §36: 41.

$o+y=2R$ §93

$x+y=o+y$ §41 Ax.

$x=o$ §43 Ax.

Q. E. D.



p H. dmdm:
1) $\angle x=\angle o$
2) $\angle y=\angle E$

Simili Demonſtratione evincitur.

$\angle y=\angle E$.

§95 Theorema 7.

Omnes Anguli x, y, o et E circa punctum aliquod E conſtituti ſunt æquales 4 Rectis.

Demonſtratio.

Producto crure uno alicujus ∠li dati v. c. EN. in F §82.

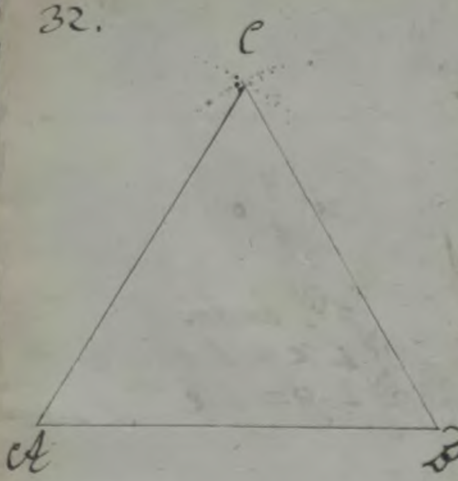
Erunt ∠li $FED+o+y=2R$
et ∠li $FEG+x=2R$ §93

$\angle FED+o+y+x+FEG=4R$. §42 Ax.

Sed ∠∠ $FED+FEG=\angle E$ §47 Ax

Ergo $\angle o+y+x+E=4R$. §10 Ax.

Q. E. D.



§96 Problema I

Super data Recta Linea AB term nata Triangulum æquilaterum deſcr bere.

Reſolutio.

1) Centro A radio et AB deſcribe Circu lum vel qui praxi ſufficit arcum §8

2) Centro B radio BA deſcribe æqualem §83. 26. vel arcum.

3) Junge mutuæ Interſectionis pun tum C cum A et B. §81 D. F.

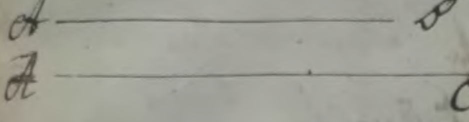
Demonſtratio

AB = AC } p. C. §26.
AB = BC }

AB = AC = BC. §41 Ax.

Ergo Δlum ABC eſt æquilaterum §55. Q. E. D.

§97. Problema II



Datis duabus rectis inæqualibus AB et AC terminatis Triangulum æquicrurum deſcribere.

Resolutio.

1) Aſſumta AC baſeos loco centro A intervallo AD deſcribe Circulum vel Arcum. §83.

2) Centro C intervallo eodem interſeca priorem §83. 36. 80.

3) Junge mutuo interſectionis punctum F cum A et C §81. DF.

Demonstratio.

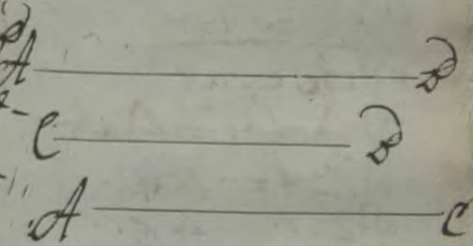
FA = FC. §81

Ergo Δlum FCA eſt æquicrurum §57.

Q. E. D.

§98. Problema III

Datis tribus rectis inæqualibus AD, DC, CA terminatis, quarum duæ quælibet tertia majores ſunt, ſcalenum deſcribere Triangulum.



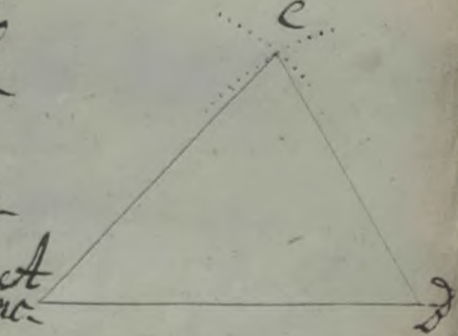
Resolutio et Demonſtratio.

1) Aſſumta una datorum v. g. AD pro baſi, centro A intervallo AC deſcribe Circulum vel Arcum §83.

2) Centro D radio DC interſeca priorem.

3) Junctis mutuæ interſectionis punctis §81.

DF. p §58.



§99. Theorema 8.

Si in duobus Triangulis ABC et abc, Angulus unus et latera ipſum intercipientia utrumque utriusque æqualia fuerint, tota Triangula æqualia ſunt, latus reliquum reliquo, et ∠li æqualibus lateribus oppoſiti æquales ſunt.

h.e. Si in Δlis ABC, abc

1) $\angle B = \angle b$
2) $BA = ba$
3) $BC = bc$

erit 1) $AC = ac$
2) $\Delta ABC = \Delta abc$
3) $\angle A = \angle a$
4) $\angle C = \angle c$.

Demonſtratio.

Concipe Δlum unum ſuperimponi alteri, quia

$AB = ab$ p. H.

Ergo AB congruit ab. §89.

adeoque Punctum A cadit in a } 7
Punctum B cadit in b }

∠lus $B = \angle b$ } p. H. Ergo
et $BC = bc$ }

Punctum C cadit in c. §7.

Ergo $AC = ac$ §90. Q.E.I.

Tota igitur Perimeter Trianguli unius ABC congruit Perimetro Δli alterius abc. Ergo

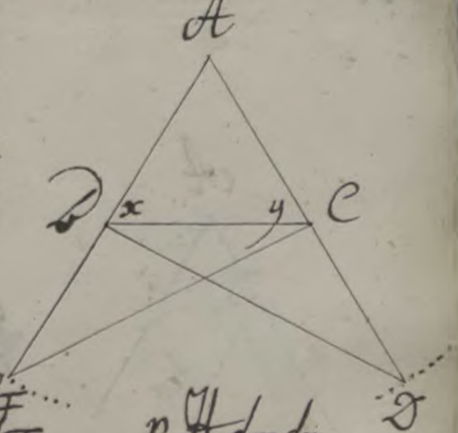
$\Delta ABC = \Delta abc$. §86. Q.E.II.

Ergo et $\angle A = \angle a$ } 87
$\angle C = \angle c$ }

Q.E.III et IV D.

§100 Theorema 9.

Isoscelium Triangulorum ABC. qui ad Basin sunt Anguli x et y sunt inter se aequales. Et Productis aequalibus rectis AB, AC, qui sub Basi sunt Anguli CBF et BCD sunt aequales.



p.H dmdm

1) $\angle FBC = \angle BCD$

2) $\angle x = \angle y$.

Demonstratio.

In productis Lateribus AB et AC p.H. Accipe aequalia intervalla BF, CD §83. 26 Junge FC et BD. §81

Hinc quia.

$AF = AD$ p.C.

$AC = AB$ p.H.

et $\angle A = \angle A$ §40 Ax.

$$\left.\begin{array}{l}\angle ACF = \angle ABD\\ \angle F = \angle D\\ FC = BD\end{array}\right\}\text{§99}$$

Cumque $AF = AD$ p.C,

et $BA = CA$ p.H

$FB = DC$. §43. Ax.

$\angle FBC = \angle BCD$ §99 Q.E.I.

Sed et

$\angle DBC = \angle FCB$ §c.

cumque $\angle ABD = \angle ACF$ p.d.

$\angle x = \angle y$ §43 Ax. Q.E.II.D.

Paullo aliter demonstratio

$x = y$

$\angle x + FBC = 2R$

$\angle y + BCD = 2R$

$\angle x + FBC = \angle y + BCD$, §41 Ax.

sed $FBC = BCD$ p.d

$\angle x = \angle y$ §43 Ax.

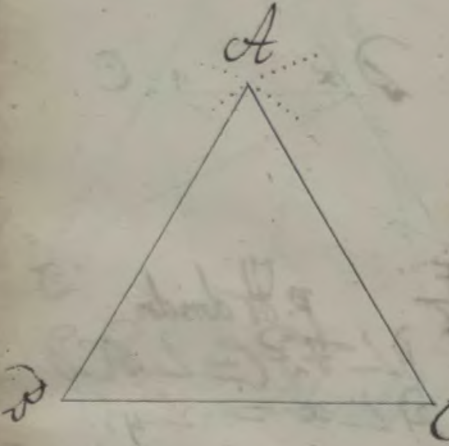
Q.E.II.D.

§101 Scholion

In Triangulo autem æquicruro dicitur Latus reliquus duobus inæquale Basis

§102. Corollarium.

Cum Triangulum æquilaterum sit etiam æquicrurum §55. 57, quæ demonstravimus §100 valent etiam de æquilatero.



§103. Theorema 10.

In Triangulo æquilatero ABC omnes $\angle$li sunt inter se æquales

p. H. dmdm:

$\angle A = \angle B = \angle C.$

Demonstratio.

$AB = AC$ §55

$\angle B = \angle C$ §100. 102.

$AC = BC$. §55.

$\angle B = \angle A$. §100. 102.

$\angle A = \angle B = C$. §41 Ar.

Q. E. D.

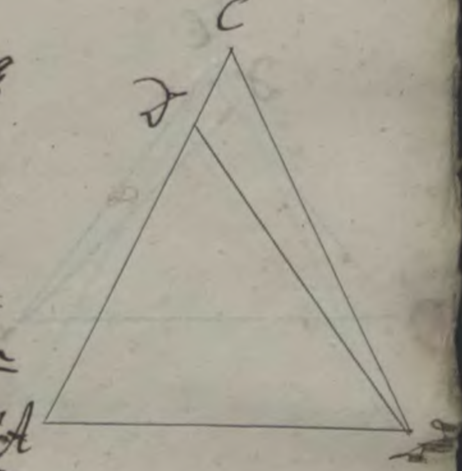
§104. Corollarium.

Triangulum æquilaterum est etiam æquiangulum.

§105. Theorema 11.

Super eadem recta linea, duabus eisdem rectis lineis, duæ aliæ lineæ rectæ æquales, utraq; utriq; non constituentur ad aliud atq; aliud punctum ad easdem partes, eosdemq; Terminos, cum duabus initio ductis rectis lineis habentes. h.e. Interprete Clavio ad Eucl. l. 7.

"Super recta linea AB constitutæ
" ad punctum quodvis C, duas rec-
" tas AC et BC. Dico: super eandem
" rectam AB versus partem eandem
" C non posse ad aliud punctum v.c.
ad D constitui duas alias rectas lineas, quæ sint æquales lineis AC et CB utraq; utriq; np. AC ipsi AD, quæ eundem habent Terminum A; et BC ipsi BD, quæ eundem etiam Terminum possident B.



Demonstratio

Sint enim si fieri potest rectæ

$$\left.\begin{array}{l} AC = AD \\ \text{et } BC = BD \end{array}\right\} \text{p H. assumtam.}$$

Indequidem Punctum D erit

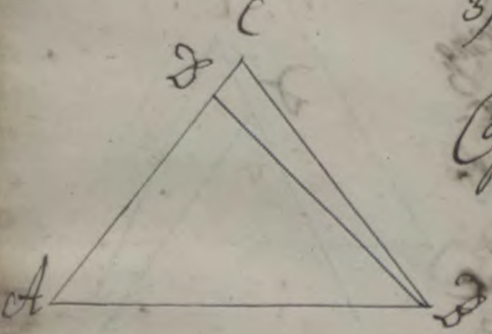
1) Vel in alterutra Rectarum AC aut CB; ita ut Recta AD in ipsam AC aut BD in ipsam CB cadat.

2) Vel intra Triangulum ACB

3) Vel extra idem Triangulum ACB.

Esto itaq; in

Casu Imo Punctum D in alterutra Rectarum AC aut CB v.g. in CA,



Ergo

AC > AD §12. 13. Ar.

Sed AC = AD p. H. ass.

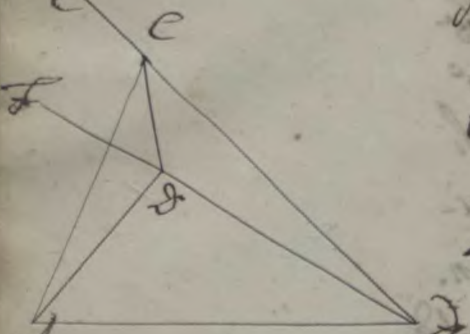
f. Q. E. A. p §47. Ar.

Casu 2do

Punctum D intra Triangulum ADC

junge CD et DA §81.

et produc Latera CD, DB in E et F §82.



AD = AC p. H. ass.

Δ ADC est aequicrurum §57

∠ ACD = ∠ ADC §100

∠ ACD ∠ r. ∠lo DCE. §12 ar.

∠ ADC ∠ r. ∠lo DCE §4b ar.

Verum

∠lus CDF est pars ipsius ∠ ADC.

Ergo
∠ CDF multo minor ∠lo DCE
Porro:
CB = DB p. Hypoth.
Δ CBD eſt æquicrurum §67.
∠ DCE = ∠ CDF §100 ſed
∠ DCE > ∠ CDF p.d.
J. Q. E. A.

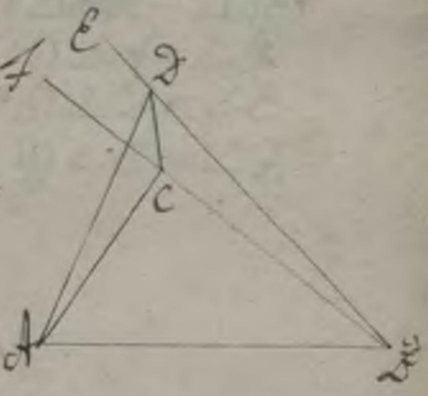
Eundem Angulum CDF modo minorem modo majorem eſſe alterollo DCE.

Caſu 3tio:

Punctum D extra Triangulum ABC unde quidem talem ſitum habebit ut

1) Vna linea ſuper alteram cadat ceu eſt in Figura prima, mutatis ſolummodo Literis D in C et C in D. Ex quo tamen ipſo idem Caſus 1mi abſurdum facile innoteſcat.

2) Poſteriores duæ lineæ ambiant priores duas, ut figura ſecunda tranſpoſitis duntaxat Litteris.



D in C, et C in D. Ex quo tamen et ipso in 2di Casus absurdum delabimur.

3) Altera Linea posteriorum v.c. AD secet alteram priorum v.c. BC.

Duc igitur DC § 61.

AC = AD p. H. assumtā.

ΔACD est aequicrurum § 51.

Ergo ∠ACD = ∠CDA. § 100.

Sed ∠CDA <r. ∠lo CDB. § 47. Ar.

~~Cumq.~~ ∠ACD ~~<r ∠lo~~ CDB. § 46. Ar.

Cumq. ∠BCD sit pars ipsius ACD.

Ergo ∠BCD multo minor ∠lo CDB.

Rursus cum

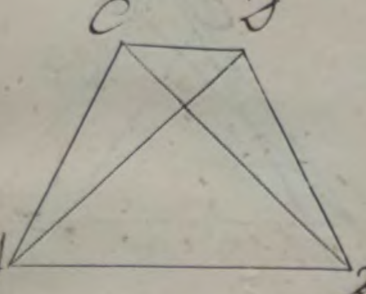
CB = DB p. H. ass.

adeoq. ΔCBD aequicrurum § 57.

Ergo ∠BCD = ∠CDB § 100.

Sed ∠BCD <r ∠lo CDB. p. d.

Q. E. A.



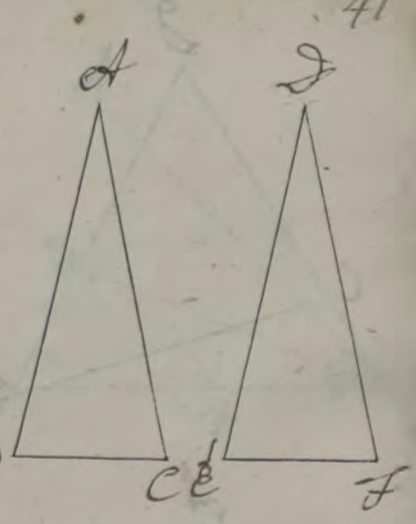
Non ergo AC = AD
et CB = DB.

quemcunque etiam punctum D ad partes ipsius C situm obtineat

Q. E. D.

§ 106 Theorema 12.

Si duo Triangula ABC, DEF habuerint duo Latera AB, AC duobus Lateribus DE et DF utrumque utrique aequalia; habuerint vero et Basin BC Basi EF aequalem: Triangula inter se aequalia sunt; et ∠li aequalibus lateribus interceptis aequales sunt.

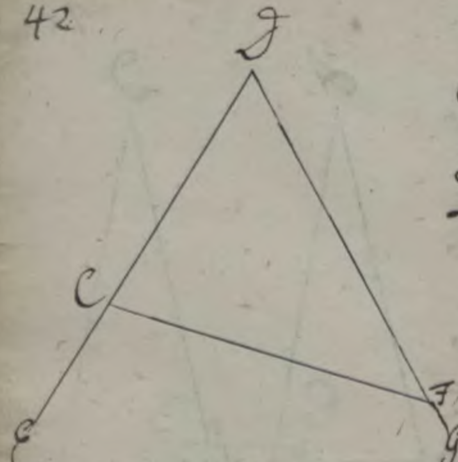


Demonstratio.

$BC = EF$. p. H.
Ergo BC congruit EF. § 89
Est vero et $\left.\begin{matrix} AB = DE \\ AC = DF \end{matrix}\right\}$ p. H.
cadet itaque punctum A in D. § 105.
Ergo $\Delta ABC = \Delta DEF$ § 86
Ergo et $\left.\begin{matrix} \angle A = \angle D \\ \angle B = \angle E \\ \angle C = \angle F \end{matrix}\right\}$ § 87.

Q. E. D.

H. e. si in duobus Triangulis
$AB = DE$
$AC = DF$
$BC = EF$ erit
1) $\Delta ABC = \Delta DEF$
2) $\angle A = \angle D$
3) $\angle B = \angle E$
4) $\angle C = \angle F$.



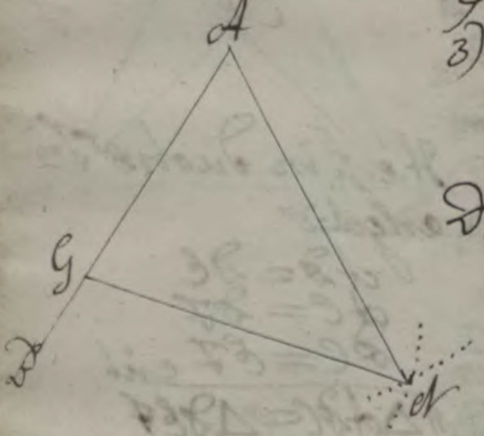
§107. Problema IV

Ad datam rectam Lineam AB datumque in ea punctum A, dato ∠lo rectilineo D aequalem ∠lum rectilineum A constituere.

Resolutio.

1) Duc utcunque Rectam CF §81 Latera DE et DG secantem
2) Fac AG = DC §26.
3) super AG describe Triangulum AGN aequilaterum alteri Triangulo DCF. §98.

Dico ∠lum A = ∠lo D. Q.E.F.



Demonstratio

$$\left.\begin{array}{l} AG = DC \\ GN = CF \\ NA = FD \end{array}\right\} \text{p.C.}$$

$$\angle A = \angle D \quad \text{§10b}$$

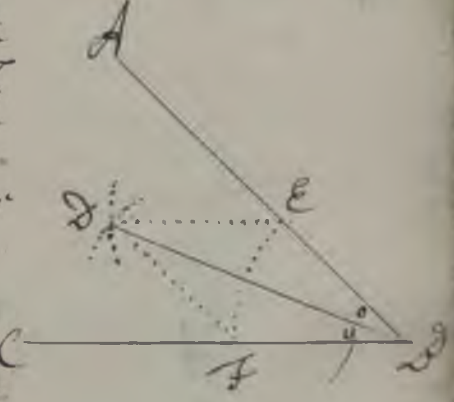
Q.E.D.

§108 Problema V.

Propositum Angulum AB bisecare.

Resolutio

1) Ex Vertice ∠li D radio quovis describe arcum secantem crura §83 in E et F.
2) Centris E et F radio eodem, vel alio quocunque æquali tamen, fac intersectionem in D. §80.
3) Junge puncta D et D recta DD §81. D.F.



Demonstratio.

Ductis ED et DF §81

erit $ED = DF$ } p.C. §26.

$ED = DF$

et $DD = DD$ §40. Ar.

$\angle o = \angle y$. §106.

Verum $\angle o + y = \angle D$. §47. Ar.

Ergo $\angle o + o = \angle D$. §10 Ar.

h.e. $2 \times \angle o = \angle D$

adeoque $\angle o = \frac{1}{2} \angle D$. §45. Ar.

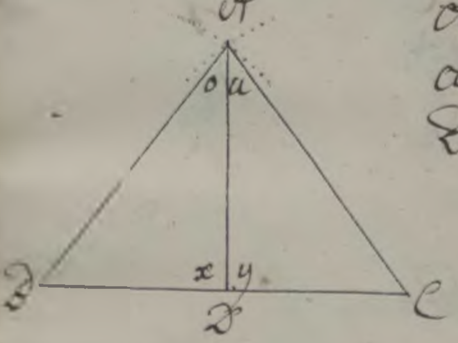
Q.E.D.

§109. Scholion.

Eodem artificio ∠lus propositus in quatuor, octo, sedecim, æquales partes secabitur, np. bisectum bisecando, bisecti bisectum bisecando etc. Methodus autem Regula et Circino ∠lum secandi in imperatas partes quotcunque hactenus Geometras latuit!

§110. Theorema 13.

Si Trianguli aequicruri ABC Angulum A bisecet basi oppositum recta AD, secante et basin BC



Dico: 1) $\triangle ABD = \triangle ACD$
2) $BD = DC$
3) $AD \perp$lem ad BC.

Demonstratio.

$\angle o = \angle u$ p. H.
$BA = AC$ p. H. §57.
$AD = AD$ §40 Ar.

$\triangle ABD = \triangle ADC$ §99.
$BD = DC$. §c
Q. E. I. et II.
cumque et $\angle x = \angle y$ §c.
Ergo $AD \perp$ ad BC. §44. 38.
Q. E. III. D.

§111. Corollarium.

Valet etiam Theorema demonstratum de Triangulo aequilatero.

§112 Problema VI.

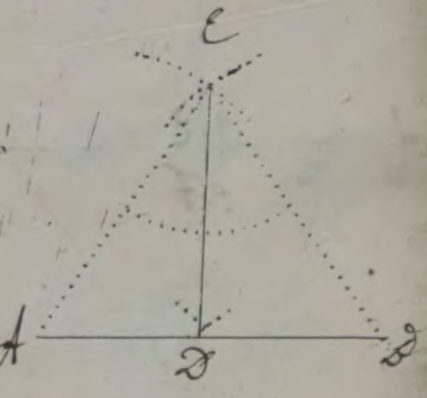
Rectam AB bifariam secare atq; in eius medio h. e. bisectionis puncto perpendiculum excitare.

Resolutio 1.

1) Super AB describe radio arbitrario Triangulum æquicrurum ABC §97.
2) Oppositum basi ∠lum C biseca recta CD secante AB basin §108.

Dico CD ⊥lem ad AB } §110.
et AD = DB

Q. E. F. et D.



Resolutio 2da.

1) Radiis arbitrariis, sed iisdem Centris A et B fac intersectiones in C §83.
2) Centris iisdem A et B radiis iisdem vel aliis æqualibus tamen fac intersectiones in E &c.

Duc Rectam CE §81. Dico
1) AF = FB
2) CF ⊥ ad AB.

Q. E. I.

Demonstratio

Junctis A et C, A et E, D et C §81.

erit $\left.\begin{array}{l} AC = CD \\ AE = ED \end{array}\right\}$ §26.

$CE = CE$ §40. At

$\angle o = \angle x$ §106.

D Sed $AC = CD$ §26.

$\left.\begin{array}{l} AF = FD \\ CF \perp ad AD \end{array}\right\}$ §110.

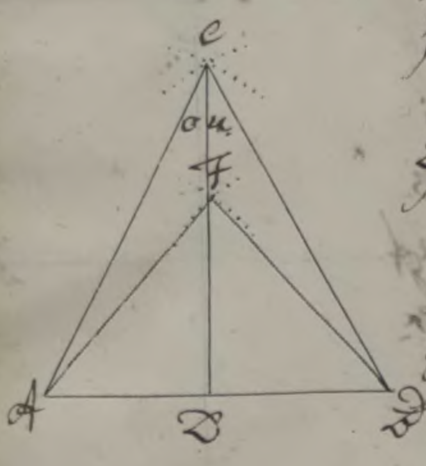
Q. E. D.

Resolutio 3.

Quod ſi non ſuffecerit interſectionib
in ſpatium

1) Factis uti Nbr. 1. Resolutionis 2do
Centris A et D intersectionibus in C
2) Radiis aliis æqualibus tamen ad pa
tes ipſius C facias alias intersection
centris iisdem A et D in F. §83.
3) Applicata in C et F Regula ducta
tam CF producendam ad AD in D
§81. 82. Dico:

1) AD = DD.
2) CD ⊥lem ad AD.



$AC = CB$ } §26
$AF = FB$ }
$CF = CF$ §40 Ax.

$\angle o = \angle u$ §106
$AC = CB$ p.d.

$AD = DB$ }
$CD \perp$lis ad AB } §110

Q.E.D.

§113 Theorema 14.

Si Trianguli ABC latus unum CA aut BA continuetur in D aut H erit ∠lus externus DAB major quolibet opposito interno B vel C.

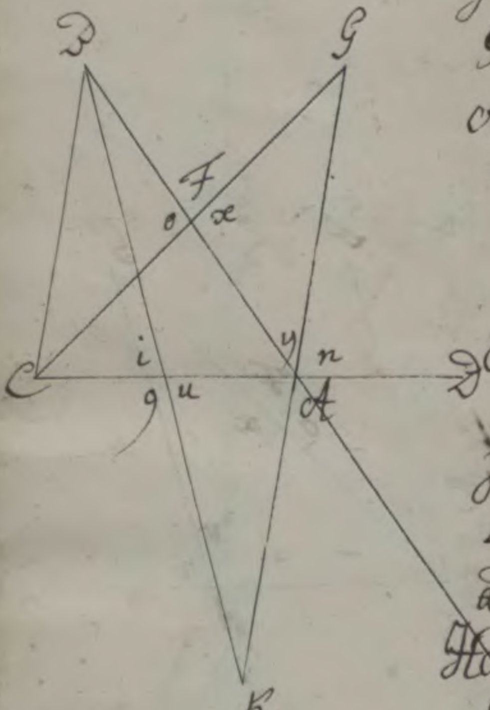
Demonstratio.

Bisecto latere BA §112 duc CF §81. producendam ut FG = FC §82.26. junge AG §81. Quia

$FG = FC$ p.c.
$FA = FB$ p.c.
$\angle o = \angle x$ §94

$\angle y = \angle B$ §99
Sed $\angle y+n > \angle$lo y §47 }
Ergo $\angle y+n > \angle$lo B §46 } Ax:
$\angle y+n = \angle$lo DAB §47 }

$\angle DAB > \angle$lo B §46 Ax.
Q.E.I.



Porro
eadem, ut ante ratione, AC bifariam
seca in I §112. duc IK §81. producendam
§82. ut $IK = IB$. §26. tandem duc
AK §81.

Quia $BI = IK$ p. C.
$CI = IA$
et $\angle i = \angle u$ §94

Ergo $\angle C = \angle IAK$ §99.
$\angle IAK + KAH > \angle$lo IAK. §47.
sed $\angle IAK + KAH = \angle$lo IAH. §c.

$\angle IAH > \angle$lo IAK §46 Ar.
adeoque et $\angle C < \angle$lo IAH §c.
Atqui $\angle IAH = \angle BAD$ §94.
Ergo $\angle C < \angle$lo BAD. §46. Ar.
Q.E.D.

§114 Theorema 15.
Si duo Triangula BAC et EDG dua
∠los B et C duobus ∠lis E et G aequa
les habuerint, utrumque utrique un
latus uni lateri aequale sive quod
aequalibus adjacet ∠lis B C et E G
sive quod uni aequalium ∠lorum

subtenditur, Reliqua latera reliquis lateribus aequalia utrumq utriq et reliquum ∠um reliquo ∠o aequalem habebunt.

h. e.
Si in duobus Triangulis ADC, DEG.

1) $\angle D = \angle E$	II. $\angle D = \angle E$
$\angle C = \angle G$	$\angle C = \angle G$
$DC = EG$, erit	$AD = ED$
1) $DA = ED$	1) $DC = EG$.
2) $AC = DG$	2) $AC = DG$.
3) $\Delta ADC = \Delta DEG$	3) $\Delta ADC = \Delta DEG$.
4) $\angle A = \angle D$	4) $\angle A = \angle D$.

Demonstratio.

Membrum I.

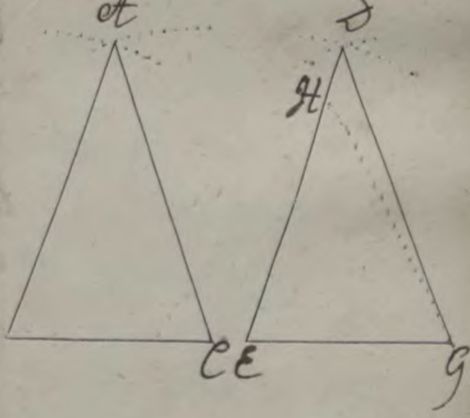
Aut $AD < DE$
Aut $AD > DE$ } § 8 Arit.
Aut $AD = DE$

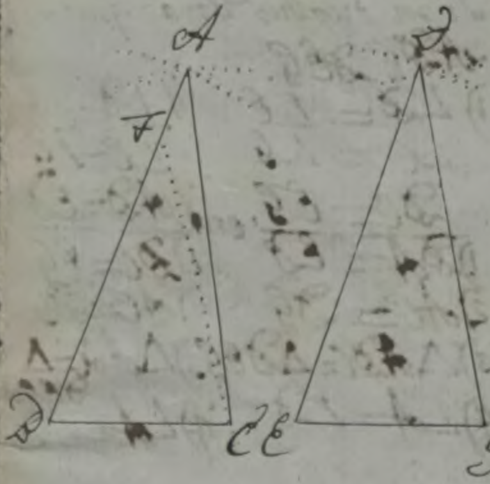
Ponamus I. si fieri possit.
$AD < DE$.
Ergo pars ipsius DE aequalis erit toti AD § 12 Ar. Esto illa EH duc HG § 81

Erit itaq $AD = EH$ p. Hyps.
sed $DC = EG$. p. Hyp.
et $\angle D = \angle E$ p. eand.

$\angle C = \angle HGE$ § 99.
sed $\angle C = \angle G$ p. Hyp.

$\angle G = \angle HGE$ § 41 Ar.
J. Q. E. A. per § 47. Ar.





Ponamus 2. ſi fieri poſſit

$AB > DE.$

Abſciſſa radio DE ex AB parte BF ductaq. FC § 81 erit

$BF = ED$ § 26

quia $\left.\begin{array}{l} BC = EG \\ \text{et } \angle B = \angle E \end{array}\right\}$ p. Hθ

$\angle FCB = \angle G$ § 99

sed $\angle C = \angle G$ p. Hθ

$\angle C = \angle FCB$ § 41 Ar.

ſ. 2. C. A. § 47. Ar.

Quare cum ſub data Theorematis Hypotheſi fieri non poſſit

vel $\left.\begin{array}{l} AB < DE \\ \text{vel } AB > DE \end{array}\right\}$ pd.

Ergo utiq. $AB = DE$. § 39. Ar.

Q. E. I.

Ergo Pct. D cadit in A. § 89

Sed et $BC = EG$. p. Hθ

Ergo Pct. G cadit in C. &c

$AC = DG$ § 90

Q. E. II.

Tota igitur Perimeter Δli EDG cadit in totam Perimetrum Δli AB.

Δlum ADC = ΔEDG. §86

Ergo et $\angle A = \angle D$ §87

Q. E. [illegible] D.

Membrum 2.

Aut DC $\angle$ EG
Aut DC $\urcorner$ EG } §39 Ar
Aut DC = EG

Ponamus 1. DC $\angle$ EG.

Abscissa radio DC parte EI ex EG

erit DC = EI. §26

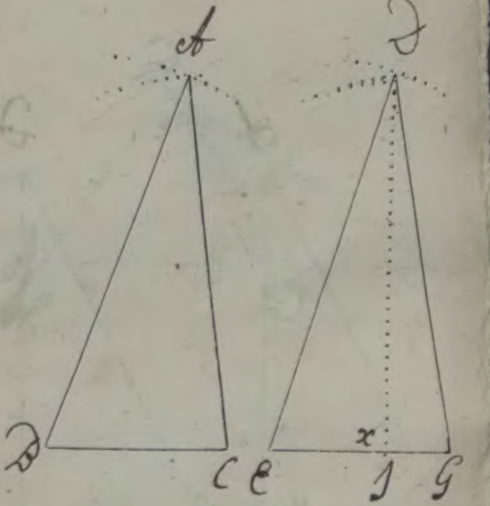
Ductaq. DI. §81

Quia AD = ED p. H. θ
DC = EI p. C. H. θ
$\angle D = \angle E$ p. H. θ

$\angle C = \angle x$ §99.
Sed $\angle C = \angle G$. p. H. θ

$\angle G = \angle x$. §41 Ar

I. Q. E. A. per §113.

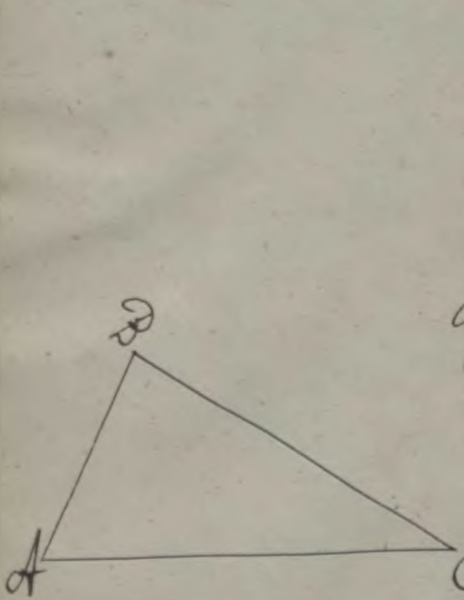


Ponamus 2° $DC > EG$
Fiat $DN = EG$ §26
Junganturque AN §81
Erit itaque $DN = EG$ p. C.
Sed $AD = ED$ p. Hyp.
$\angle D = \angle E$ p. eandem

$\angle y = \angle G$ §99.
Sed $\angle C = \angle G$ p. Hyp.

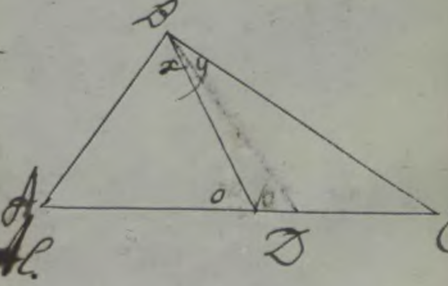
$\angle y = \angle C$ §41 Ar.
s. Q. E. A. p. §113.
Quare cum neque $DC < EG$
neque $DC > EG$
Ergo omnino $DC = EG$. §59 Ar.
Q. E. I.
Reliqua demonstrantur, uti Membra
Q. E. D.

§115. Theorema 16



In omni Triangulo ADC latus majus AC opponitur $\angle$lo majori D latus autem minus AD $\angle$lo minori C. Et contra: In omni $\triangle$lo ADC major $\angle D$ opponitur lateri majori AC minor $\angle C$ minori AD

Vel cum expressa Hypothesi. Si detur in quopiam Triangulo ABC latus majus AC opponitur illud ∠lo majori B minus autem AB minori ∠lo C. Et contra: Si detur in quopiam Triangulo ABC major ∠ B opponitur ille lateri majori AC.



Demonstratio

Latus AC $>$ AB p. H.

Fac AD $=$ AB §26.

Junge BD §61.

Ergo Δ ABD est æquicrurum §57

Ergo $\angle o = \angle x$ §100

Sed $\angle o >$ ∠lo C §113

$\angle x >$ ∠lo C §46 Ax.

Sed $\angle B = x + y$ §47 Ax.

∠B multo $>$ ∠lo C.

Q.E.I.

$\angle B >$ ∠lo C p. H.

Hinc, si non sit AC $>$ AB erit

vel 1) AC $=$ AB }
vel 2) AC $<$ AB } §81 Ax.

Supponamus 1) AC $=$ AB

Ergo $\angle B = \angle C$ §100

S.Q.C.H. quo ∠lum majorem B supponit

Supponamus 2. AC $<$ AB

Ergo $\angle C >$ ∠lo B p. M.

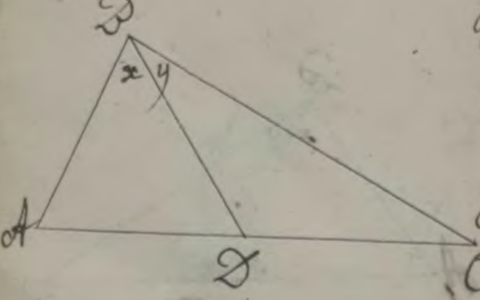
Verum et hoc contra Hypothesin, quæ majorem esse ∠lum B statuit altero C.

Quare cum non sit

vel 1) AC $=$ AB }
vel 2) AC $<$ AB } p. d.

erit AC $>$ AB §39 Ax.

Q.E.D.



§116. Theorema 17

In omni Triangulo ABD duo latera AD et DB ſimul ſumta tertio AB ſunt majora

Demonſtratio

Produc AD in C uſque donec DC = DB. Hinc quia AC = AD + DC erit AC = AD + DB. §10 Ar.

Duc BC §81 Ergo △ BDC eſt aequicrurum §57.

Ergo ∠y = ∠C. §100

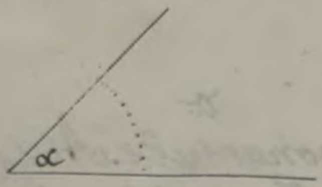
Sed ∠C $<$ ∠lis x+y §47 Ar.

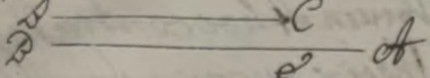
Ergo AB $<$ AC §115.

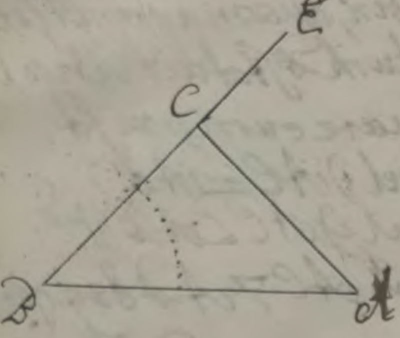
Sed AD+DB = AC p. d.

BA $<$ AD+DB §46 Ar.

Q. E. D.







§117. Problema VIII

Datis duobus Lateribus AB, BC una cum ∠lo intercepto x Triangulum conſtruere.

Reſolutio et Demonſtr.

1) Aſſumto latere uno datorum v. c. AB pro baſi, ad B conſtitue ∠lum

ad §116.

Aliter

Proclo Auctore idem Theorema demonstrant Familiares Heronis et Porphyrii ut Clavius habet ad L. 1. Prop. 20 Eucl. nullo Latere producto, hunc fere in Modum.

Esto Δlum ADC: Dico

1) $DC + DA > CA$,

2) $DC + CA > DA$.

3) $CA + DA > DC$.

Demonstratio.

Diseca $\angle$lum D Recta DD secante et basin CA §108.

Hinc ob productum in Δlo ADD Latus AD in C

$\angle y > \angle$lo x. §113.

Sed $\angle x = \angle o$ p. C.

$\angle y >^{r} \angle o$ §46. A.

Ergo $CD > Cd$. §115.

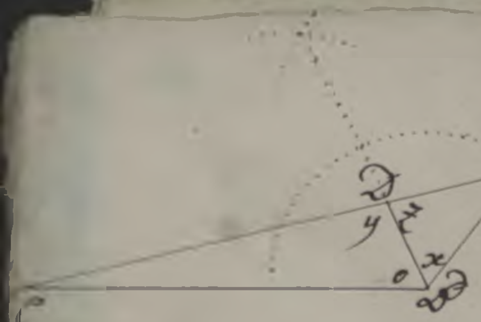
Sic et ob productum in Δlo CDD Latus CD in A

$\angle z >^{r} \angle$lo o §113.

$\angle o = \angle x$ p. C.

$\angle z >^{r} \angle x$. §46. A.

C



Ergo $BA > AD$ §115

et $CB + BA > AD + CD$ §42. et

$AD + CD = AC$. §47. A.

$CB + BA > AC$. §46. A.

Simili Discursu Q.E.I.

Ex bisectis ∠lis C et A lineis secantibus et Lateribus BA atq; BC ostenditur.

et $BC + CA > BA$

et $CA + BA > BC$. Q.E.II et III D.

ato æqualem §107.
Circino cape intervallum DC et trans-
fer in DE §26.
Duc rectam AE §81. Descriptumque erit
Triangulum quæsitum.
Q. E. F. et D.

118. Theorema 18.

inea recta AD est brevissima omni-
m, quæ intra eosdem Terminos
AD continetur.

Demonstratio

Sit curva quæcunque ACD, ductis ut-
unque subtensis AC et CD §81 erit
$AC + CD > AD$ §116.
Assumtis porro punctis quibuslibet
in curva D et E, ductisque rectis AD et
DC; CE et ED §81. erit

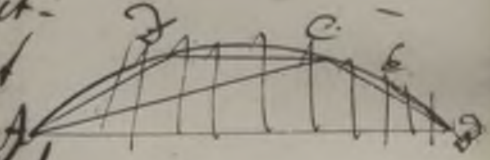
$$\left.\begin{array}{l} AD + DC > AC \\ CE + ED > CD \end{array}\right\} \S 116.$$

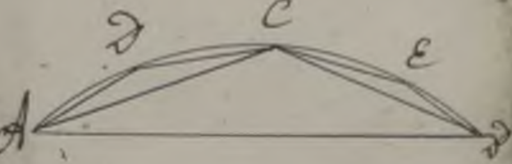
$AD + DC + CE + ED > AC + CD$ §4e Ar.

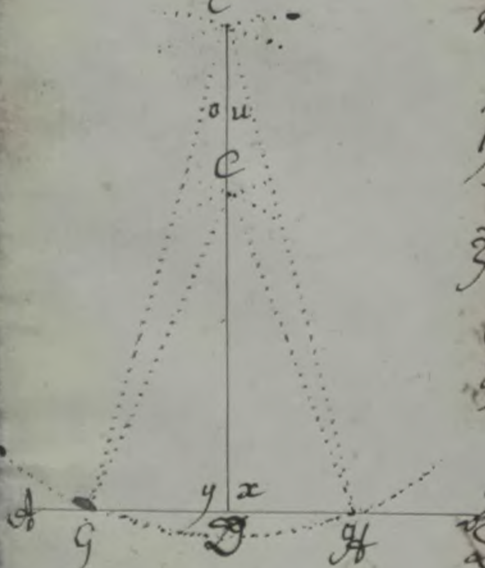
Sed $AC + CD > AD$ p. d

$AD + DC + CE + ED$ multo majores AD.

Quare cum tandem subtensæ ipsi Curvæ
coincident, adparet AD esse brevissimam
earum, quæ eosdem Terminos habent. Q. E. D.







§119. Problema VIII

A dato super Recta AB puncto C demittere normalem CD.

Resolutio

1) Centro C radio arbitrario duc arcum GH. §83.

2) Centris G et H, radio alio eodem tamen fac intersectiones in E. §c.

3) Duc, applicata in punctis E et C regula, rectam DE. §81 D. F.

Demonstratio.

Duc CG, CH, HE, GE. §81.

$$\text{Quia } CG = CH \left.\begin{matrix}\\\end{matrix}\right\} §26$$

$$GE = EH$$

$$\underline{EC = EC.} \ §39.\ Ar.$$

$$\angle o = \angle u \ §106.$$

$$\text{sed } GE = EH.\ §26.$$

$$\text{et } \underline{ED = ED.} \ §39\ Ar.$$

$$\angle y = \angle x \ §99.$$

Ergo et y et x = R. §38.

adeoque CD ⊥ ad AB. §44. 46.

Q. E. D.

no Problema IX

Ex dato in Recta AB puncto D normalem excitare.

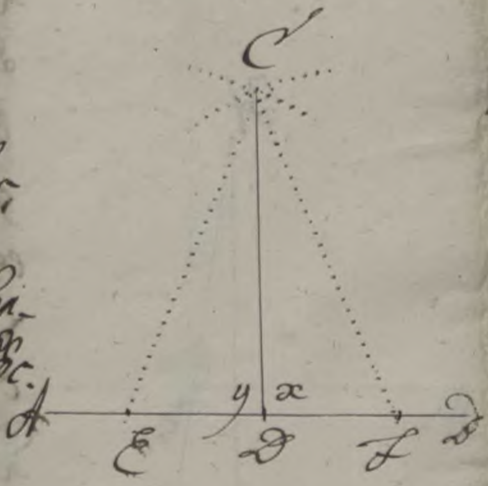
Reſolutio

Ex dato puncto D arbitrario radio fac, interſectiones in AB, E et F. §83.

alio radio arbitrario, ſed eodem Centris E et F fac interſectiones in C §c.

Duc CD §81.

D. F.



Demonſtratio

Ductis CE et CF. §81.

erit $CE = CF$ } §26
$ED = DF$ }
$CD = CD$ §39 Ax
$\angle y = \angle x$. §10b.

Sed $\angle\angle$ y et x ſunt d. p. §36

Ergo et $\angle y$ et $\angle x = R$ §33.

Ergo CD ⊥lis ad AB. §44. 46.

Q. E. D.

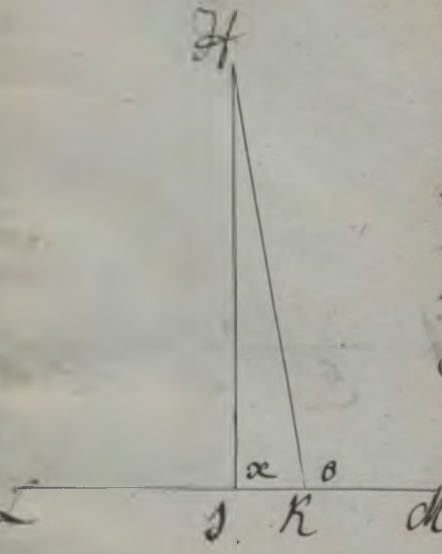
§121 Scholion

Diximus in recta excludendo extremum, quo de casu infra specialibus Problematibus agetur.

§122. Theorema 19

Ab uno puncto H ad eandem rectam LM, non nisi unica normalis HJ duci potest.

Demonstratio



Ducatur si fieri possit adhuc alia normalis HK ad LM, et erit

$\angle o = R$. §44

Sed et HJ ⊥lis ad LM. p. H.

Ergo et $\angle x = R$. §c

$\angle o = \angle x$. §92.

Verum $\angle x$ $\angle r$ $\angle o$ §113.

Ergo Recti sunt inaequales

J.Q.E.A. per §92.

§123. Theorema 20

Linea normalis HJ est brevissima omnium quae ex eodem puncto H ad eandem rectam LM duci possunt.

Demonstratio.

HJ ⊥lis ad LM p. H.

rgo $\angle x = R$. §44.

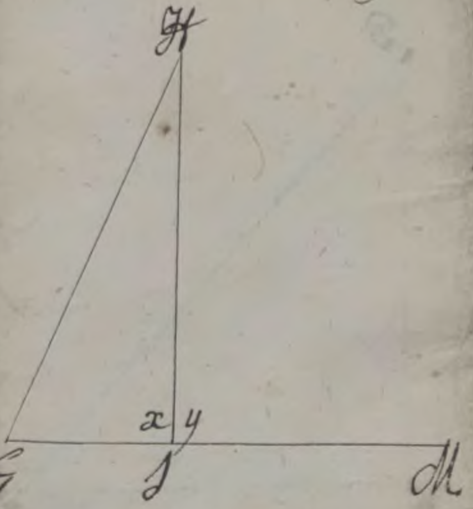
Et $\angle y = R$. §c. et 38.

$\angle x = \angle y$ §92.

ed $\angle y$ >r ∠lo L. §113.

rgo $\angle x$ >r ∠lo L. §46 bctr

rgo HL > HJ. §115. Q. E. D.

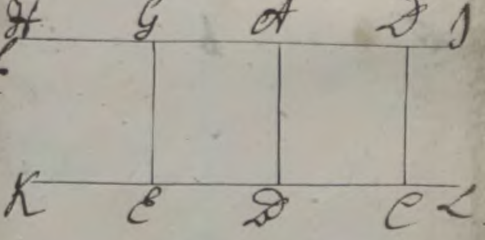


§124. Corollarium 1.

rgo distantia puncti a Linea vel Plano est recta ab illo puncto ad Lineam el Planum normalis §47.

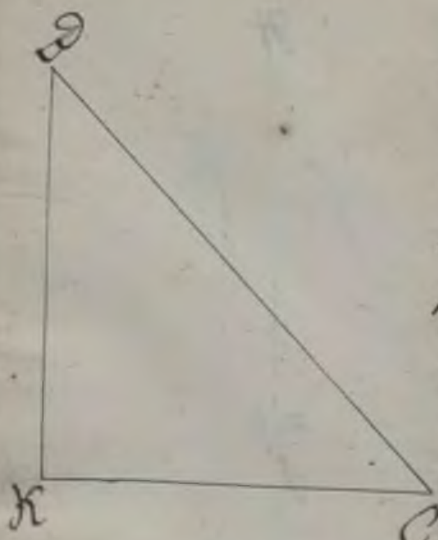
§125. Corollarium 2.

i Linea HJ fuerit ipsi KL parallela erunt perpendicula quovis ex illa in hac demissa. GE, AD, DC inter se æqualia §48.



Et contra

Si Perpendicula fuerint æqualia Lineæ ipsæ sunt Parallelæ.



§126. Corollarium 3.
Altitudo Figuræ est normalis ex Vertice in Basin demissa. §116.

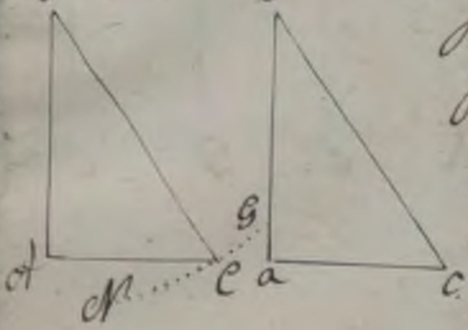
§127. Corollarium 4.
Hinc etiam in Triangulo DKC qua K = R §59 Catheti DK vel KC adeo sunt invicem normales §94. Assumta itaque KC pro Basi erit D vertex §116 et altitudo DK §126.

§128. Corollarium 5.
Idem eodem modo et de Quadrato et de Rectangulo ostenditur.

§129. Theorema 21.
In duobus Triangulis rectangulis si Hypotenusæ et Catheti æquales fuerint.

1) Triangula ipsa sunt æqualia
2) Bases.
3) Anguli æqualibus lateribus oppositi sunt æquales.

Demonstratio



Ponamus Triangulum abc super imponi alteri ADC, hinc quia

$AD = ab$ p. H.

rgo Petm a cadit in A. §89.

Petm b cadit in D §c.

ac radio $bc = DC$ p. H. centro.

arcum Gel. §83. quia et

$\angle a = \angle A = R$. p. H et §92.

ac. cadit AC §89.

ndequidem ob duarum linearum uni-cam solummodo intersectionem punctualem §80:

Petm. c. cadit in C §71. sed

$\left.\begin{array}{l}\text{Petm a . . . . . in A}\\ \text{Petm b . . . . D}\end{array}\right\}$ p.d.

rgo tota Perimeter Δli abc cadit in totam Perimetrum Δli ADC

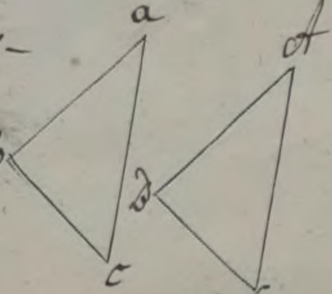
rgo Δlum abc = Δlo ADC. Q. E. I.

$ac = AC$

$\left.\begin{array}{l}\angle b = \angle D\\ \angle c = \angle C\end{array}\right\}$ Q. E. III. et IV. D. §c.

§130. Scholion.

Valet etiam Theorema de Triangulis obliquangulis, si praeter Latera $AC = ac$ et $AD = ab$ fuerit $\angle C = \angle c$.



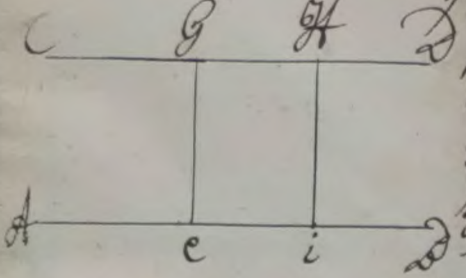
§131. Problema X.

Cum data recta AB ducere parallel lineam CD.

Resolutio.

1) Assumtis in Recta AB duobus vel pluribus punctis e, i, erige ex iis normales eG, iH. §120.
2) Excitatas fac ęquales §26.
3) Per puncta G et H duc rectam CD §81. 82.

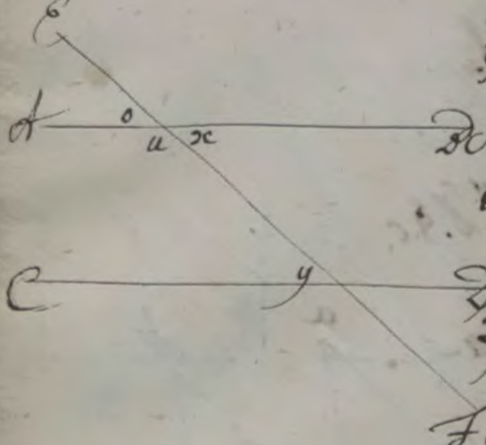
D. F.



Demonstratio.

Est enim $eG = HI = \perp li$ p. C.
Ergo AB ≈ CD §.125. Q. E. D.

§132. Theorema 22.



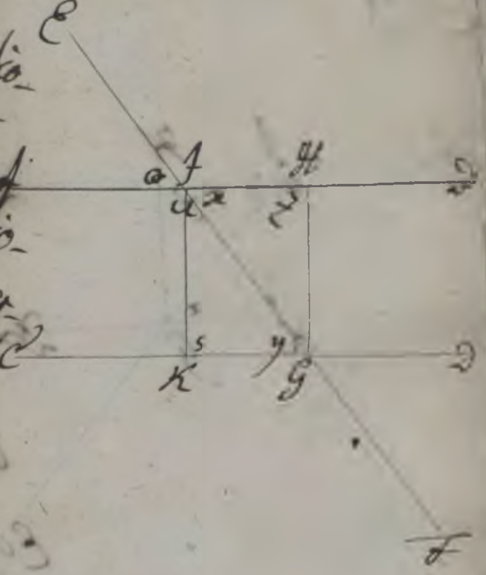
Si duas Parallelas AB, CD secet tr versa EF, erunt

1) Anguli alterni ęquales $x = y$
2) Angulus externus o ęqualis interno opposito y. $o = y$.
3) Duo ∠li interni oppositi $u + y$ ęquales duobus Rectis. $u + y = 2R$

Demonstratio

Demitte ex Inp. puncto Intersectionis Parallelo unius AB atq transversa EF ⊥lem IK. §119

Excita ex Gnp. Puncto Intersectionis alterius Parallelo CD & transversa EF ⊥lem GH. §120.



Quia $AB \approx CD$ p. H.

Ergo $HG = IK$ §125

$IG = IG$ §40 Ar.

$\angle z = \angle s$. §92.

$\angle x = \angle y$ §129.

Q.E.I.

$\angle x = \angle o$ §94.

Sed $\angle x = \angle y$ p.d.

$\angle o = \angle y$ §41 Ar.

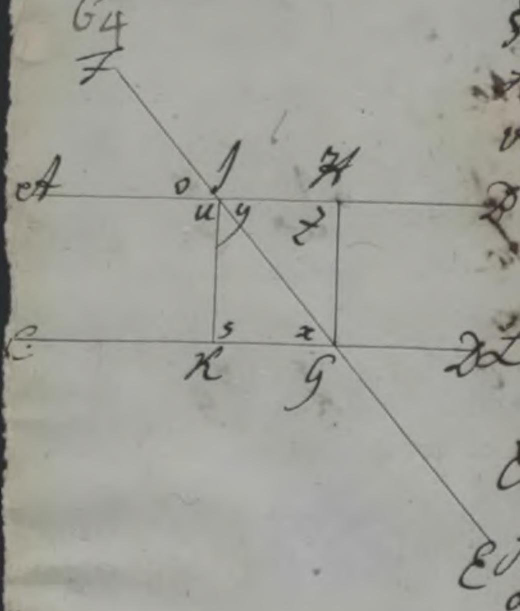
Q.E.II.

$\angle o + u = 2R$. §93.

Sed $\angle o = \angle y$ p.d.

$\angle u + y = 2R$. §10 Ar.

Q.E.III.D.



§133. Theorema 23.

Si duas Lineas AB et CD secet transversa EF in I et G, ita ut vel
1) Anguli alterni aequales
2) $\angle o = \angle x$.
3) $\angle u + x = 2R.$ erunt,
Lineae AB et CD parallelae

Demonstratio

Ex I demitte normalem IK. §119.
fac IH = KG. §26.
duc HG. §81.

Quoniam $y = x$ p. H.
et IG = IG. §40 Ar
et IH = KG p. C.

Ergo IK = HG. §99.
et $\angle s = \angle z$ §c.
sed $\angle s = R.$ p. C. et §274

Ergo $\angle z = R.$ §92.
Ergo HG ⊥lis ad AB. §44
adeoque CD ≈ AB. §125.

Q. El.

$\angle o = \angle x$ p. H.
$\angle o = \angle y$ §94.
$\angle x = \angle y$. §41. Ar.
Ergo $AB \simeq CD$ per Casum I

Q. E. II.

$u + x = 2R$. p. H.
$o + u = 2R$. §93.
$u + x = o + u$ §41. Ar.
Ergo $\angle x = \angle o$ §43. Ar.
Ergo $AB \simeq CD$. per Casum II.

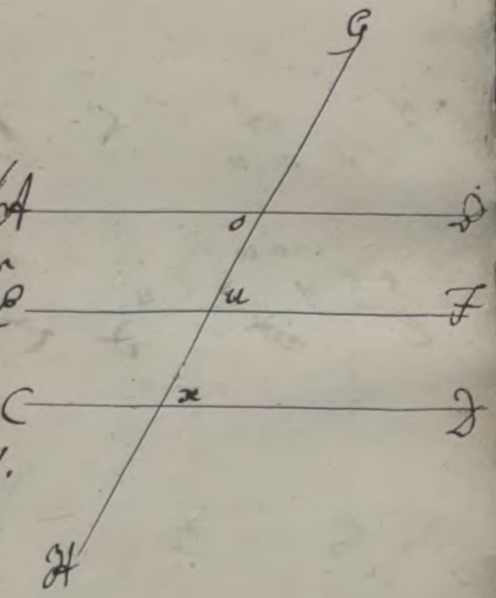
Q. E. III. D.

§134. Theorema 24.

Si duae Lineae AB et CD fuerint parallelae eidem tertiae EF et inter se sunt parallelae.

Demonstratio.

Ducta utcunque transversa GH. §81.
Quia $AB \simeq EF$ p. H.
Ergo $\angle o = \angle u$ §132. Mbr. 1.
Sed et $EF \simeq CD$ p. H.
Ergo $\angle u = \angle x$. §132. Mbr. 2.
$\angle o = \angle x$ §41. Ar.
Ergo $AB \simeq CD$. §133. Q. E. D.



h. e.
Si $AB \simeq EF$
et $CD \simeq EF$ Dico
$AB \simeq CD$.

§135. Problema XI

Per datum punctum A cum Recta AD ducere parallelam.

Resolutio et Demonstratio.

1) Per A et rectam AD duc utcunque transversam EF. §81.

2) Ad A fac $\angle$lum $o = \angle y$ §107.

3) Per G et A duc rectam NG §81.82 quae erit Parallela alteri AD §133.

Q. E. F. et D.

§136. Theorema 25.

Si HI fuerit parallela cum KL et AD normalis ad KL erit eadem DA ⊥lis ad HI.

Demonstratio.

Fac ED = DD. §26.
et excita ⊥les GE et CD ex E et D. §120. Quia.

$HI \approx KL$ p. H.
Ergo $GE = CD$ §125
$ED = DD$ p. C.
$\angle s = \angle u$ §44.92.

$DG = DC$ } §99.
et $\angle y = \angle z$ }

Verum AD $\perp$ ad KL. p. H.

Ergo $\angle o+y = R.$ §44.

$\angle o+y = x+z$ §38.

Sed $\angle y = \angle z$ p. d.

Ergo $\angle o = \angle x.$ §43. Ar.

cumque AD = AD §40. Ar.

$\angle m = \angle n$ §99.

Sed $\angle m$ et n sunt $\angle$ d. p. §36.

Ergo $\angle m + n = 2R$ §93.

adeoque $2 \times \angle m = 2R.$ §10. Ar.

et $\angle m = R.$ §45. Ar.

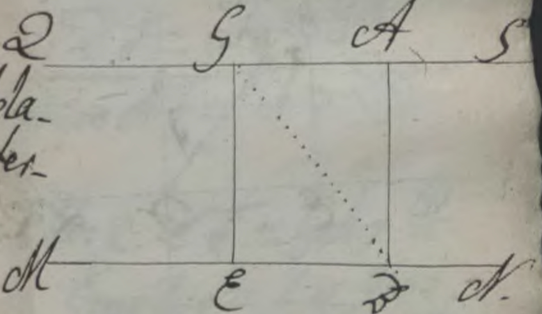
Ergo AD $\perp$lis ad HI §44.

Q. E. D.

§137. Theorema 26.

Perpendicula GE et AD aequales Parallelarum QS. et MN. partes GA et ED intercipiunt.

Demonstratio.



QS. $\simeq$ MN p. H.

Duc GD. §81.

Quia AD et GE. $\perp$les ad QS. et MN p. H.

Erit $\angle E = \angle A$ §44. 92. 136.

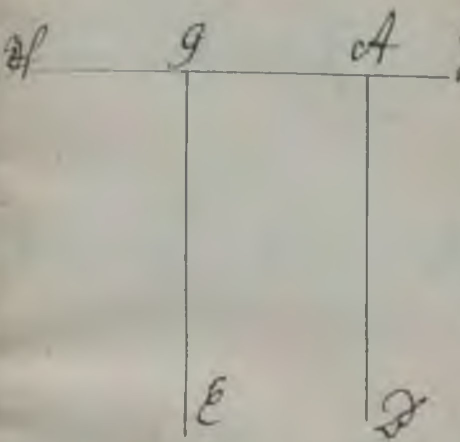
GE = AD. §125.

GD = GD §40.

GA = ED. §129. Q. E. D.

§138. Theorema 27.

Si duae Lineae EG et AD fuerint normales ad eandem tertiam HJ. sunt illae inter se parallelae.



Demonstratio

$GE \perp$ ad AD. p. H.

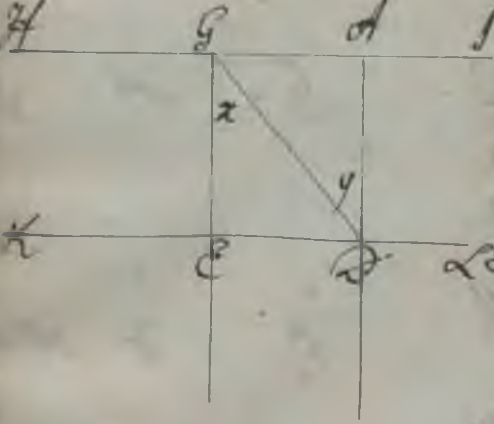
$\angle EGA = R$. §44.

$DA \perp$ ad HJ. p. H.

$\angle GAD = R$. &c.

$\angle EGA + GAD = 2R$ §42. Ar.

$EG \approx DA$. §133. Q.E.D.



Aliter

Fac $AD = EG$. §26.

Duc KL per E et D. §81.82.

itemque GD. §81

∠ Quia $HJ \approx KL$. §125.

ergo $ED = GA$ §137.

sed $DG = DG$ §39. Ar.

et $GE = AD$. p. C.

Ergo $\angle x = \angle y$. §106.

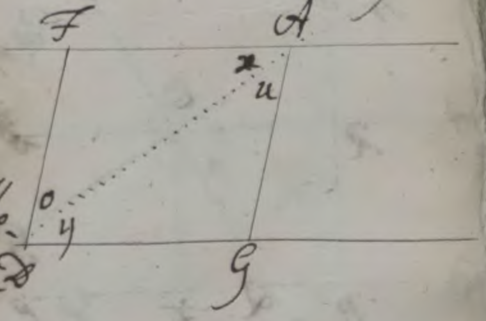
Ergo $GE \approx AD$. §.133. Q.E.D.

§139. Theorema 28.

Parallelo DF et GA inter easdem Parallelas FA et DG sunt œquales.

Et contra

Si DF et GA sunt œquales et parallelæ erunt etiam FA et DG parallelæ et œquales



Demonstratio.

Mbr. 1. Duc. DA. §81.

Quia DF $\approx$ GA p. H.

∴ $\angle o = \angle y$. §132.

Sed et AF $\approx$ DG. p. H.

∴ $\angle y = \angle x$. §c.

AD = AD. §39. Ar.

DF = GA. §114. Q. E. 1.

h. e.

1) Si DF $\approx$ GA et FA = atq $\approx$ DG

Dico DF = GA.

2) DF = atq $\approx$ GA

Dico AF = et $\approx$ DG.

Aliter

Duc DA. §81.

Quia AF $\approx$ GD. p. H.

$\angle x = \angle y$. §132.

Sed et AF = GD. p. H.

et DA = AG. §99 Q. E. 1.

Mbr 2. DF $\approx$ GA p. H.

Ergo $\angle o = \angle u$ §132.

DF = GA. p. H.

DA = DA §39. Ar.

FA = DG. §99.

cumq. $y = \angle x$ §c.

Ergo FA $\approx$ DG. §133.

Q. E. II. D.

§140. Theorema 29.

Si recta NO secat duas rectas alias AG et DE in C et Q ita ut duo ∠li interni oppositi HCO et DQN majores fuerint duobus Rectis, lineae AG et DE versus illam plagam divergunt.

Demonstratio.

$\angle\angle$ HCO + DQN $>$ res 2R. p. H.

Ergo AG non est $\eqsim$ cum DE. §132.

Per C. age AD $\eqsim$lam cum DE. §135.

Ergo $\angle$ ACO + DQN $=$ 2R. §132.

Sed $\angle$ HCO + DQN $>$ 2R. p. H.

$\angle$ ACO + DQN $<$ res $\angle$ HCO + DQN. §46 Ar.

et $\angle$ ACO $<$ r. $\angle$lo HCO. §43 Ar.

Ergo AC cadit intra spatium HCD. Ex Recto HC puncto quovis S demitte normalem SP ad DE. §119. similiter ex alio adhuc puncto v.e. C demitte item CF ad DE. &c. Ergo

RP $=$ CF. §125.

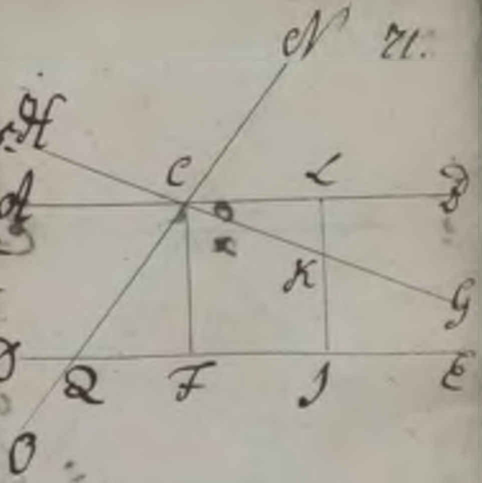
SP $>$ RP. §47. Ar.

SP $>$ CF §46. Ar.

☆ ☆

Sunt autem CF et SP distantiae rectarum AG et DE. §124. eaeq. crescentes p.d. Ergo lineae AG et DE ab ea parte divergunt. §11. Q.E.D.

☆ ☆

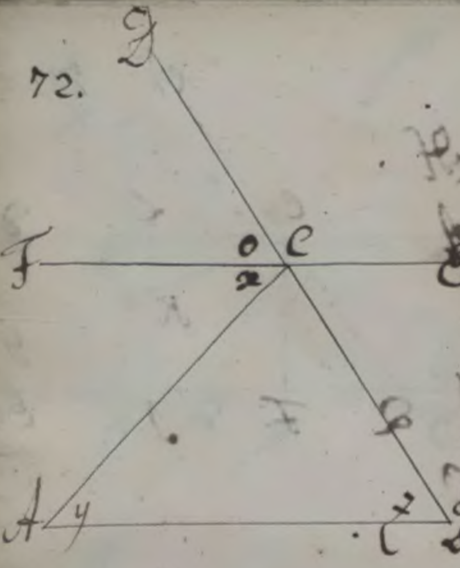


§141 Theorema 30.

Si duas Lineas HG et DE secet transversa NO in C et Q ita ut ∠ GCO et CQN simul sumti sint duobus Rectis minores, lineæ GC et QE versus illam plagam convergunt.

Demonstratio.

Duc AB = cum DE per C §136.
Ergo ∠ BCQ + CQN = 2 R §132.
Sed ∠ GCO + CQN < 2 R p. H.
∠ BCQ + CQN > ∠ GCO + CQN subtr.
Ergo ∠ BCQ > ∠lo GCO §43 Ar. cadit
Extra spatium GCQE igitur recta CG.
Ex puncto C demitte ⊥ ad QE §119.
itemque aliam ex alio quovis recto
CG v. c. K puncto et continuandum in L §82.
Quia CF = LI §125.
et IK < LI §47 Ar.
Ergo IK < CF §46 Ar.
Sunt autem CF et IK distantiæ rectarum HG et DE §124 eoque decrescentes p. d. Ergo DE et HG sunt lineæ convergentes §50
Q.E.D.



§142. Theorema 31.

Si Trianguli cujuscunq ADC latus unum CD continuetur in D erit ∠lus externus aequalis duobus internis oppositis z et y simul sumtis.

Demonstratio.

Age per C cum AD ≈ CF §138 inde quidem 1) CA Secans Parallelas AD et FE. Ergo

$\angle x = y$ §132

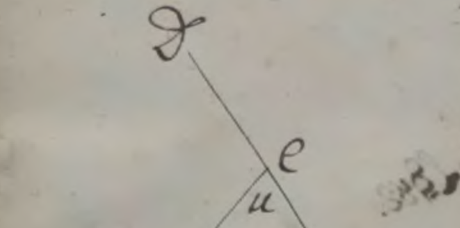
2) CD Secans Parallelas AD et FE. Ergo

$\angle o = z$ §c.

$\angle x + o = \angle y + z$ §42.

$\angle x + o = \angle ACD$ §47 } Ar.

$\angle ACD = \angle y + z$ §41. Q.E.D.



§143. Theorema 32.

In omni Triangulo ADC tres anguli junctim sumti $u+y+z$ sunt aequales duobus Rectis.

Demonstratio.

Producto Latere quolibet v.c BC in D §82 erit

$\angle ACD = \angle y+z$ §142.

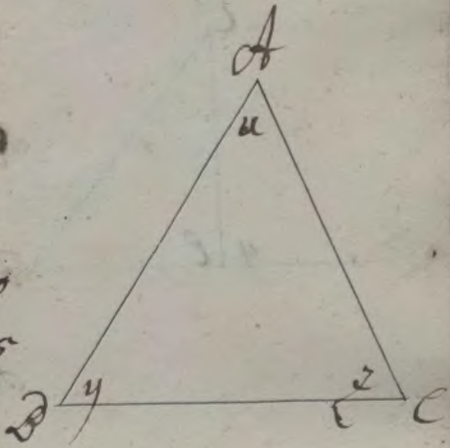
Sed $\angle u = \angle u$ §40. Ar

$\angle ACD+u = \angle u+y+z$ §42. Ar.

$\angle ACD+u = 2R$. §93.

$\angle u+y+z = 2R$ §41. Ar.

Q. E. D.



§144. Corollarium 1.

Hinc cujuscunq. Trianguli et ABC duo Anguli duobus Rectis sunt minores omnifariam sumti

Nam:

$y+z+u = 2R$: §143

$z+u = 2R-y$ §43. Ar.

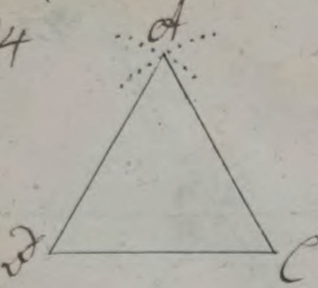
Sed $2R > 2R-y$. §47. Ar.

$z+u < 2R$. §46. Ar.

§145. Corollarium 2.

In Triangulo aequilatero ABC quilibet et $\angle$lus aequalis est $\frac{2}{3}R$.

Nam:



$A + B + C = 2R$. §143.

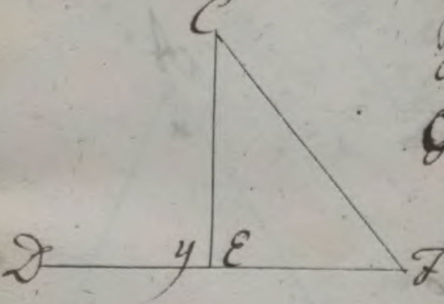
cumque $A = B = C$. §103

Ergo $3 \times \angle A = 2R$. §10. Ar.

Ergo $\angle A = \frac{2}{3} R$. §45. Ar.

§146. Corollarium 3.

In Triangulo rectangulo CEF non nisi Angulus unicus actu rectus est.



Sit enim si fieri possit.

$\angle F = R$ et

$\angle E = R$

$\angle F + E = 2R$. §42 Ar.

Sed $\angle F + E + C = 2R$ §143.

$\angle F + E = \angle F + E + C$. §41 Ar.

J. Q. E. A. per §47 Ar.

Aliter:

Sit $\angle E = R$.

$\angle F = R$. p. H. ass.

$\angle F + E = 2R$. §.

Ergo $CE \approx CF$. §133

Nullum igitur spatium terminabunt rectae CE, CF, EF.

J. Q. E. C. H.

§147. Corollarium 4

In Triangulo rectangulo CEF reliqui duo $\angle$li æquales sunt Recto junctim sumti, et uterq. acutus est. Quia enim

$$\angle C+E+F = 2R. \quad §143$$
$$\angle E = R. \quad p. H$$
$$\angle C+F = R \quad §43 \text{ Ax.}$$

Ergo $\angle F = R-C.$ } §c.
et $\angle C = R-F$ }

adeoq. acuti §40

Aliter.

Produc basin Trianguli rectanguli EF in D §82. Cum sit

$\angle E = R$ p. H.

Ergo et $\angle y = R$. §38.

Verum $\angle y > \angle$lo F } §113.
$\angle y > \angle$lo C }

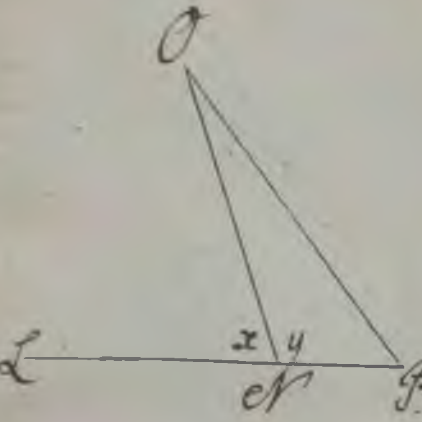
Ergo acuti §40.

§148. Corollarium 5.

Hinc in Triangulo rectangulo Angulus maximus est rectus §147.

§149. Corollarium

Et latus maximum Hypotenusa §115.



§150. Corollarium 7.

In Triangulo obtusangulo unicus tantum obtusus est. Nam sit

$\angle P >$ ior R. p. H. ap.
et $\angle y >$ ior R. p. H.

$\angle P + \angle y >$ res 2R. §42 Ar.
Sed $\angle P + y + O = 2R$. §143

$\angle P + y + O <$ res $P + y$ §46 Ari
f. Q. E. A. per §47. Ar.

Aliter.

Sint ut ante y et P duo obtusi ad Re tam NP p. H. Quoniam $\angle y + P >$ re 2R. §42. Ar.
Ergo divergent rectæ NO et PO ab ill parte, qua sunt $\angle$li duobus Rectis majores §140 nunquam spatium terminaturæ.

§151. Corollarium 8.

In Triangulo obtusangulo ONP duo reliqui $\angle$li sunt acuti
Producto enim latere PN in L §82. erit:

$x+y=2R$. §93.

$\angle y+\theta+P=2R$. §142.

$\angle y+\theta+P=x+y$. §.40 Ar.

rgo $\angle \theta+P=x$. §43 Ar.

cumque $\angle x+y=2R$ p.d.

et $\angle y > r\,R$. p. H.

$\angle x < r\,R$. §43 Ar.

$\angle \theta+P < res\,R$ §46 Ar. h.e.

$\angle \theta+P$ sunt acuti §40.

Aliter:

roducto NP in L. §82. erit

$x+y=2R$. §93.

$\angle y > r\,R$. p. H.

$\angle x < r\,R$. §43 Ar.

sed $\angle x = \angle \theta+P$ §142.

$\angle \theta+P < res\,R$ §46 Ar.

adeoque acuti §40.

§152. Corollarium 9.

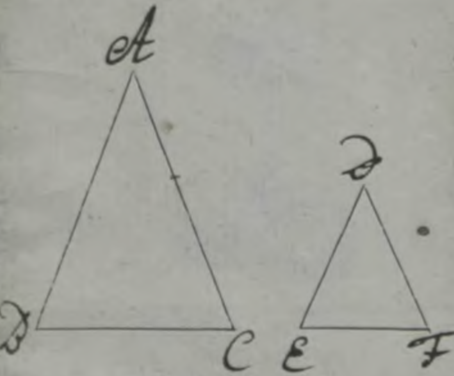
Hinc in Triangulo obtusangulo

lus obtusus est maximus §151

§153. Corollarium 10.

Latus autem maximum quod

lo obtuso opponitur §115.



§154. Corollarium 11.

Si in duobus Triangulis ABC et
DEF summa duorum Angulorum
unius A+B aequalis fuerit summae
duorum Angulorum alterius D+E
etiam reliquus ∠lus C aequalis est
reliquo ∠lo F eidem.

$$\left.\begin{array}{l} A+B+C = 2R. \\ D+E+F = 2R \end{array}\right\} \text{§143.}$$

$$A+B+C = D+E+F \text{ §41 Ar.}$$

$$A+B = D+E \text{ p. H.}$$

$$\angle C = \angle F \text{ §43 Ar.}$$

§156. Corollarium 12.

Inde etiam patet, si in duobus Tri
gulis duo ∠li unius A C B fuerint
aequales duobus ∠lis alterius D et
uterq; utriq; fore etiam tertium C
residuum aequalem tertio F residu

§156. Corollarium 13.

Et si in quocunq; Trigono duo ∠li
cogniti fuerint, notus quoq; est
tertius auferendo summam cogn
torum ex duobus Rectis.

§156. Corollarium 14.

Tandem si in Trigono aequicruro ADC Angulus Basi oppositus C datus fuerit, datur et Summa Angulorum ad Basin A+D: cumque sit A = ∠D §100 uterque datur bisecando summam.

Eodem modo patet si unus Baseos Angulorum A datus fuerit, angulum Basi oppositum C innotescere auferendo Summam Angulorum Baseos $A+D = 2 \times A$ e duobus Rectis.

Nam $A+D+C = 2R$. §143.

$\underline{A+D = 2 \times A}$ §100

$2 \times A + C = 2R$. §10 Ax.

Ergo $\angle C = 2R - 2A$ §43 A.

§158 Problema XII

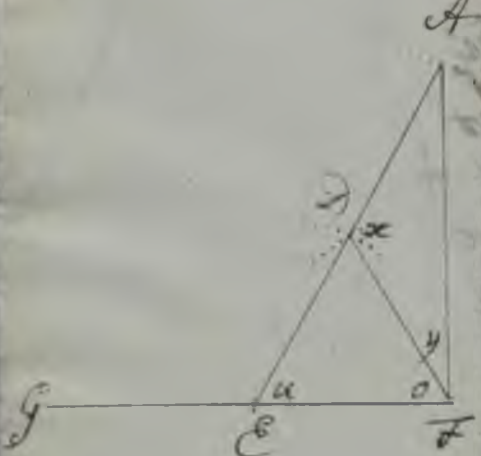
In Extremitate Rectae FG normalem excitare.

Resolutio.

1) Accipe Circino quodvis intervallum Rectae FG v.g. E·F.

2) Super illo describe Triangul
æquilaterum §96. EDF.
3) Producto latere ED §82.
4) Fac AD = DF. §26.
5) Duc AF. §81

DF.



Demonstratio.

$\triangle$ DEF est æquilaterum p. C.

Ergo $\angle o = \frac{2}{3}R.$ } 145.
et $\angle u = \frac{2}{3}R.$

$\angle o+u = \frac{4}{3}R.$ §42 Ar.
$\angle o+u = x.$ §142.

$\angle x = \frac{4}{3}R.$ §41 Ar

DA = DF. p. C.

Ergo $\triangle$ DAF est æquicrurum. §87.
Ergo $\angle y = \frac{1}{3}R.$ §157.
cumque $\angle o = \frac{2}{3}R.$ p. d

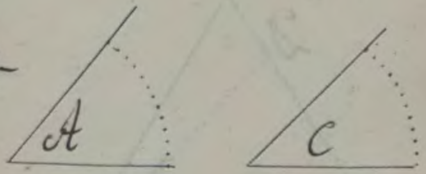
$\angle y+o = \frac{3}{3}R.$ §42 } Ar
$\frac{3}{3}R = R.$ §197.

sed $\angle o+y = \angle F.$ §47 } Ar.
$\angle F = R$ §41

Ergo AF ⊥lis ad GF. §44.
Q. E. D.

§ 159. Problema XIII

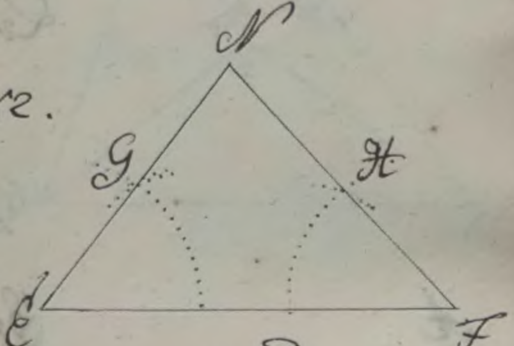
Datis Recta EF et Angulis A et C ipſi adjacentibus, qui junctim ſumti duobus rectis ſunt minores conſtruere Triangulum.



Resolutio & Demonſtratio.

1) Ad punctum E fac ∠lum A §107
2) Ad pctm F alterum C. §c.
3) Productis cruribus EG, FH. ad interſectionis pctm. N. §82.
4) Dico Triangulum EFN eſſe deſcribendum §141.

Q.E.F. et D.



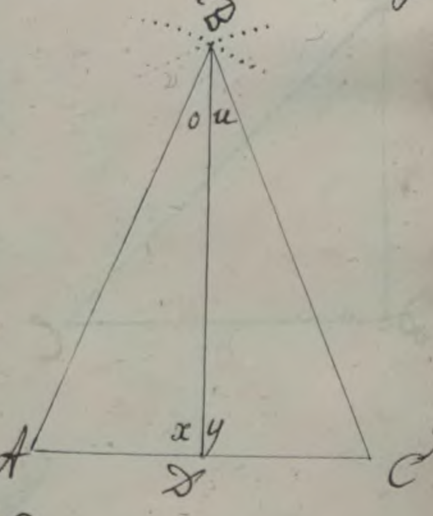
§ 169. Theorema 33

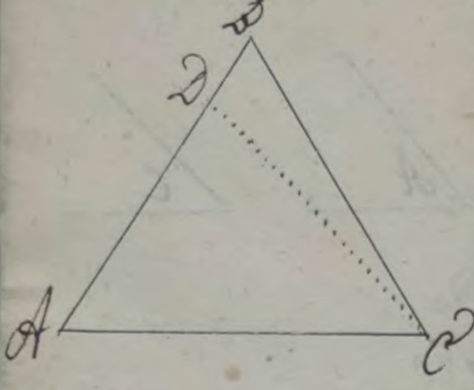
Si in Triangulo ABC ∠li ad Baſin ſunt æquales et ſub æqualibus ∠lis ſubtenſa latera æqualia ſunt.

Demonſtratio

Diſecto ∠lo B. §108. Duc BD. §81

Cum itaq; ſit ∠A = ∠C. p. H
∠o = ∠u p. C.
Ergo ∠x = ∠y §155
ſed BD = BD §30 A
AB = BC. §114. Q.E.D.





§161 Scholion.

Euclides Theorema hoc per Indirectum demonstrat L. 1. P. 6. h. m. Supponamus, manente tamen Angulorum A et C aequalitate fieri posse ut $AB > BC$ aut contra: facitaq.

$$AD = BC \quad §26$$

atque duc DC — §81.

Quia $AD \overset{b}{=} BC$ p. C.

$$AC = AC \quad §39\ A.$$

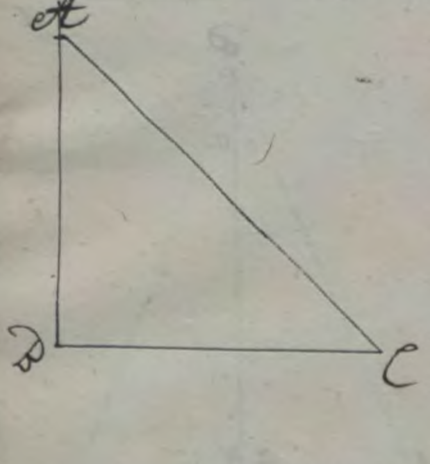
$$\underline{\angle A = \angle C.} \ \text{p. H.}$$

$$\triangle ADC = \triangle ACB \quad §99.$$

J. Q. E. A. p. §47 A.

§162. Corollarium.

Quodsi in Triangulo rectangulo ABC duo reliqui Anguli aequales fuerint, semirecti sunt et Triangulum rectangulum est aequicrurum §156. 57.



§163. Theorema 34.

Si super Trianguli ABC latere uno BC ab extremitatibus B et C duae rectae lineae BD et CD interius constitutæ fuerint, hæ constitutæ, reliquis Trianguli lateribus BA et AC minores quidem sunt, majorem autem Angulum continent.

Demonstratio.

Produc BD in E §82.

$CE + ED > DC$ §116.

$BD = BD$ §40. Ar.

$CE + ED + BD = DC + BD$ §42. Ar

h.e. $CE + ED > DC + BD$ §47. Ar

Porro:

$BA + AE > BE$ §116.

$EC = EC$ §40 A.

$BA + AE + EC > BE + EC$ §41. Ar.

h.e. $BA + AC > BE + EC$ §47. Ar.

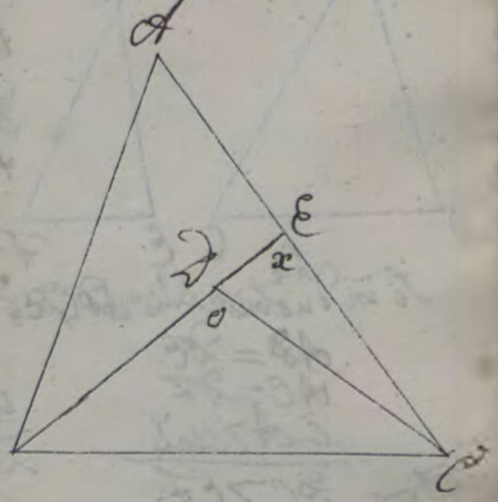
sed $CE + EB > DC + BD$ p.d.

$BA + AC > DC + BD$

Q.E.1.

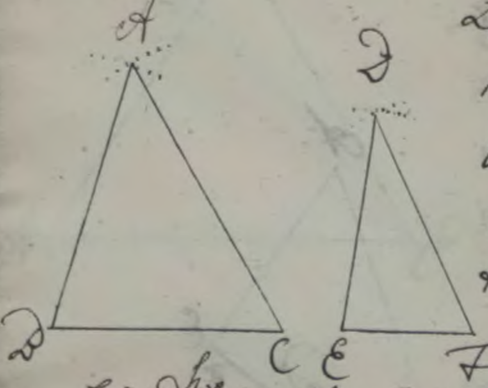
$\angle o > \angle lo\, x$ ⎫ §113.
$\angle x > \angle lo\, A$ ⎭

$\angle o$ multo major $\angle A$. Q.E.II.D.



§164. Theorema 35.

Si duo Triangula BAC, EDF duo Latera duobus Lateribus æquali habuerint, utrumque utrique, Angulum vero Angulo majorem sub rectis æqualibus Lineis contentum et Basin majorem habebun



Si in duobus h.e. Δlis ABC, DEF
AB = DE.
AC = DF
∠A > ∠lo D

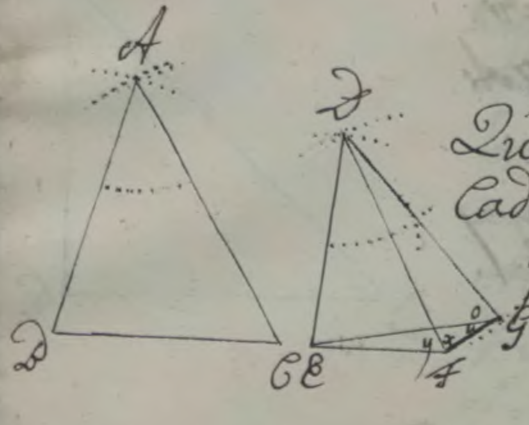
Ergo BC > EF.

Demonstratio.

Ad DE in pcto D fac p. §107
∠lum EDG = ∠lo A.
itemque DG = DF = AC §26.
Ductis itaque FG et EG §81.
Recta EG cadet vel
1) supra
2) in
3) infra } rectam EF.

Quare in casu 1mo
Cadat EG supra EF.
quia AB = DE
AC = DG } p. H et C.
~~[illegible]~~
∠A = ∠EDG p. C.

BC = EG. §99.



Est autem DF = DG p. C.
Ergo ∠x = ∠u+o. §100 G. et 47 Ar.
∠u ∠r ∠lis° u+o §47. A.

∠x ⌝r ∠lo u. §46. Ar.
Sed ∠x ∠r ∠lis° x+y §47. Ar

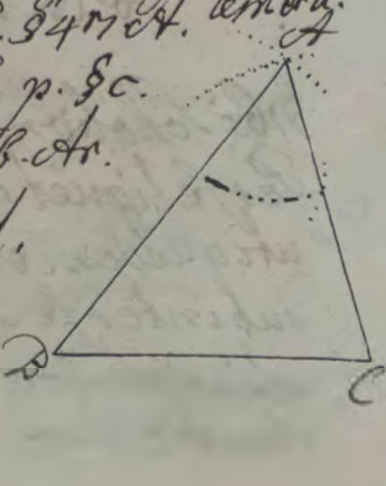
∠u multo minor ∠lis° x+y
i.e. ∠u multo minor ∠F §47. Ar
Ergo in Triangulo EFG.
Latus° EG ⌝ EF §115
Verum EG = DC p. d.

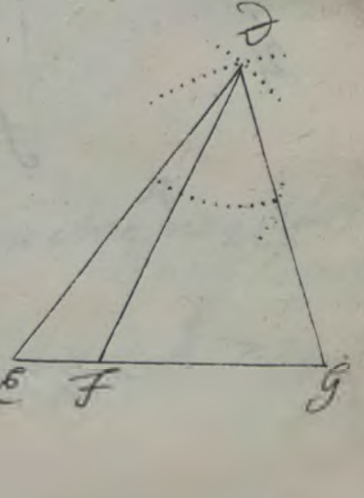
DC ⌝ EF. §46. Ar
Q.E.I.

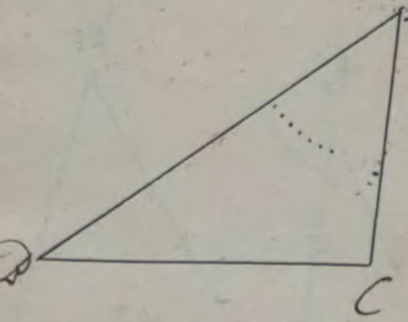
In Casu IIdo
Cadat EG in EF.
Quia ante DC = EG p. C. genera
et EG = EF + FG. §47. A. lemdd.
adeoque EF ∠ EF + FG. p. §c.

EF ∠ EG §46. Ar.
Sed EG = DC. p. d.

EF ∠ DC.







In Casu III^{tio}

Cadat EG infra EF.

Produc rectas DF et DG in H et I.

§82. quia ap. C. et D generalem

DC = EG

DF = DG p. C.

Ergo ∠u = ∠FGI. §100

Sed ∠FGI $\urcorner$r ∠lo FGE §47 Ar

∠u $\urcorner$r ∠lo FGE §46. Ar.

Sed ∠u ∠r ∠lis u + o §47. Ar

Ergo ∠FGE multo minor ∠u + o

h. e. ∠FGE m. minor ∠F.

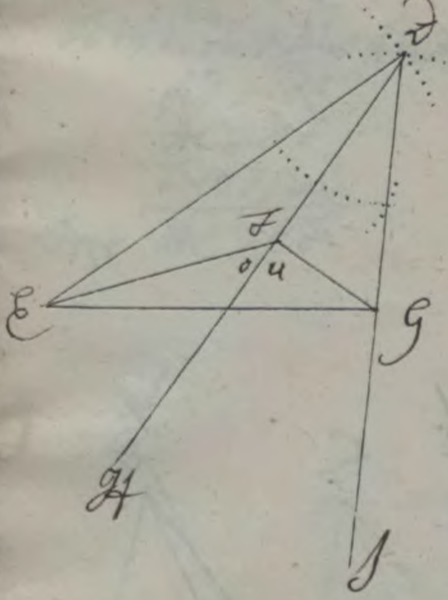
Quare in Triangulo EFG.

Latus EG $\urcorner$ EF. §115.

DC = EG p. d.

DC $\urcorner$ EF. §46. Ar.

Q. E. III. D.

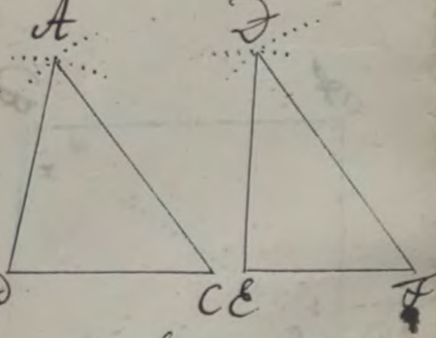


§165 Scholion

Per se liquet hoc ipso theoremate uti quiddem et Hypothesis eloquitur duo inter se △la comparari alias enim coincidit Demonstratio cum §115.

§166. Theorema 36.

Si duo Triangula ABC, DEF duo latera et BA, AC duobus Lateribus DE, DF aequalia habuerint utrumque utrique, Basin vero BC, Basi EF majorem et ∠lum A sub aequalibus rectis lineis contentum angulo D majorem habebunt.



h. e.

Si in duobus ∆lis ABC, DEF
$BA = DE$
$AC = DF$
Sed $BC > EF$
erit ∠A > ∠lo D.

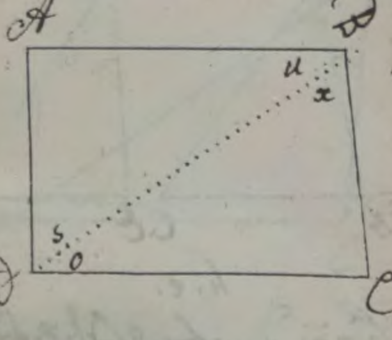
Demonstratio.

Dantur tres Casus, aut enim
1) ∠A = ∠D
2) ∠A < ∠lo D
3) ∠A > ∠lo D Hinc in

Casu 1mo
Sit ∠A = ∠D p. H. ass.
quia $BA = DE$ } p. H.
$AC = DF$ }
Ergo $BC = EF$. § 99. q. E. C. H.

Casu 2do
Sit ∠A < ∠lo D. p. H. ass.
quia $BA = DE$ } p. H.
$AC = DF$ }
$BC < EF$ § 164 c. H. quo Basin BC majorem Basi EF ponit.

Quia itaque neque Casus 1mus nec Casus IIdus
Ergo utique ∠A > ∠lo D.
Q. E. D.



§167. Theorema 37.

In Parallelogrammis ABCD later opposita sunt aequalia AB et CD item AD et BC. Etsi in figuris quadrilateris ABCD latera opposita fuerint aequalia, sunt illa Parallelogram̄a

Demonstratio.

Duc Diagonalem DB §81.74.

ABCD est Pllgmum p. H.

Ergo $AB \cong DC$, $AD \cong BC$ §72.

Ergo $\angle o = \angle u$, $\angle s = \angle x$ §133.

sed $DB = DB$ §40 Ar.

$AB = DC$, $AD = BC$ §105

Q. E. I.

$AB = DC$, $AD = BC$ p. H.

$DB = DB$ §40. Ar

$\angle o = \angle u$, $\angle s = \angle x$ §106.

Hinc $AB \cong DC$, $AD \cong BC$ §133.

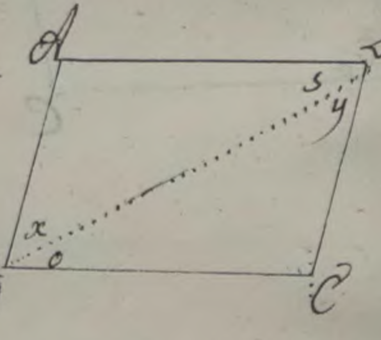
Ergo ABCD est Pllgmum §72.

Q. E. IIdo

§168. Corollarium.

Quia in Quadrato, Oblongo, Rhombo et Rhomboide Latera opposita sunt æqualia §68–71 erunt Quadratum, Oblongum, Rhombus et Rhomboides Parallelogram̄a. §167.

§169. Theorema 38.



Diagonalis dividit Parallelogramma in duas partes æquales. Anguli diagonaliter oppositi sunt æquales. Anguli vero ad idem Latus oppositi sunt æquales 2 Rectis, et tandem duo qualibet Latera sunt Diagonali majora.

Demonstratio

ABCD est Plgm p. H.

Ergo $\left.\begin{array}{l} AB = DC \\ AD = BC \end{array}\right\}$ §167

et DB = DB §40.

$\triangle DAB = \triangle BCD$. §106.

Q. E. 1.

Dmdm.

1) $\triangle ABD = \triangle DCB$

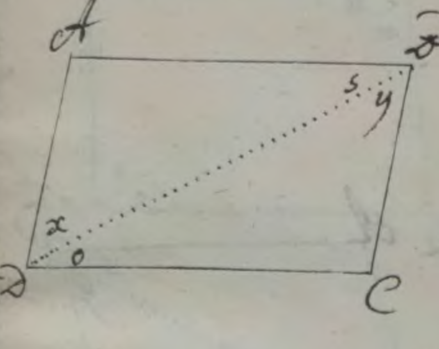
2) $\angle D = \angle B$ et $\angle A = \angle C$.

3) $\left.\begin{array}{l} B+C \\ C+D \\ D+A \\ A+B \end{array}\right\} = 2R.$

4) $AB + AD > DB$

Quia $\triangle ADB = \triangle DCB$ p.d.

$$\left.\begin{array}{l}\angle A = \angle C\\ \angle o = \angle s\\ \angle x = \angle y\end{array}\right\}\ \S 106.$$



cumque $\angle o + x = \angle s + y$ §42. A.
Ergo $\angle D = \angle B$. §42. A.

Q.E. II.

Quia $AB \approx DC$. p.H.
Ergo $\angle D + C = 2R$. §132.
sed $D = B$ p.d.

$\angle B + C = 2R$. §10. A.
sed $\angle C = A$. p.d.

$\angle B + A = 2R$. §10. A.
cumque $B = D$. p.d.

$\angle D + A = 2R$. §10 A.

Q.E. III

Tandem

$$\left.\begin{array}{l}AB + AD > DB\\ DC + CB > DB\end{array}\right\}\ \S 116.$$

Q.E. IV. D.

§170. Problema XIV.

Super data Recta AB terminata Quadratum construere.

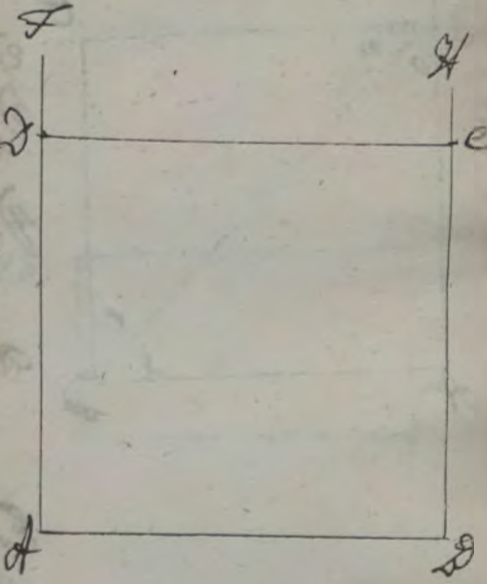
Resolutio.

1) In extremitatibus A et B propositae rectae, excita ⊥les AF et BH. §158.

2) Fac AD = BC = AB. §26.

3) Duc DC §41.

Dico ABCD esse Quadratum.



Demonstratio.

BH ⊥ ad AB }
DA ⊥ ad AB } p. C.

Ergo DA ≈ CB §138.

Sed DA = CB p. C.

AB ≈ DC }
et AB = DC } §139.

Sed AB = BC p. C.

Ergo AB = BC = DC = CA §41. A.

adeoq ABCD est Pllgm. §167.

Ergo ∠∠ B + A = 2R. §169.

Sed ∠ A = R. p. C.

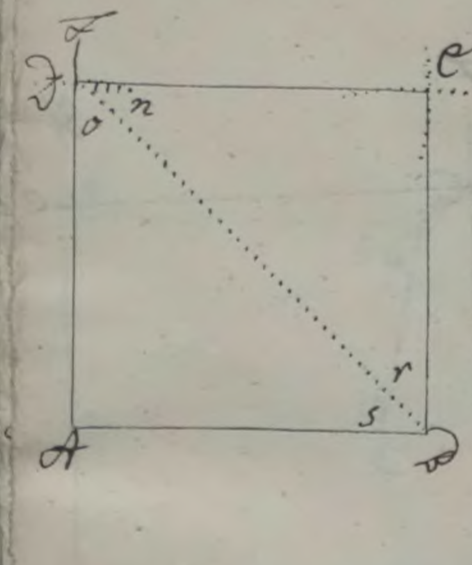
∠B = R. §43. Ar.

Cumq ∠A = ∠C = R §169. §92.

et ∠B = R. p. C.

Ergo ∠A = ∠B = ∠C = D. §92.

Adeoq ABCD est Quadratum §68. Q. E. D.



Aliter.

1) In Extremitate A rectae AB excita ⊥lem AF §158.

2) Fac AD = AB §26 et

3) Centris A et B eodem radio AB et tra um intersectiones in C §83.

4) Duc DC et CB §81

D. T.

Demonstratio.

AB = AD = DC = CB p. c.

Ergo ABCD est Pllgm §162.

Duc diagonalem DB. §87

Ergo △ DAB = △ DCB. §169

∠A = ∠C. §106.

Sed ∠A = R.

∠C = R. §92.

Cumque

DA = AB et CD = CB p. c.

∠o = ∠s = ½ R. §100. 162.

∠n = ∠r = ½ R. §§. c. c.

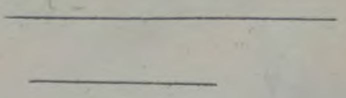
∠o+n = ∠s+r = R. §42. Ar

h. e. ∠D = ∠B §47. Ar et 92.

Ergo ABCD est Quadratum §60. Q. E. D.

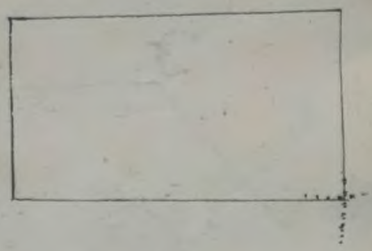
§171. Problema. XV

Datis duabus rectis AB et AC Oblongum construere.



Resolutio.

1) Junge AB et AC ad ∠ R. §158.
2) Centro C radio AB fac Arcum.
3) Factum interseca centro B radio AC §83
4) Duc DC et DB. §81. D. f.



Demonstratio.

$AB = CD \} p.C.$

$CA = DB \}$

Ergo AD est Pllgm §167.

Ergo A + B = 2R. §169.

Sed A = R. p.C.

∠B = R. §43. Ar.

∠B+D = 2R. §169.

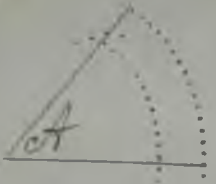
∠D = R. §43. A.

∠D+C. = 2R. §169.

∠C = R. §43. Ar.

Ergo Figura descripta est oblongum §70.

Q.E.D.



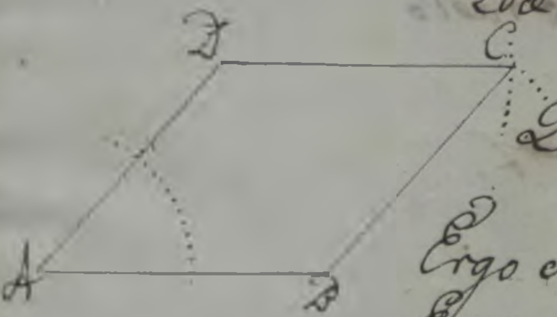
§172. Problema XVI

Data recta AD et ∠lo obliquo Rh[illegible]bum construere.

Resolutio.

1) Ad rectam AD fac ∠lum A. §107.

2) Reliqua fac ut mbr. 2.3.4. Resolutio 2da §170. precipimus.



Demonstratio

Quia $DC = AD$ ⎫ p.C.
$AD = BC$ ⎭

Ergo ABCD est Pllgm §167

Ergo et $A + B = 2R$. §169.

Sed $\angle A =$ obliquo p.H.

$\angle B =$ obliquo

Simili ratiocinio ostendetur ∠los C et D esse obliquos. Inde quidem Figura descripta est Rhombus. §69. Q.E.D.

§173. Problema XVII

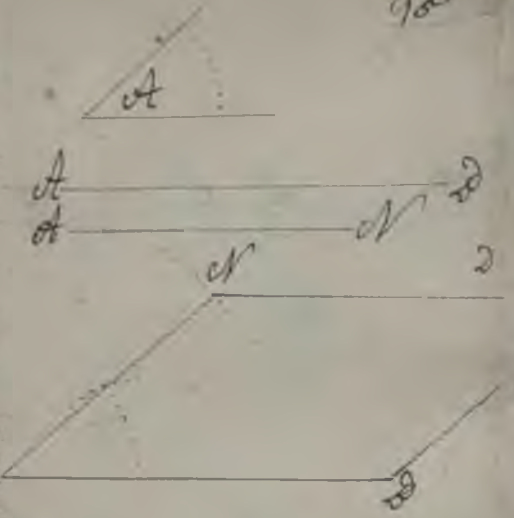
Datis duabus Rectis DC et AD et ∠lo obliquo A Rhomboidem construere.

Resolutio.

1) Ad Latus AD constitue ∠um A §107.
2) Fac AN = AN et
3) Reliqua ex legibus Mbr. 2.3.4. Resolutionis §171. D. F.

Demonstratio.

Eadem est quae §172. Q. E. D.



§174. Theorema 39.

Parallelogramma BD et BF super eadem Basi BC et in iisdem Parallelis BC et AF constituta sunt inter se aequalia. s. q. i. e. Parallelogramma BD et BF super eadem Basi BC et ejusdem Altitudinis sunt inter se aequalia.

Demonstratio

AD = BC } p. H.
BC = EF }

AD = EF §41 Ar.
DE = DE §40 Ar.

AE = DF §42 Ar.

Porro:

$$\left.\begin{array}{l} AB = DC \\ \text{et } BE = CF \end{array}\right\} \begin{array}{l} §.17 \\ \text{p. H.} \end{array}$$

$\Delta\, ABE = \Delta\, DCF$. §106.

$\Delta\, DGE = \Delta\, DGE$. §40 Ar

$\Delta\, ABE - \Delta\, DGE = \Delta\, DCF - \Delta\, DG$

h. e Trapez ABGD = Trapezio CGEF.

$\Delta^{r}\, GBC = \Delta^{r}\, GBC$. §40 Ar

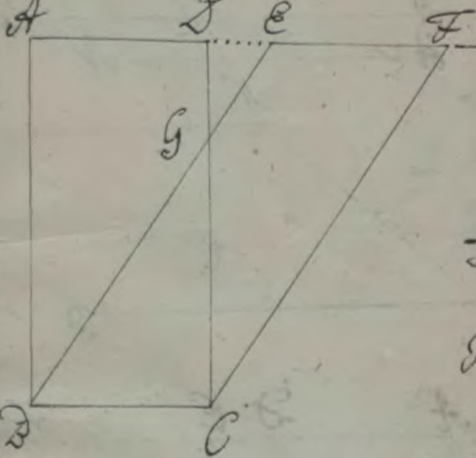
Tr ABGD + GBC = Tr. CGEF + GBC. §47

h. e.

Pllgm ABCD = Pllgo BCFE.

§47. Ar.

Q. E. D.



§175. Scholion.

Quare, si Latus AB Parallelogrammi rectanguli fieri intelligatur perpendiculariter per totam BC aut conversim BC per totam AB prodibit eo motu Area Rectanguli AC. Hinc dicitur fieri Rectangulum ex multiplicatione duorum Laterum contiguorum. h. e. AB × BC. cf. Barrow Eucl. L. I. Prop. 35

Quod si supponas cujuslibet Parallelogrammi DF dimensionem invenies. Cum enim

$DF = AC$. p. §174.

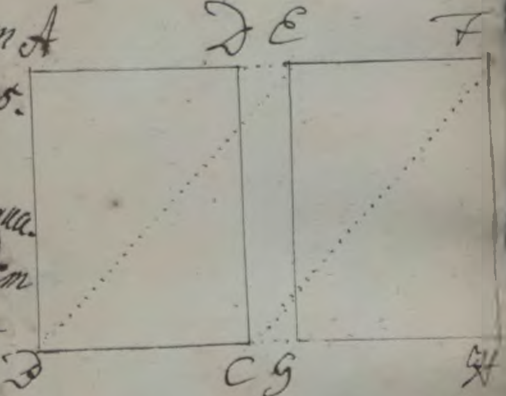
et $AD \times DC = AC$. p. d.

Ergo $DF = AD \times DC$ §41. Ar.

h. e. Area cujuslibet Parallelogrammi obliquanguli, aequatur facto ex Altitudine in basin alterius rectanguli intra easdem Parallelas constituti ejusdem baseos.

§176. Theorema. 40.

Parallelograma DE et FG super aequalibus basibus DC et GH atq. in iisdem parallelis DH et AF constituta sunt aequalia.



Demonstratio.

Duc DE et CF. §81.

$DC = GH$. p. H.

$EF = GH$ p. H.

$DC = EF$. §41 Ar.

Sed $DC \approx EF$ p. H.

Ergo DE et CF = et ≈ la §139.

*

* Ergo DF est Pllgm §167 aut 72.

Ergo.

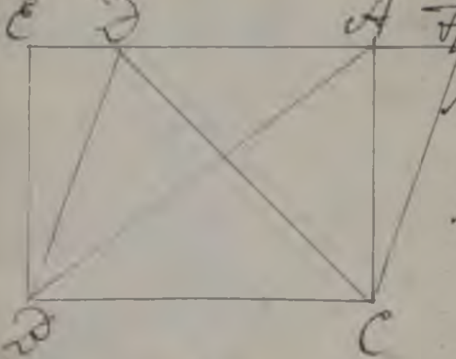
$DF = DD$ } §174.
et $DF = EH$ }

$DD = EH$ §41 Ar.

Q. E. D.

§177. Theorema 41

Triangula BCA et BCD ſuper eadem Baſi BC conſtituta atq intra eaſdem Parallelas BC et ſunt inter ſe æqualia



Demonſtratio.

Per B age parallelam cum AC npr BE. §135.

Ergo $\triangle$ BCA $= \frac{1}{2}$ Pllgo CE. §16

et $2\times\triangle$ BCA $=$ Pllgo CE. §44 A

Per C age ≈lam CF cum BD §135

Ergo $\triangle$ BDC $= \frac{1}{2}$ Pllgo BF. §16

et $2\times\triangle$ BDC $=$ Pllgo BF. §44

Eſt autem BF $=$ CE §174.

cumq CE $= 2\times\triangle$ BCA

et BF $= 2\times\triangle$ BDC } p.d.

$2\times\triangle$ BCA $= 2\times\triangle$ BDC §41. Ar.

$\triangle$ BCA $= \triangle$ BDC. §45. Ar.

Q.E.D.

§178. Theorema 42.

Triangula BCA et CFD ſuper æqualibus Baſibus BC et GF conſtitu et in iiſdem ≈lis GA et BF inter ſe ſunt æqualia.

Demonstratio.

Duc BG ≈ cum AC. §135.

eritq; Trigonum $BCA = \frac{1}{2}$ Pllgo GC. §169.

et $2\times\triangle BCA =$ Pllgo GC. §44. Ar.

Duc HF ≈ cum DE. §135.

erit uti ante.

$2\times\triangle EDF =$ Pllgo EH. §169.

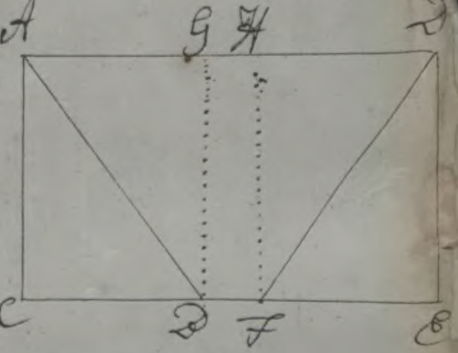
Verum $GC = EH$ §176.

$$\left.\begin{array}{l}\text{Cumq } GC = 2\times\triangle BCA \\ \text{Et } EH = 2\times\triangle EDF\end{array}\right\}\text{p.d.}$$

$2\times\triangle BCA = 2\times\triangle EDF$. §41. Ar.

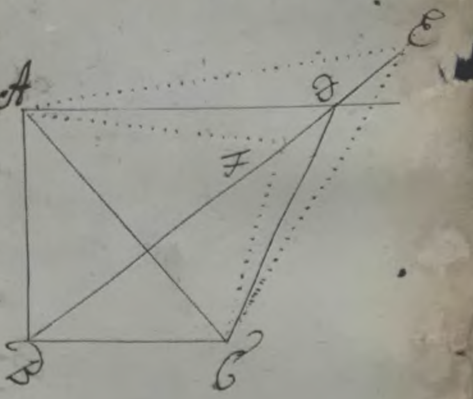
$\triangle BCA = \triangle EDF$. §45. Ar.

Q.E.D.



§179. Theorema 43.

Triangula aequalia ABC et DBC. super eadem basi BC et ad easdem partes constituta sunt etiam in iisdem ≈lis BC et AD.



Demonstratio.

Dantur tres casus aut enim

1) AD ≈ BC aut

2) AF ≈ BC aut

3) AE ≈ BC ita scilicet ut pctm. D vel supra D cadat in E vel infra

in F quo tandem aut AE aut AD aut AF sit ≂la ipsi BC.

Ponamus ergo in casu

I^mo supra cadere et coire cum DB in E. Duc EC §81.

Quia AE ≂ BC p. Hyp. erit $\Delta BAC = \Delta BEC$ §177.

Sed $\Delta BAC = \Delta BDC$ p. Hyp. Theor.

$\Delta BEC = \Delta BDC$. §41. Ar.

$\Delta BEC = \Delta BDC + DEC$ §47. Ar.

$\Delta BDC = \Delta$lis $BDC + DEC$ §41 Ar.

J. Q. E. A. p §47 Ar.

II^do Parallelam cum BC ductam per A infra cadere in F hoc est esse AF. Duc FC. §81. Ergo

$\Delta ABC = \Delta BFC$. §177.

$\Delta ABC = \Delta BDC$ p. Hyp.

$\Delta BFC = \Delta BDC$ §41. Ar.

$\Delta BFC = \Delta BDC - \Delta FDC$

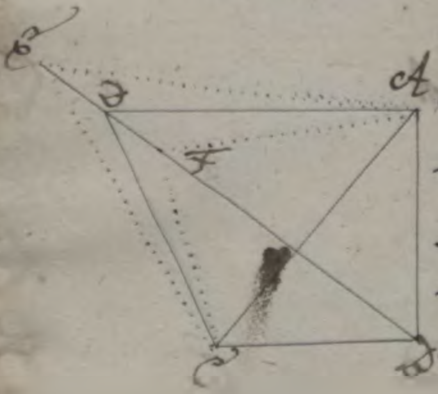
$\Delta BDC = \Delta BDC - \Delta FDC$. §41 Ar.

J. Q. E. A. p. 47 Ar.

Quare cum neq AE ≂ BC (p. d.)
neq AF ≂ BC

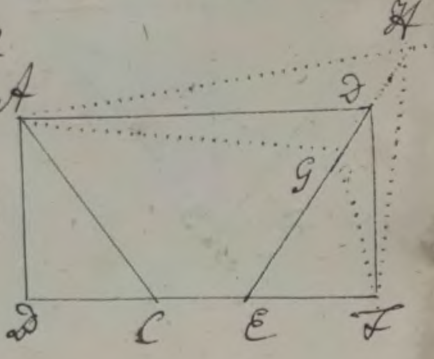
Ergo omnino AD ≂ BC.

Q. E. D.



§180. Theorema 44

Triangula æqualia BCA et EFD super æqualibus basibus BC et EF et ad easdem partes constituta sunt in iisdem ≈lis



Demonstratio.

Si negas AD ≈ BF. dabitur alia recta
vel AH ≈ BF
vel AG ≈ BF In casu ergo

I Produc ED in H §82
et duc rectam HF. §81 eritque

$\triangle$ ABC = $\triangle$ EHF §178.
$\triangle$ ABC = $\triangle$ EDF p. Hyp.

$\triangle$ EHF = $\triangle$ EDF §41. Ar.
J. Q. E. A. p §47. Ar.

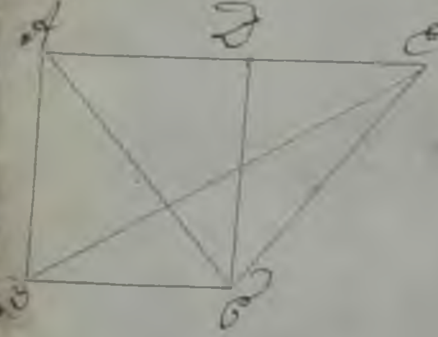
II $\triangle$ ABC = $\triangle$ EGF. §178
$\triangle$ ABC = $\triangle$ EDF p. Hyp.

$\triangle$ EGF = $\triangle$ EDF §41 Ar.
J. Q. E. A. p §47 Ar

Cum itaque neque AH ≈ BF } p. d.
neque AG ≈ BF }

Ergo omnino AD ≈ BF.

Q. E. D.



§181 Theorema 45.

Si Parallelogrammum BDum Triangulo BCE eandem basin BC habuerit, sinq; iisdem =lis fuerit AE et BC, duplum erit Pllgm BD ipsius Δ BCE.

Demonstratio

Duc rectam AC. §81.

Ergo $2 \times \Delta$ ABC $=$ Pllgo BD. §169.

Δ ABC $= \frac{1}{2}$ Pllgo BD. §45 Ar.

sed Δ ABC $= \Delta$ BCE. §177.

$\frac{1}{2}$ Pllgm BD $= \Delta$ BCE. §41 Ar.

Pllgm BD $= 2 \times \Delta$ BCE §44 Ar.

Q.E.D.

Vel paullo brevius.

Ducta ut ante AC §81.

erit $2 \times \Delta$ ABC $=$ Pllgo BD. §169

sed Δ ABC $= \Delta$ BCE. §177

$2 \times \Delta$ BCE $=$ Pllgo BD. §41 Ar.

Q.E.D.

§182. Scholion.

Inde quidem facillimo negotio producitur Area Trigoni cujuslibet. Cum enim Area Pllgmi

BD producatur ex altitudine in Baſin §175. hoc ſi factum biſecetur, inventa erit Area Trigoni §181. Quare in genere ſi Baſis = b.

Altitudo = a erit.

$$\text{Area Trigoni} = \frac{a \times b}{2} \left. \begin{aligned} &= \frac{a}{2} \times b \\ &= a \times \frac{b}{2} \end{aligned} \right\} \text{§185. Ar.}$$

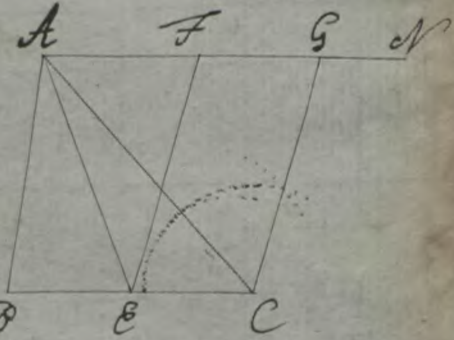
§183. Problema XVIII

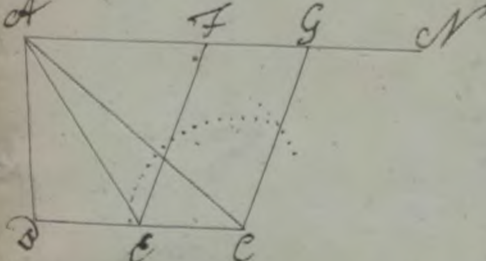
Dato Trigono ABC æquale Prallelogrammum EG conſtituere in dato Angulo rectilineo D.

Reſolutio.

1) Per A duc AN ≏lam ipſi BC §135.
2) Ad C conſtitue ∠lum BCG dato D æqualem §107.
3) Biſecta Baſi BC in E. §112.
4) Per E age EF ≏lam ipſi CG §135.

D. F.





Demonstratio

∠lus ECG = ∠D p. h.

A praeterea EC ≈ FG } p. c.
CG ≈ EF }

EG est Pllgmum §72.

Duc rectam AC. §81.

Ergo.

2×ΔAEC = ▱EG. §181

Sed ED = EC p. c.

Ergo ΔADE = ΔAEC §178.

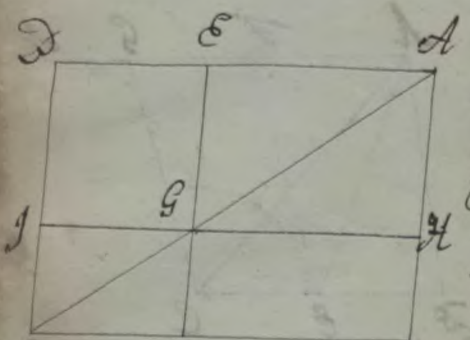
ΔADC = ΔADE + AEC.

ΔADC = 2×ΔAEC. §10. Ar.

Ergo

ΔADC = Pllgo EG. §41 Ar

Q. E. D.



§184. Theorema 46

In omni Parallelogrammo DB complementa DG et GB eorum quae circa diametrum AC sunt Parallelogrammorum HE et FI inter se sunt aequalia

Demonstratio

BD est Pllgm p. H.

Ergo ΔACD = ΔABC. §169.

HE eſt Pllgm p. H.
△ AGH = △ AEG &c.

△ ACD − △ AGH = △ ADH − △ AEG. §43. Ar.
h. e. Trap. HGCD = Trp. EGCD.
Sed et FD eſt Pllgm p. H.
Ergo △ GFC = △ GCI §169.
Ergo Trap. HGCD − △ FGC = Trp. EGCD − △ GCI. §43. Ar.
h. e. Pllgm DG = Pllgo GD
Q. E. D.

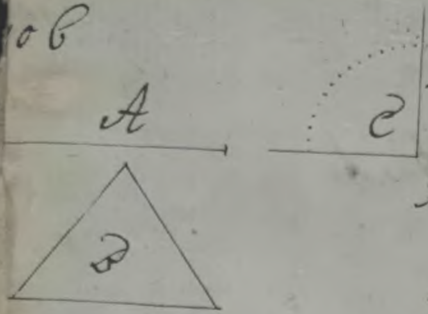
§185. Problema XIX

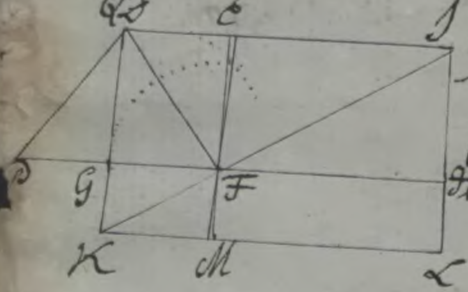
Ad datam rectam Lineam A, dato Triangulo D æquale Pllgm MH applicare in dato ∠lo rectilineo. C. ſ. qi. c.
Data ſit recta Linea A, datum triangulum D et ∠lus Rectilineus C oportet conſtituere Parallelogrammum, æquale Triangulo D habens ∠lum æqualem ∠lo C et unum Latus æquale Lateri A dato. ita Clavius ad Prop. 44. L. 1. Eucl.

Reſolutio

1) ſumto △lo D = FPQ. §98.

0 b





2) Fac Parallelogrammum aequale sub ∠lo C, GE. §183.

3) Lateri FG pone indirectum FH = A §82.83.

4) Per H age IL ≈ cum EF §135 cui occurrat DE producta in I. §82

5) Per IF ducta recta occurrat DG producta in K §82.

6) Per punctum K duc KL ≈ GH §135. cui occurrant EF et IH in M et L §82 producta

Dico MH esse Pllgm quaesitum

Demonstratio

EKI est Pllgm p.C.
Ergo MH = FD §184
Sed FD = ΔFPQ. p.C.
MH = ΔFPQ. §41 Ar

Porro.
∠GFE = ∠C p.C.
∠GFE = ∠MFH §94
∠C = ∠MFH. §41 Ar.
Cumque et FH = A p.C.

✱
Indequidem Pllgm MH
habet 1) Aream = Δlo B
2) Latus FH = A
3) ∠MFH = ∠lo C.
Q.E.D.

Ad §

Sunto Polygona P, Q R similia similiter descripta super Rectas AB, CD, EF continue ppales.

Quia $AB : CD = CD : EF$ p. H.

Ergo $AB : EF = AB^2 : CD^2$ § 189 Ar.

Sed $AB^2 : CD^2 = P : Q$. § 377.

Ergo $P : Q = AB : EF$. § 144 Ar.

Q.E.D.

Quia $AB : CD = CD : EF$ p. H.

Ergo $AB^2 : CD^2 = CD^2 : EF^2$ § 187. Ar.

Et $AB : EF = AB^2 : CD^2$ § 189 Ar

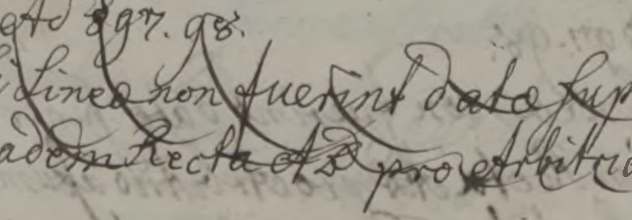
Sed $CD^2 : EF^2 = Q : R$. § 377.

Ergo $Q : R = AB : R$. § 144. Ar.

Q.E.D.

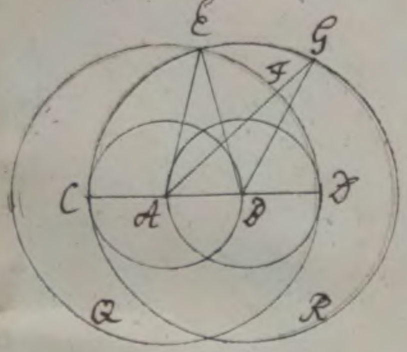
Ad §97. 98.

Si lineae non fuerint datae ſuper eadem Recta AB pro Arbitrio



Ad §97.98.

Si Lineæ non fuerint datæ ſuper eadem Recta AD pro Arbitrio aſſumta deſcribetur Triangulum et æquicrurum et ſcalenum hm



1) Centris A et D Radio AD deſcribe Circulus. §83.

2) Produc Rectam AD utring: in C et B §82 ad Interſectionem ſaſcum Sphia.

3) Centro A Radio AD et Centro D Radio CD deſcribe Circulos ſemet invicem ſecantes in E. §83.

4) Junctis EA et ED §81
Dico Δlum AED eſſe æquicrurum

Porro.

5) Duc Rectam quamvis ex A quæ non cadat in mutuam Interſectionem Circulorum Radiis CD AD deſcriptorum ſecantem Sphiam EDQ in F ad Sphiam alteram ECR in G respect.

6) Junctis G et D §81
Dico Δ ADG eſſe Scalenum.

Demonstratio

Membrum I.

$AB = AB$ §40 A.

$CA = BD$. §26

$CB = AD$ §42 A.

Sed $CB = BE$ }
et $AD = AE$ } §26

$AE = BE$ §41 Ar.

Sed $CB > AB$ §25.

$CB = EB$ p. d

$AB < BE$ }
et $AB < AE$ } §46. Ar.

Ergo △lum EAB aequicrurum §57.

Q.E.I.

Membr II.

$AF = AE$ §26

$AE = EB$ p. d

$EB = BG$ §26

$AF = BG$ §41 Ar.

Sed $AF < AG$ §47 Ar

$AG > BG$. §46. Ar

Porro $AB < EB$ p. d.

$EB = BG$ p. d

$AB < BG$. §46 A.

Ergo △ ABG Scalenum §58 Q.E.II D

Proclo utramq. Propositionem
Clavius ad Eucl. L. 1. Prop. 1.
Euphorbo Phrygi posteriorem
Cl. Richardus ad Eucl. l. c.
Auctoritate Diog. Laërt.
p. m 17 inductus tribuit

ad pag 108,, 115

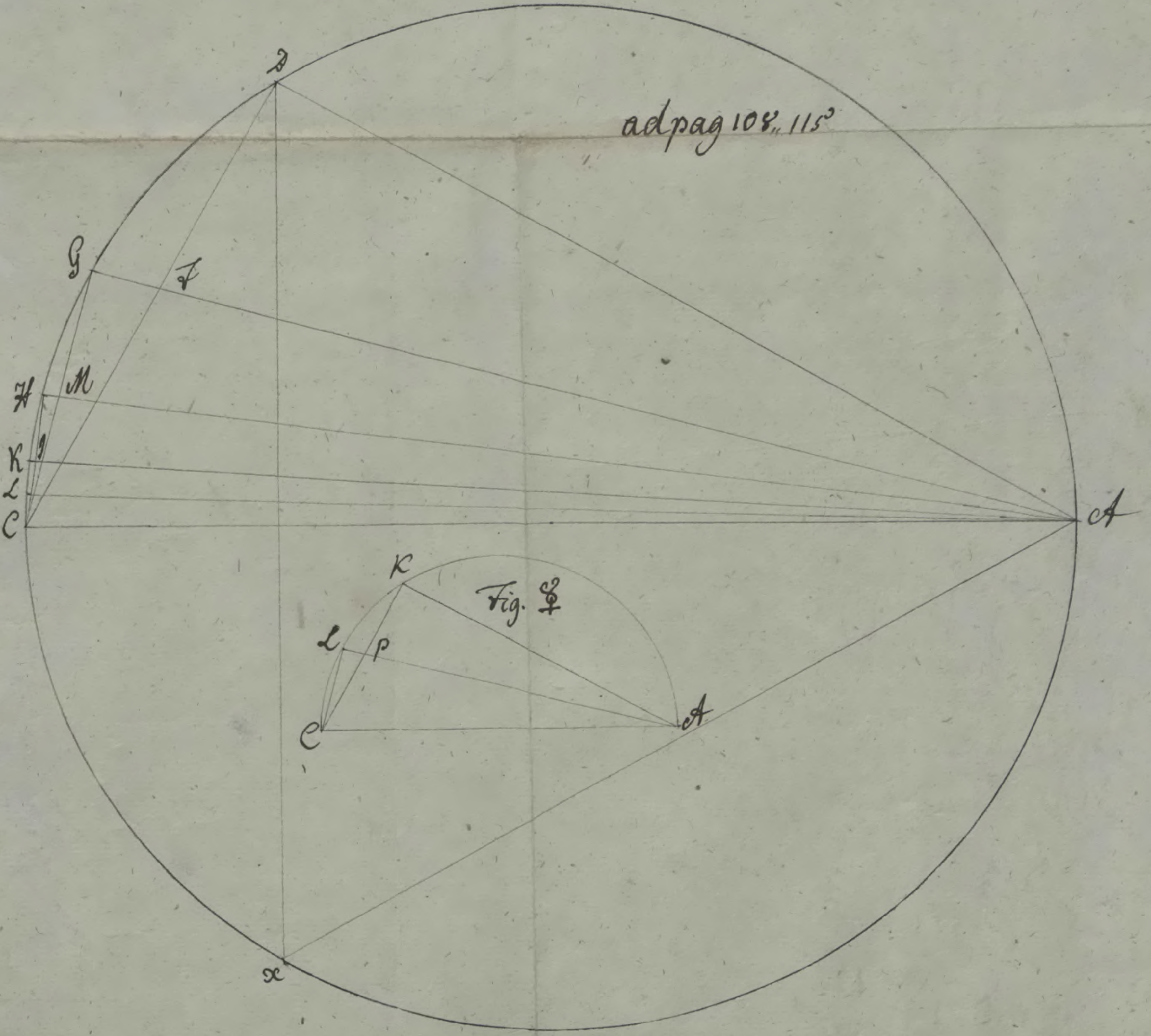
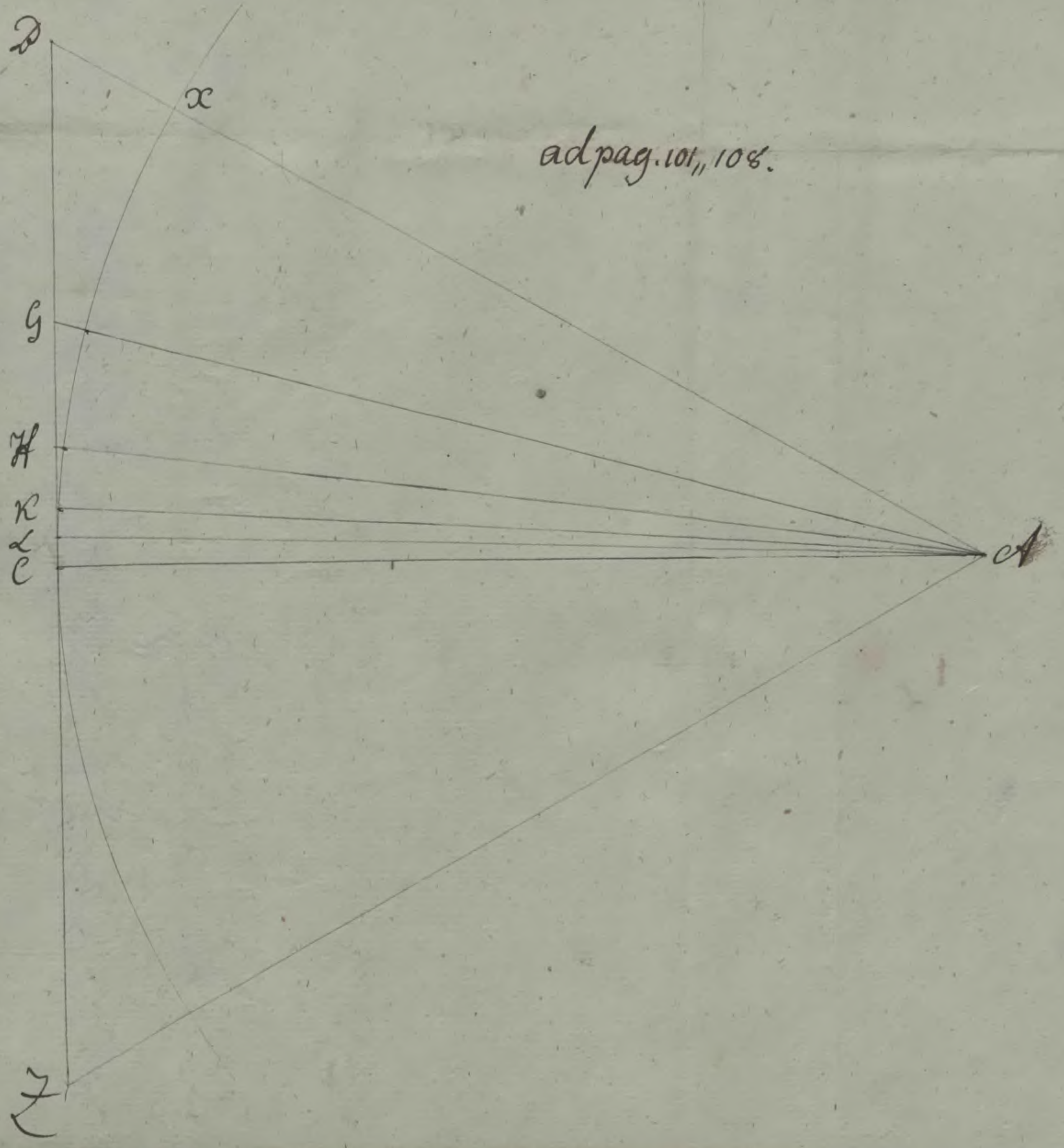
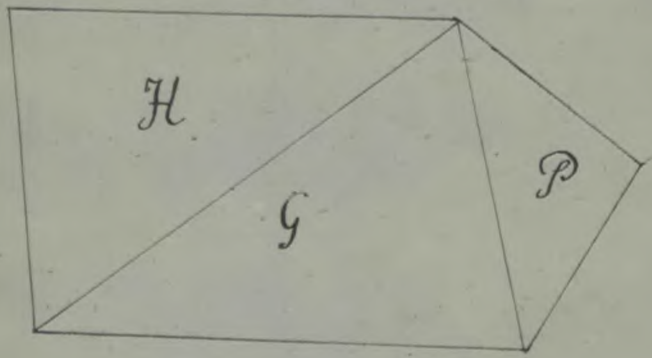
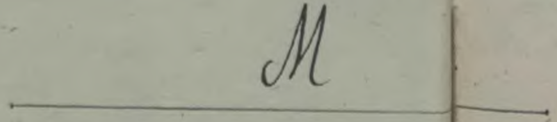


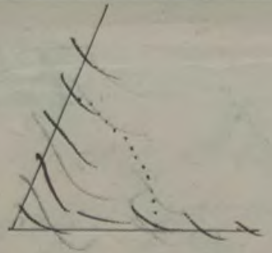
Fig. ♀

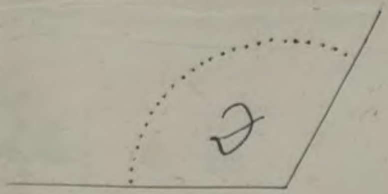


ad pag. 101„ 108.


H
G
P


M

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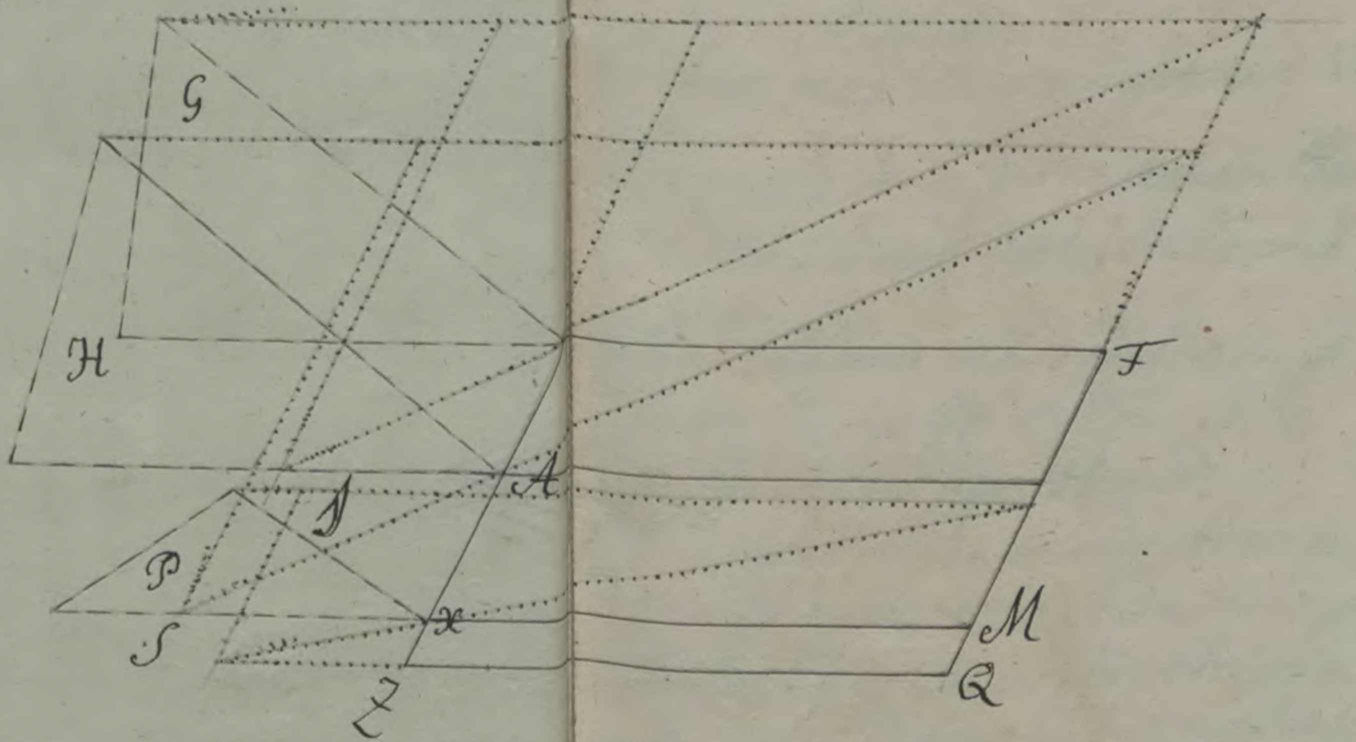
G
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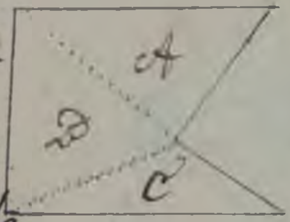
§186 Problema XX

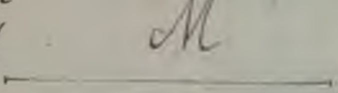
Ad datam rectam lineam M dato rectilineo ADC æquale Pllgm constituere in dato ∠lo rectilineo D.



Resolutio.

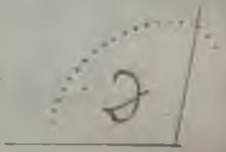
1) Datum rectilineum resolve in Δla
§81

2) Triangulo A constitue æquale Pllgm FH sub ∠lo dato D et latere EF = M

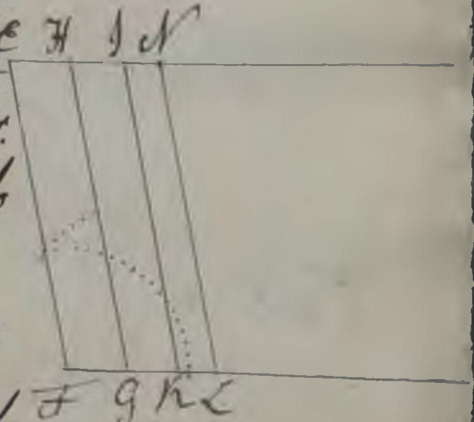


§185.

3) Idem eodem modo fiat cum D super Recta GH = EF = M. &c. et ∠lo ut sit Pllgm GI = D.



4) Eadem Methodo procede etiam cum C ut sit KN = C &c. et ita deinceps, si Rectilineum in plura Triangula ductis diagonalibus resolvatur. Dico Pllgm FN = Rectilineo dato ADC sub ∠lo dato D.



Demonstratio

Cum ex ipso §185. pateat EG, HK, IL esse Pllgra æqualia Δlis A, D, C, sub ∠lo et latere datis, id solum demonstrandum est FN constituere unum illud Pllgm aequale tribus illi EG, HK et IL. Quod patebit ostendendo EH ipsi HI et IN itemq FG, GK et KL in directum esse sitas atq parallelas.

$\angle F = \angle D$ p.C.

$\angle o = \angle D$ p.C.

$\angle F = \angle o$ §41 Ar

$\angle y = \angle y$ §40 } Ar

$\angle F+y = \angle o+y$ §42 }

Est autem EF ≈ HG p.C.

Sed $\angle F+y = 2R$ §132

$\angle o+y = 2R$ §41 Ar

Ergo FG et GK in directum sitae 8gs.

Porro, quia FH et HK Pllaga p.C.

$\angle x = \angle F$ §169.

$\angle F = \angle o$ p.C. et d.

$\angle x = \angle o$ §41. Ar.

$\angle o = \angle u$ §169.

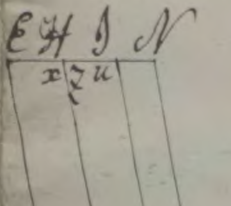
$\angle x = \angle u$ §41 } Ar.

$\angle z = \angle z$ §40 }

$\angle x+z = \angle u+z$ §42 }

$\angle u+z = 2R$ §132.

$\angle x+z = 2R$ §41 Ar



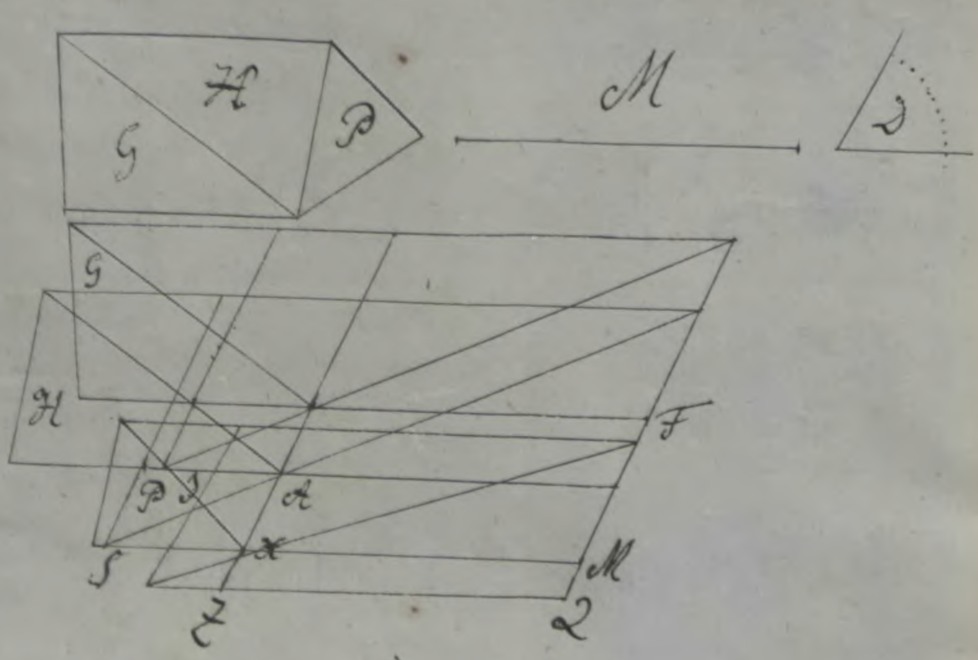
EH et HI in directum sitae §93. Ergo
Q.E.I.

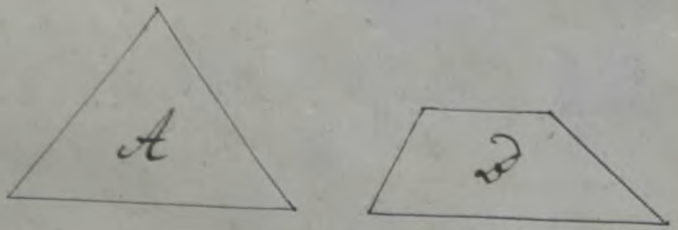
Est autem $\angle F = \angle x$ p.d.
et $s = \angle z$ §169.
$\angle F + s = \angle x + z$ §42 Ar
Sed $\angle x + z = 2R$ p.d.
$\angle F + s = 2R$ §41 Ar
Ergo $EF \approx IK$ §133.

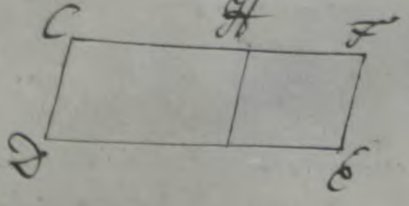
Cumque $\angle y + F = 2R$ p.d.
et $\angle y = \angle E$ §169
$\angle F + E = 2R$ §10 A
Ergo et $EI \approx FK$ §133.
Ergo ~~E~~ FI Parallgm §72.
Q.E.II.

Quare cum $FI = FH + GI$ §42 Ar
et $FH = A$ } p.C.
$GI = D$ }
$FH + GI = A + D$ §42 Ar
$FI = A + D$ §41 Ar.
Q.E.III.

Simili omnino modo evincitur
EI in directum IN } positas
FK in directum KL }
atque $EN \approx FL$
$EF \approx NL$ et tandem fit
$FN = A + D + C$ Q.E.D.







§187. Scholion 1.

Praxis Problematis §186 pendet ab iterata praxi §185. Uti quidem Clavius ad P. 45. L. 1. Eucl. Schol. docet. h. m. absolvenda:

1) Resoluto Rectilineo GHP in Δla §8 factoq Pllgo AF = Δlo G. §185.

2) Super AS vel ipsa vel producta vel imminuta, prout scil. Trianguli sequentis basis postulat ex pcto A describe Δlum H §98. facq omnia ut Mbr 1. eritq Pllgm AM = ΔH.

3) Sic etiam ex puncto x super xs vel ipsa etc. descripto Δlo P fac omnia ut Mbr. 1 ut sit XQ = Δlo P. §

Inde quidem vi Demonstrationis § 186 Pllgm FQ = Δlis G, H, P. hoc est toti Rectilineo.

§188. Scholion 2.

Hinc etiam facile invenitur Excessus HE, quo rectilineum aliquod A superat rectilineum minus D, nimirum si ad quamvis rectam applicentur §187

☆

☆ Etenim Pllgm CF = A } p. C.
Pllgm HD = D }
Pllgm HE = A − D §43 Ar.
cf. Barrow L. 1. P. 45. Eucl.

§189. Theorema 47.

In Triangulis rectangulis ABC Quadratum Hypotenusae AC aequale est Quadratis, quae a Lateribus rectum ∠um continentibus AK et BH describuntur, simulsumtis.

Demonstratio.

$\angle ABC = R$. p. H

$\angle ABK = R$ p. C. §64.

$\angle ABC + ABK = 2R$. §41 Ax

Ergo BC et BK in directum sita §93.

AL ≈ KC §68. 72.

Duc CL et FB. §81

Quia $\angle o = R$ } §68

$\angle s = R$ }

$\angle o = \angle s$ §92

$\angle CAB = \angle CAB$ §40 Ax

$\angle FAB = \angle CAL$ §42 Ax

sed $FA = AC$ } §68.

$AB = AL$ }

$\triangle FAB = \triangle CAL$ §99 et

$2 \times \triangle FAB = 2 \times \triangle ACL$. §44 Ax.

Per B age ≈lam BE cum FA aut DC. §136.

Ergo:

* *

* *

Pllgm $EA = 2 \times \triangle FAB$ } §181

Pllgm $AK = 2 \times \triangle CAL$ }

sed $2 \times \triangle FAB = 2 \times \triangle LAC$ p. d.

Pllgm EA = Pllgo AK §41 Ax

sed $AK = AB^2$ §40. Ax.

Pllgm $EA = AB^2$ §41 Ax.

Eodem plane discursu ductis AH et BD demonstratur

$DG = CB^2$

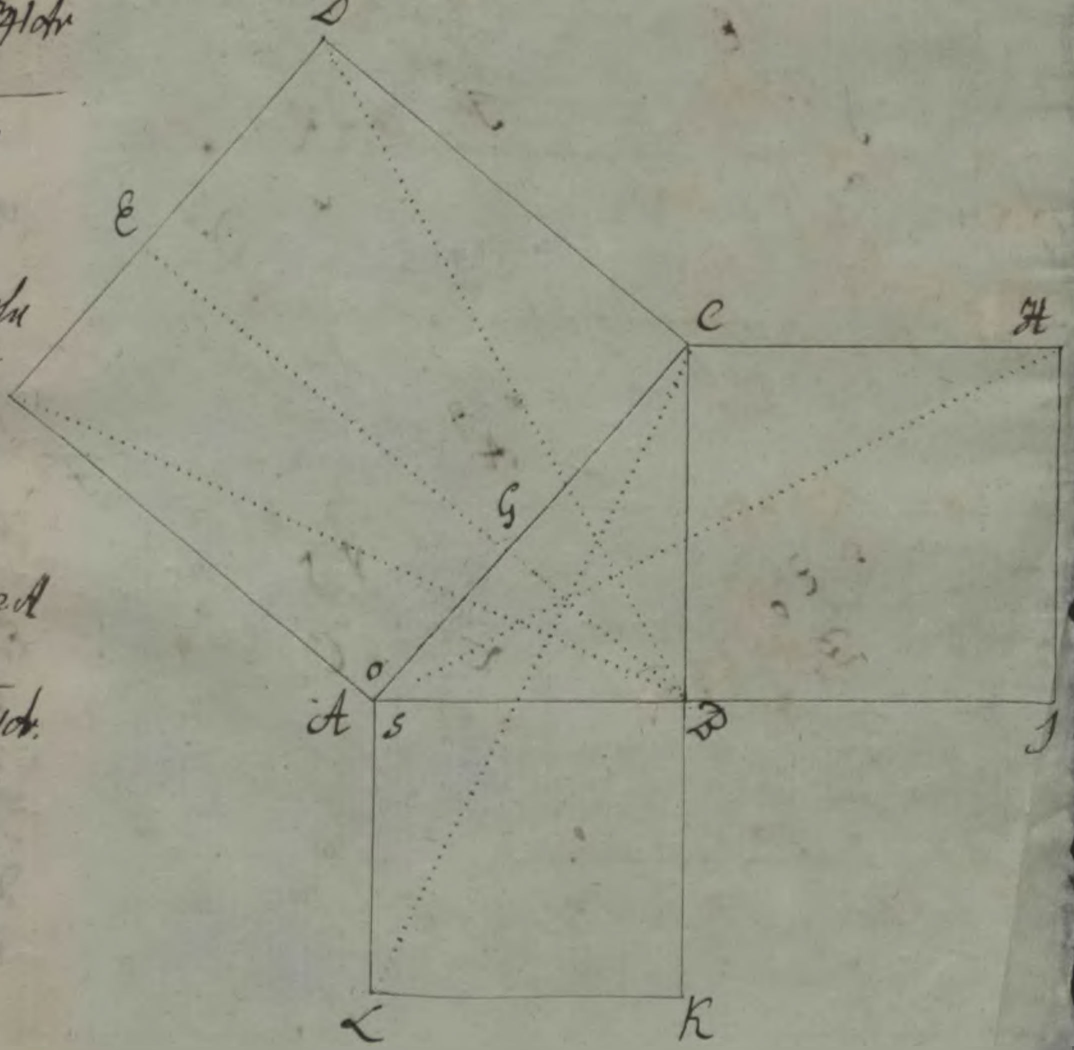
cumque $EA = AB^2$ p. d.

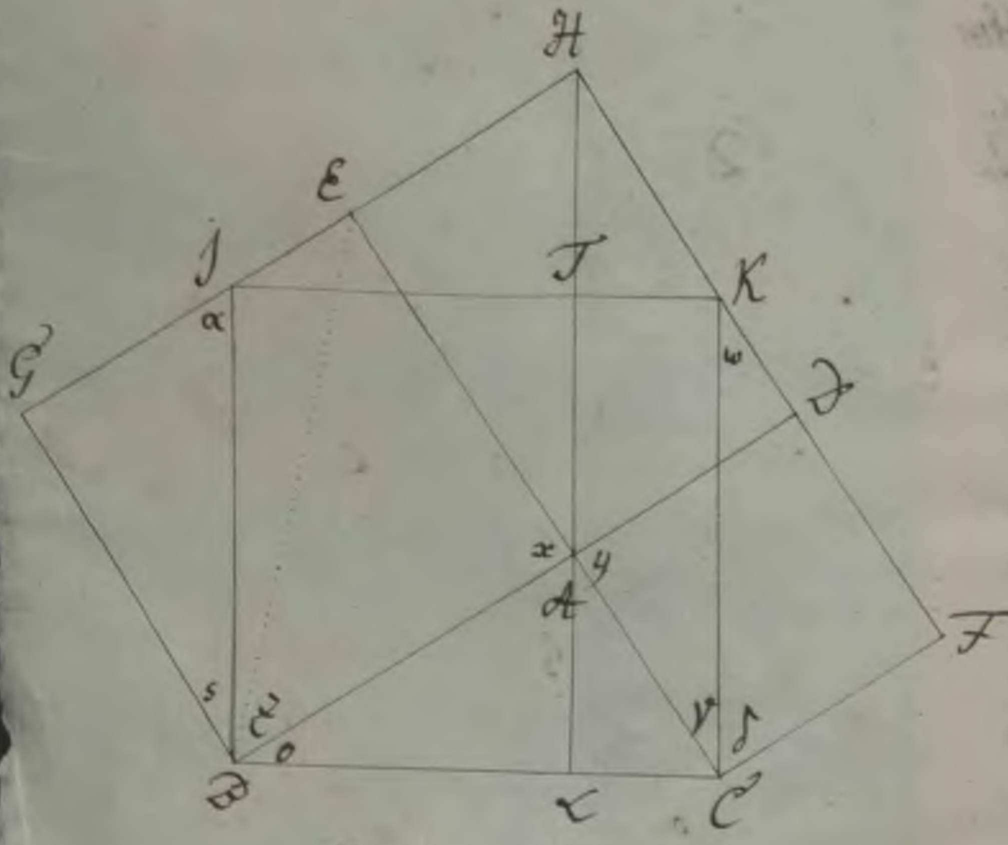
$DG + EA = CB^2 + AB^2$ §42 A

$DG + EA = AC^2$ §40 Ax

$AC^2 = CB^2 + AB^2$ §41 Ax.

Q. E. D.





* *

Ergo GE et GD convergunt
§ 191. Idem similiter ostendit
de DF et CF coituris in F.
Ergo et G et AF ℓlga §72.
Ergo GE = AD } §167.
et = GD }
EA = AD p. C.

AD = GD = GE = EA §41 Ax.
Porro A + x = 2R §93
Sed ∠A = R. p. H.

∠x = R §43 Ar.
cumque ∠x = ∠G §169.

∠G = R. §92.
Sed et ∠G + GEA = 2R §169

∠GEA = R. §43 Ar.
Ergo et ∠GDA = R. §169.
Ergo ADGE est Quadratum
§68.

Similiter discursu osten-
ditur ADFC esse Quadra-
tum.

§190 Scholion 1.
Nobilissimum Theorema hoc m
plici adhuc ratione variata ta-
men ex parte constructione ma
ille Geometra Clavius ad L. 1. P. 4
Eucl. demonstratum ivit adver
Peletarium, qui tale quid absq
Proportionibus fieri posse negave
rat. h. m. Constructo ut ante Tria
gulo rectangulo BAC produc Late
BA et AC ad partes ∠li recti §88
ut AD = AC } §26
AE = AB }
per puncta G, D, D, E age rectas cumque
et CA coeuntes in F et G §135.
Coituras autem EG et DG patet qui
GE ≈ BA p. C. Ergo
∠E = ∠A §132 M. 11
Sed et GD ≈ EA
∠GDA = ∠A §132 M. 1.
Sed ∠A = R.

∠GDA = ∠E = R. §92
Ducta ED §81.
∠lus GED < R. } §47 A
et ∠lus GDE < R. }

∠GED + GDE < res 2R. §42 Ar.

* *

Porro.

Productis Lateribus GE, et FD ad concursum in H §80. in extremitatibus Rectae AD G excita normales CK et DI secantes FH et HG in K et I. §158.

Cum enim $\angle G = R$. p.d.

$\angle \varepsilon < R$. §47 et

$\angle G + \varepsilon <$ res $2R$. §42 A.

onvergentitaq GI et DI §141. ut necessario alicubi secet DI ipsam GI.

imiliter illud de CK et KF patet

Ducta igitur IK §81

$\angle \zeta + o = R$ p.C. et §44.

ed $\angle \varepsilon + \zeta = R$ p.d.

$\angle \zeta + o = \angle \varepsilon + \zeta$ §41 A vel §92.

$\angle o = \angle \varepsilon$ §43. Ar.

et $AD = DG$ p.d.

$\angle A = \angle G$ §92. et p.d.

$DC = ID$ }
et $AC = GI$ } §114.

Porro: $\gamma + \beta = R$. §44

$\gamma + \delta = R$. p.d.

$\gamma + \beta = \gamma + \delta$ §41 Ar

✠

✠ $\angle \beta = \angle \delta$ §43 Ar

sed $AC = CF$ p.d.

et $\angle A = \angle F$ §92.

$DC = CK$ }
$AD = KF$ } §114.

Quare cum $DC = ID$ }
$DC = CK$ } p.d.

$ID = CK$ §41 Ar

Cumq. ID et CK ⊥les ad DC. p.C.

Ergo ID ∥ CK §138

Ergo et IK = et ∥ DC §139

Inde quidem ICest Pllgm §72 habens singula latera aequalia.

Ergo tandem et

$\angle \zeta + o = \angle IKC$ }
$\angle \beta + \gamma = \angle KID$ } §169

sed $\zeta + o = \beta + \gamma = R$. p.C.

$\angle IKC = \angle KID$ §41 Ar

Ergo singuli ∠li Recti

Adeoq. IKCD est Quadratum §68.

Qua constructionis Demonstratione praemissa ostendendum:

$DC^2 = AD^2 + AC^2$.

Duc per H et A rectam HL. § 81

Quia $HG \approx ED$ } p.C.
$EC \approx HF$ }

Ergo ED est Pllgm § 72.

Ergo $EH = AD$ § 167.
$AD = AC$ p.d.
$EH = AC$ § 41 Ar
Sed $AC = GI$ p.d.
$EH = GI$. § 41 Ar
$IE = IE$ § 40 } Ar
$IH = EG$. § 42 }
$EG = AD$ p.d.
$IH = AD$ § 41 Ar. Sed et
$IH \approx AD$ p.C.
$ID =$ et $\approx HA$ § 139.

Quare cum

I In Pllgis AD^2 et HD eadem basi AD
et $HG \approx AD$ p.C.
$AD^2 = HD$ § 174

II In Pllgis DH et IL eadem basi ID
et $ID \approx HA$ p.d.
$HD = IL$ § 174

Ergo $AD^2 = IL$. § 41 Ar.

×

Tandem: × $HD = EF$ § 167
$EF = AD$ p.C.
$AD = KF$ p.d.
$HD = KF$ § 41 Ar.
$KD = KD$ § 30 Ar.
$HK = DF$ § 43 Ar
$DF = AC$ p.d.
$HK = AC$ § 41 Ar
$HK \approx AC$ p.C.
$HA =$ et $\approx KC$. § 139.

Inde quidem

I) In Pllgis AC^2 et HC ead. basi AC
et $AC \approx HK$ p.C.
$AC^2 = HC$ § 174

II In Pllgis HC et TC ead. basi CK
et $HA \approx CK$ p.d.
$HC = TC$. § 174
$AC^2 = TC$. § 41 Ar
Sed $AD^2 = IL$. p.d.
$AD^2 + AC^2 = TC + IL$ § 42 Ar.
$CD^2 = TC + IL$ § 47 not Ar
$CD^2 = AD^2 + AC^2$ § 41 Ar

Q.E.D.

§191 Scholion 2.

Facile adparet elegantiſſimam eſſe allatam Demonſtrationem ex parte tamen arduam ut ipſe Clavius non diffiteatur l.c. Euclideam ſimpliciorem magisque expeditam eſſe. Alias ejusdem Theorematis Demonſtrationes ſine Proportionibus concinnatas vide ap. eundem Clavium l.c. adde Claud. Richardum in Comment. in Euclid. ad L. 1. P. 47. Krüger in den Anmerkungen über d. H. Hfr. von Wolfs Geometrie.

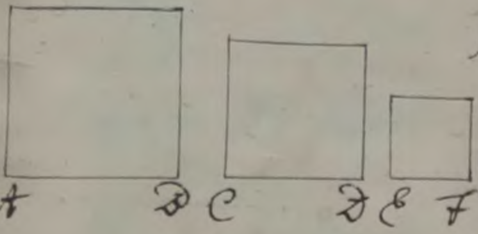
§192. Scholion 3.

Demonſtratum Theorema ab Inventore Pythagoricum dici ejusque ampliſſimum per univerſam Matheſin uſum patere notum in vulgus eſt, Auctoremque in Deos gratum Hecatomben, ſecundum alias Bovem immolaſſe dicunt.

§193. Problema XXI

Datis quotcunque Quadratis AB^2, CD^2, EF^2, unum omnibus aequale construe

Resolutio.



1) Latera Quadratorum AB^2 et CD^2 np. AB et CD junge ad ∠los R. ut sit $NM = AB$ $LM = CD$. §158.

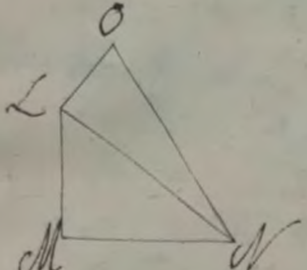
2) Duc Hypotenusam LN §81

3) In Extremitate illius L aut N excita normalem $LO = EF$. §158. 2b.

4) Duc ON. §81.

Dico $ON^2 = AB^2 + CD^2 + EF^2$.

Demonstratio.



Δ LMN est Rlglum p. C.

$LM^2 + MN^2 = LN^2$. §189.

Sed $MN = AB$ $LM = CD$ p. C.

Ergo $MN^2 = AB^2$. $LM^2 = CD^2$. §44 et §175. Geom.

Ergo $AB^2 + CD^2 = LN^2$ §10. Ar.

$\triangle \theta L N$ est R∠gulum p. C.

Ergo $\theta N^2 = LN^2 + L\theta^2$. §189.

sed $\theta L = EF$. p. C.

Ergo $\theta L^2 = EF^2$. §44 Ar. et 175 G.

cumque $LN^2 = AD^2 + CD^2$ p. d.

$LN^2 + \theta L^2 = AD^2 + CD^2 + EF^2$. §42. Ar.

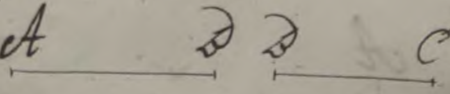
Adeoque

$\theta N^2 = AD^2 + CD^2 + EF^2$. §41 Ar.

Q. E. D.

§194. Problema XXII

Datis duabus rectis inaequalibus AD et CD exhibere Quadratum, quo Quadratum majoris excedit Quadratum minoris DC.



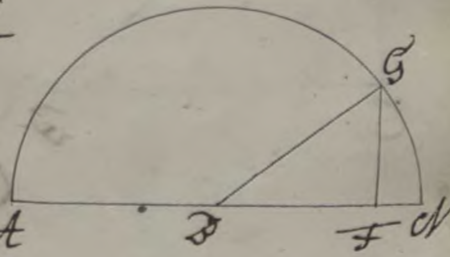
Resolutio

1) Centro D intervallo DA describe semicirculum AGN. §85.

2) Ex D transfer DF = DC. §26.

3) Ex F erige ⊥lem §120. GF secantem Peripheriam. Dico:

$GF^2 = AD^2 - DC^2$.



Demonstratio.

Duc DG. §81.

Quia Trigonum DGF R∠gl. p. C.

Ergo $DG^2 = DF^2 + FG^2$. §189.

Sed $DF = DC$. $DG = AD$ p. C.

Ergo $DF^2 = DC^2$. $DG^2 = AD^2$

§44. A. et 175. G.

Ergo

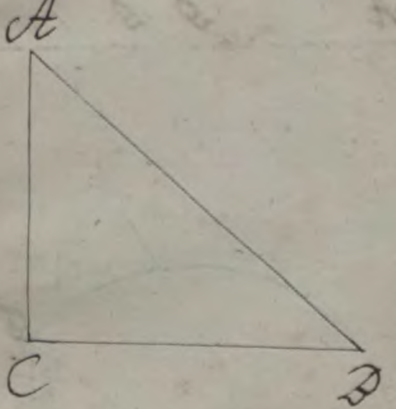
$AD^2 = DC^2 + FG^2$. §10. Ar.

$AD^2 - DC^2 = FG^2$. §43. Ar.

Q. E. D.

§195. Problema XXIII

Notis duobus Trianguli rectanguli Lateribus invenire tertium.



Resolutio et Demonstratio.

Dantur duo Casus, aut enim

1) Hypotenusa aut

2) Alterutra Cathetorum quaeritur.

Quare in

Casu I. Quia

$AD^2 = AC^2 + CD^2$. §189

Ergo $AD = \sqrt{AC^2 + CD^2}$

h. e. Ex summa Quadratorum Laterum ∠um Rectum intercipientium extrahe Radicem quadraticam & — Inventa exhibet Hypotenusam.

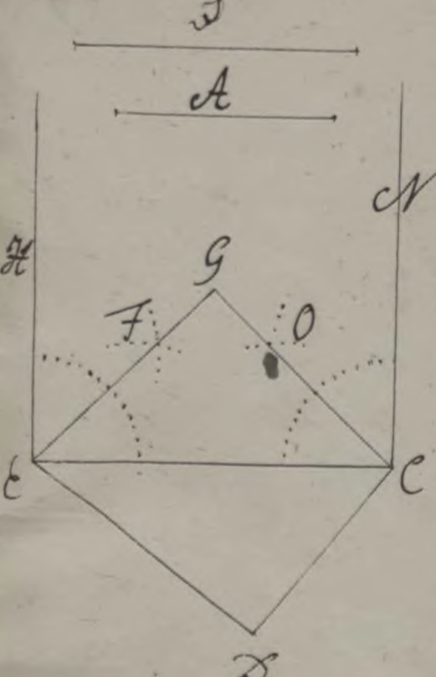
Q. E. I.

Casu 2do Quia
$AD^2 = AC^2 + DC^2$. §189.
Ergo $AD^2 - AC^2 = DC^2$. §43 Ar
et $\sqrt{AD^2 - AC^2} = DC$.
Ex differentia Quadratorum Hypotenusæ et Catheti data extracta radix quadratica æqualis est Lateri quæsito.

Q. E. I. F. et D.

§196. Problema XXIV

Propositis duobus Quadratis inæqualibus, invenire duo alia Quadrata, quæ et æqualia sint inter se et simul sumta æqualia duobus inæqualibus propositis.



Resolutio.

Sint A et D Latera Quadratorum propositorum inaequalium.

1) Junctis A = CD et D = DE ad ∠R. §158.

2) Duc CE §81.

3) Excita in C et E ⊥res §158.

4) Rectos ∠los NCE et CEH biseca §108 rectis CO et EF in G concurrentibus §82.

Dico 1) $GE^2 + GC^2 = CD^2 + DE^2$.

2) $GE^2 = GC^2$.

Demonstratio

△ CDE est R∠lum p. C.

$CD^2 + DE^2 = CE^2$ §189.

Porro $\left.\begin{array}{l}\angle OCE = \frac{1}{2}R \\ \angle FEC = \frac{1}{2}R\end{array}\right\}$ p. C.

$\angle OCE + FEC = R$. §42 Ar.

sed R. ∠r 2 R. §40. Ar.

$\angle OCE + FEC$ ∠res 2R. §46. Ar.

Ergo GE ~~et~~ GC convergunt §141.

Ergo GCE est △lum §58 idq aequicrurum §160 atq rectangulum §157.

★

★ Ergo

$CE^2 = CG^2 + GE^2$ §189.

sed $CE^2 = CD^2 + DE^2$ p. d.

$CG^2 + GE^2 = CD^2 + DE^2$. §41 Ar.

Q.E.I.

$CG = GE$. §160.

Ergo $CG^2 = GE^2$. §44 Ar. et 175 G.

Q.E.II D.

§197. Theorema 48.

Quadratorum aequalium AD^2 et CD^2 aequalia sunt Latera AD et CD.

Demonstratio.

Aut $AD = CD$ }
aut $AD < CD$ } §39 Ar.
aut $AD > CD$ }

Ergo in

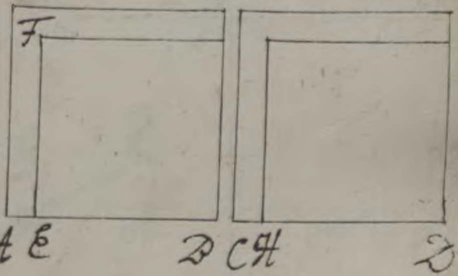
Casu 1 Esto $AD > CD$ p. H assi.
Fac $DE = CD$ §26.
Descripto latere DE Quadrato §170.
erit $DE^2 = CD^2$ §44. Ar. 175. G.
Sed $AD^2 = CD^2$ p. H. Th.

$AD^2 = DE^2$ §41. Ar
J. Q. E. A. §47 Ar.

IIdo Esto $AD < CD$ p. H assi.
Et simili modo ostendetur.
$DH^2 = CD^2$
J. Q. E. A. §47 A.

Quare cum neq $AD > CD$ } p. d.
neq $AD < CD$ }

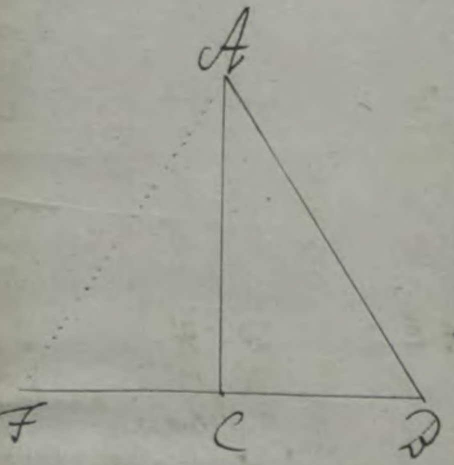
Ergo omnino $AD = CD$.
Q. E. D.



h. e dmdm.
si $AD^2 = CD^2$
fore $AD = CD$.

§198. Theorema 49.

Si Quadratum ab uno Trianguli latere AD fuerit aequale duobus reliquorum Laterum Quadratis, $\angle$ ACD quem reliqua Latera continent rectus est.



h.e.
Si $AD^2 = AC^2 + CD^2$
erit $\angle ACD = R.$

Demonstratio.

Duc $FC \perp$ ad AC. §158.

facque $FC = CD$ §26.

Junge AF. §81.

Ergo $\triangle FCA$ est R∠glum §59.

Ergo $AF^2 = FC^2 + CA^2$. §189.

sed $CD = FC$ p.C.

Ergo $CD^2 = FC^2$. §44. Ar. 175 Geom.

Ergo $AF^2 = CD^2 + CA^2$. §10 Ar.

sed $AD^2 = CD^2 + CA^2$ p.H.

$AF^2 = AD^2$. §41 Ar

et $AF = AD$. §197

$FC = CD$ p.C.

$AC = AC$. §40 Ar

$\angle ACF = \angle ACD$. §106.

$\angle ACF = R.$ p.C.

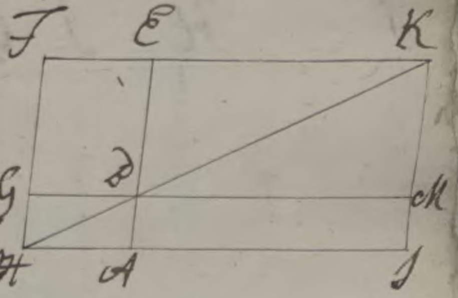
$\angle ACD = R.$ §92.

Q.E.D.

Caput II^{dum}

De rectangulorum affectionibus

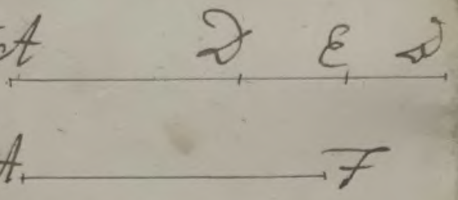
§199. Definitio LVIII



In omni parallelogrammo FHIK unum quodque eorum, quae circa Diametrum HK illius sunt Parallelogrammorum EM, GA cum duobus complementis dicitur Gnomon. Sic DF+DI+GA h.e. EHMI est
DF+DI+EM h.e. GKAI Gnomon.

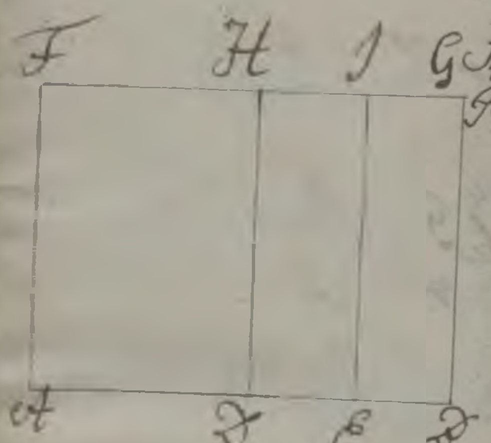
§200. Theorema 50.

Si fuerint duae rectae Lineae AD, AF, seceturque ipsarum altera AD in quotcunque segmenta AD, DE, ED Rectangulum comprehensum sub illis duabus rectis lineis AD et AF, aequale est eis quae sub insecta AF et quolibet segmentorum AD, DE, ED comprehenduntur Rectangulis.



Demonstratio.

In Extremitatum altera A vel D



excita normalem AF §158.
Per F eum AB itemq.
Per D, E et B alias age ≈las cum AF
np. FG, DH, EI, BG. §135.
Sunt ergo AG, AH, HE, et EG rect∠gla
Pllgrma. §72. 79

Ergo.

$AG = AB \times AF$. §175.

$AG = AF \times AD + DH \times DE + EI \times EB$ §80. atq §41 Ar.

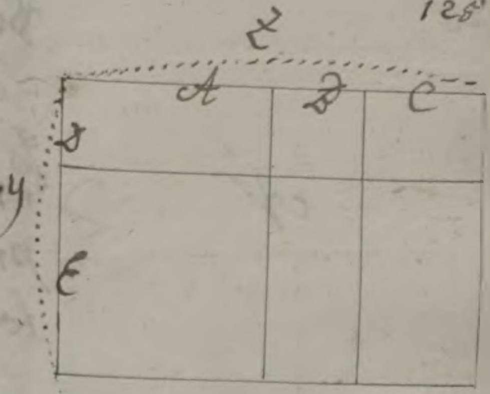
Sed $DH = EI = AF$. §167. atq §4 Ar

$AB \times AF = AF \times AD + AF \times DE + AF \times EB$ §41. 10 Ar

Q. E. D.

§201. Scholion 1.
Hinc si fuerint quaecunq duae rectae Lineae secenturq ambae in partes quotcunq, idem proveniet ex multiplicatione totius in totam, quod ex Multiplicatione partium omnium in partes omnes utriusq rectae Rectangulum.

Esto Recta $= Z = A + B + C$.
Esto alia $= y = D + E$.
Quia $Z = A + B + C$ p. H
et $D = D$. §40. Ar.
$Z \times D = A \times D + B \times D + C \times D$ §200.
Et quia $Z = A + B + C$ p. H.
atq. $E = E$. §40. Ar
$Z \times E = A \times E + B \times E + C \times E$. §200
$Z \times D + Z \times E = A \times D + B \times D + C \times D + A \times E + B \times E + C \times E$. §42. Ar
$Z \times D + Z \times E = Z \times y$. §200
$Z \times y = A \times D + B \times D + C \times D + A \times E + B \times E + C \times E$. §41. Ar.



§202. Scholion 2.

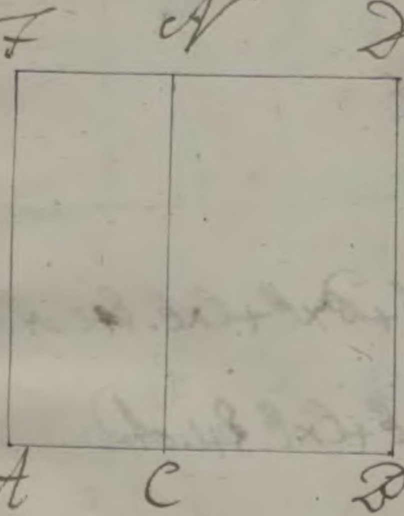
Cum Propositiones X. primo hujus Capitis etiam in Numeris valeant, iisdem illustrabimus.

Esto ad §200. $AB = 17 = 5 + 9 + 3$.
$AF = 6 = 6$
$AB \times AF = 102 = 30 + 54 + 18$.
$= 102$.

Esto ad §201. $Z = 11 = 5 + 2 + 4$
$y = 7 = 4 + 3$
$Z \times y = 77 = 20 + 8 + 16 + 15 + 6 + 12$.
$= 77$

§203. Theorema 57.

Si recta Linea AB secta sit utcunque in C, Rectangula, quae sub tota AB et quolibet Segmentorum AC, CB comprehenduntur aequalia sunt ei quod a tota AB fit, Quadrato.



Demonstratio.

Fac Quadratum Lateris AB. §170 cum DB aut AF duc ∥ CN per C §12

Hinc ~~Quadratum ca~~

$CN = DB$. §167

$DB = AB$ §68. et p. l.

$CN = AB$. §41 Ar

$AC = AC$ §40. Ar

$CN \times AC = AB \times AC$. §44. Ar. 175 G.

simili discursu.

$CN \times CB = AB \times CB$. §. §c. c.

$CN \times AC + CN \times CB = AB \times AC + AB \times CB$ §42

Sed $CN \times AC + CN \times CB = CN \times AB$. §200

$CN \times AB = AB \times AC + AB \times CB$. §41 Ar

sed $CN = AB$. p. d.

$AB \times AB = AB \times AC + AB \times CB$

$AB^2 = AB \times AC + AB \times CB$ §10. Ar

Q. E. D

r. H. dmdm

$AB^2 = AB \times AC + AB \times CB$

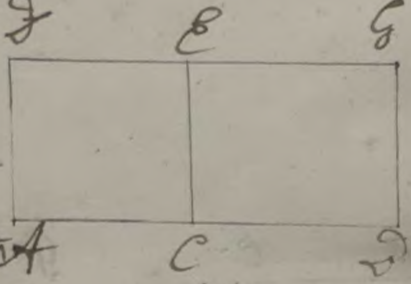
§204. Scholion.

Esto AB = 15. AC = 7 Ergo
CB = 8. Ergo.

$225 = 15 \times 7 + 15 \times 8.$
$= 105 + 120.$
$\equiv 225.$

§205. Theorema 52.

Si Recta AB, secta sit utcunq; in C Rectangulum sub tota AB et uno segmentorum AC aut CB comprehensum, æquale est illi, quod sub segmentis AC et CB comprehenditur Rectangulo, et illi, quod a prædicto segmento AC aut CB describitur Quadrato.



p. H dmdm.

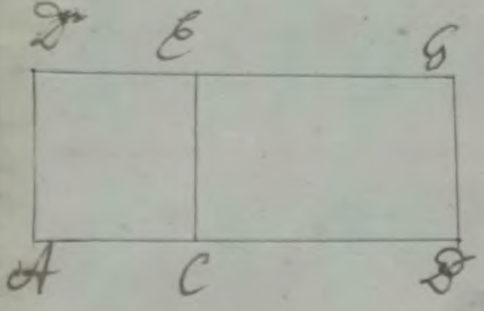
1) $AB \times AC = AC^2 + AC \times CB$
2) $AB \times BC = BC^2 + BC \times AC.$

Demonstratio.

Ad ∠los R. statue in extremitate rectæ AB, AD = AC §158.
Per puncta D. C, B age ∥las cum AB et AD. §155. coëuntes in G et E.

Ergo

AE, AG, CG sunt R∠gla §175.170.
Ergo et AD = EC. §167.



Ergo

$AG = AE + CG$. §47. Ar.

ſed $AG = AB \times AD$. §175.

cumque $AD = AC$. p. C.

Ergo $AG = AB \times AC$. §10. Ar.

Eſt vero et

$AE = AD \times AC$. §105.

$= AC^2$.

Tandem

$CG = EC \times CB$. §175.

ſed $EC = AD$. p. d.

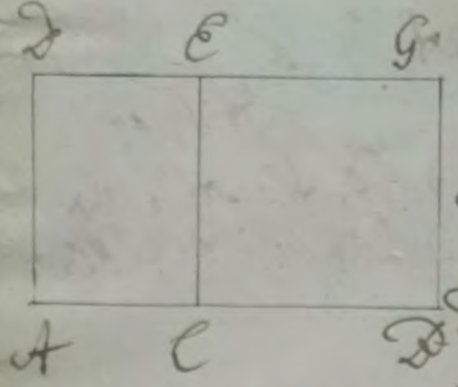
$CG = AD \times CB$

$= AC \times CB$ §8.

Ergo per §10. Ar. ſubſtituendo.

$AB \times AC = AC^2 + AC \times CB$

Q. E. I.



Simili prorſus diſcurſu mutatis in conſtructione mutandis oſtenditur quoque

$AB \times BC = BC^2 + BC \times CA$.

Q. E. II. D.

§206. Scholion.

Esto $AD = 13$. $AC = 5$. Ergo
$CD = 8$.

Ergo cum

I. $AD \times AC = AC^2 + AC \times CD$. erit
$13 \times 5 = 25 + 5 \times 8$. h. e
$65 = 25 + 40$
$= 65$.

II. $AD \times DC = DC^2 + CD \times AC$.
$13 \times 8 = 64 + 8 \times 5$
$104 = 64 + 40$
$= 104$.

§207. Theorema 53

i Linea recta AD secta sit utcun-
q in C, Quadratum, quod a tota
AD describitur, aequale est et illis
quae a segmentis AC, CD describun-
tur Quadratis et ei quod bis sub
Segmentis AC et CD comprehendi-
tur Rectangulo

p. Hdmdm;
$AD^2 = AC^2 + CD^2 + 2 \times AC \times CD$

Demonstratio.

Descripto Latere AD Quadrato §170.
duc Diagonalem CD. §81.

Quia $\angle A = R$. p. C. et §68.

et $\underline{EA = AB}$ p. C. et §c.

$\angle y = \angle s$. §100

Ergo $\angle y = \frac{1}{2} R$. §162.

Per C age ≈ CF cum EA aut DB }
Per G age ≈ HI cum AB aut ED } §130

Ergo $\angle \theta = \angle A$ §132.

$\underline{\angle A = R}$. p. C.

$\angle \theta = R$. §92.

cumq $\angle y = \frac{1}{2} R$. p. d.

Ergo $\angle u = \frac{1}{2} R$. §157.

Ergo $HE = HG$. §160.

ſed $\underline{HG = AC}$. §167. 139.

$HE = AC$. §41. Ar

ſimili Diſcurſu demonſtrabo Triangulum EFG eſſe ad Z Rectangulum idq; æquicrurum Angulos α et w ſemirectos latusq; $EF = AC$

$\angle y + \alpha = R$ }
$\angle u + w = R$ } §42 Ar

cumq et $\angle \theta = \angle A = R$. p. d

Ergo $HI = HG^2$

h. e. $HI = AC^2$ §41 Ar

Hoc ipso modo demonstrabitur
$CI = CD^2$. Tandem cum
$AG = AC \times CG$. §.175.
erit $AG = AC \times CD$ §68.
Sed $AG = GD$. §184
$GD = AC \times CD$. §41. Ar.

Verum.

$AD = HF + CI + AG + GD$ §47. Ar

h. e.

$AD^2 = AC^2 + CD^2 + 2 \times AC \times CD$. §10. Ar

Q. E. D.

§208. Scholion.

Sit $AD = 11$. $AC = 9$. Ergo
$CD = 2$.

Quia.

$AD^2 = AC^2 + CD^2 + 2 \times AC \times CD$. §107

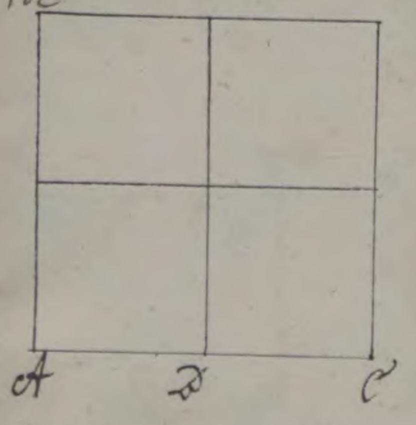
Ergo.

$121 = 81 + 4 + 2 \times 18.$
$= 81 + 4 + 36.$
$= 121.$ Similiter in aliis.

§209. Corollarium.

Ex Demonstratione §107. liquet Parallelogramma circa Diametrum HF et CI esse Quadrata.

132



§210. Corollarium 2.

Vlterius liquet Diametrum cujusv Quadrati ∠los bisecare.

§.211. Corollarium 3.

Esto $AD = DC = \frac{1}{2} AC$.

Dico $AD^2 = \frac{1}{4} AC^2$.

Quia enim

$AC^2 = AD^2 + DC^2 + 2 \times AD \times DC$ §107

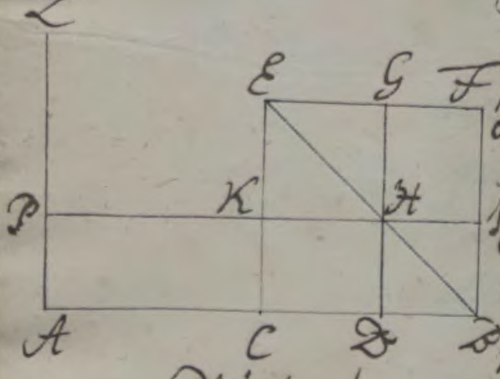
Ergo $AC^2 = AD^2 + AD^2 + 2 \times AD \times AD$ §10 Ar

h.e. $AC^2 = 4 \times AD^2$. §47 Ar

Ergo $\frac{1}{4} AC^2 = AD^2$. §45. Ar.

§212. Theorema 54.

Si recta Linea AB secetur in aequalia AC et CB, et in non aequali AD et DB; Rectangulum sub inaequalibus segmentis AD et DB comprehensum, una cum Quadrato, quod fit ab intermedia secti num CD, aequale est ei, quod a dimidia CB describitur, Quadra



P.H dm dm.

$CB^2 = AD \times DB + CD^2$

Demonstratio.

Descripto Latere CB Quadrato §170.
ductaq. Diogonali BL. §81.
Per B et A age ≈las DG et AL cum
EC. Per H age ≈lam IP cum EF qut
DA secantem AL in P. §135. Ergo
Figurae descriptae sunt Pllgma §72.
atq. KG et DI Quadrata §209.

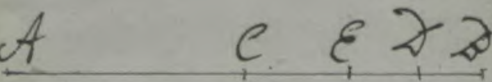
ß Hinc $CH = HF$ §184
$DI = DI$. §40. Ar.
$CI = DF$. §42. Ar.
Sed $AC = CB$. p. H.
$AB \approx IP$. p. C.
$AK = CI$. §176.
$AK = DF$. §41. Ar.
$CH = CH$. §40. Ar.
$AH =$ Gnom KDG. §42. Ar.
$KG = KG$. §40. Ar
$AH + KG = KG +$ Gn. KDG. §42. Ar
$CB^2 = KG +$ Gn. KDG. §47. Ar
$CB^2 = AH + KG$. §41. Ar.
cumq $AH = AD \times DB$. §175. Cy
ß et $KG = CD^2$. §209. Ergo.
$CB^2 = AD \times DB + CD^2$ §10. Ar.

Q. E. D.

134

§213. Scholion.

Esto $AB = 16$ Ergo $AC = CB = 8$.
Esto $AD = 12$ Ergo $DB = 4$
Quare, cum et $CD = 4$.
$CB^2 = AD \times DB + CD^2$. Ergo
$64 = 12 \times 4 + 16$.
$= 48 + 16$
$= 64$



§214. Corollarium 1.

Ponamus inter punctum æqualis sectionis C et inæqualis D aliud adhuc esse E, Dico Rectangulum ex segmentis AE et EB quæ fiunt a puncto E bisectioni propiore, majus esse Rectangulo ex segmentis AD et DB quæ fiunt a pcto D, a bisectione remotiore. h. e. Dico

$AE \times EB > AD \times DB$.

Nam.

$AE \times EB + CE^2 = CB^2$ §212.
$AE \times EB = CB^2 - CE^2$ §43 Ar.
$AD \times DB + CD^2 = CB^2$ §212.
$AD \times DB = CB^2 - CD^2$ §43 Ar.

Enimvero $CB^2 = CB^2$. §40 Ar.
cumque $CE < CD$ p. H. $CE^2 < CD^2$. §44 Ar. 175. G.

$CB^2 - CE^2 > CB^2 - CD^2$. §43. Ar.
Sed $CB^2 - CE^2 = AE \times EB$. p.d.

$AE \times EB > CB^2 - CD^2$. §46 Ar.
Cumque $AD \times DB = CB^2 - CD^2$ p.d

Ergo $AE \times EB > AD \times DB$. §.c.

§215. Corollarium 2.

Ex adverso autem Summa Quadratorum, quæ fiunt ex Segmentis a puncto D, a Bisectionis puncto C remotiore, major est Aggregato Quadratorum, quæ fiunt ex Segmentis a puncto E. Bisectionis puncto C, propiore. h.e.

$$AD^2 + DB^2 > AE^2 + EB^2.$$

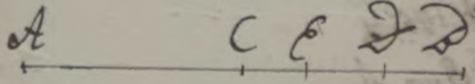
Nam

$$\left.\begin{array}{l} AB^2 = AD^2 + 2 \times AD \times DB + DB^2. \\ AB^2 = AE + 2 \times AE \times EB + EB^2. \end{array}\right\} \text{§207.}$$

$AD^2 + 2 \times AD \times DB + DB^2 = AE^2 + 2 \times AE \times EB + EB^2$. §41 Ar.
Sed $AD \times DB < AE \times EB$. §214.
Ergo $2 \times AD \times DB < 2 \times AE \times EB$. §44. Ar.

$AD^2 + DB^2 > AE^2 + EB^2$. §43. Ar.

§216. Corollarium 3.



Tandem, quia $AD^2+2\times AD\times DB+DB^2 = AE^2+2\times AE\times EB+$
p.d. ad §.215. Ergo
$AD^2+DB^2-AE^2-EB^2 = 2\times AE\times EB-2\times A$
$\times DB$. §43. Ar

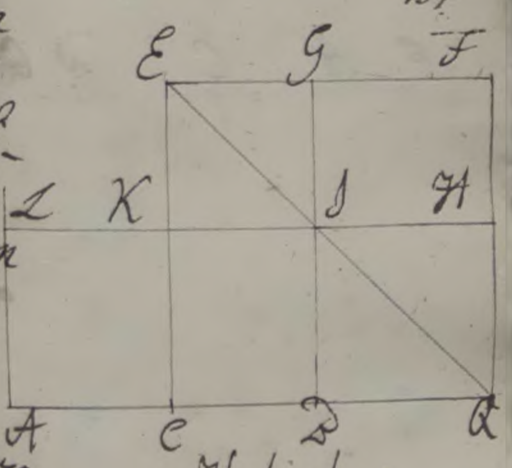
§217. Scholion.

Esto $AD=9$ $DB=3$.
$AE=8$ $EB=4$. Ergo
Quia 1. $AE\times EB > AD\times DB$. §214.
$32 > 27$.
2. $AD^2+DB^2 > AE^2+EB^2$ §215.
$81+9 > 64+16$
$90 > 80$.
3. $AD^2+DB^2-AE^2-EB^2 = 2\times AE\times EB-$
$2\times AD\times DB$ §.216.
$81+9-64-16 = 2\times 32-2\times 27$
$90-80 = 64-54$.
$10 = 10$

§218. Theorema 55.

Si recta AB bifariam secetur in C et illi recta quapiam linea BD indirectum adjiciatur; Rectangulum, quod est sub tota AD

cum adjecta DQ et adjecta DQ una
rum Quadrato, quod fit a dimidia
secta CD aequale est Quadrato, a Linea quae tum ex dimidia CD tum
ex adjecta DQ componitur, tanquam
ab una CQ descripto.



Demonstratio.

Descripto super CQ Quadrato §170.
duc Diametrum EQ. §81.
Per D et A age GD et LA ≈las cum
per I age HL ≈ cum EF aut AQ
secantem LA in L. §c.

p. H dmdm:
$AQ \times QD + CD^2 = CQ^2$
QF. §135.

Ergo

$KG = CD^2$ } §209. 207.
$DH = DQ^2$ }

Est autem

$CI = IF$ §184.

cumque $AL = CD$ p. H
A^p $AQ \approx LH$ p. C.

$AK = CI$ §176.

$AK + CI = CI + IF$ §22. Ar

$DQ^2 = DQ^2$ §40. Ar

$AH = $ Gnom. GQK. §42. Ar

$CD^2 = CD^2$ §40. Ar

$AH + CD^2 = CQ^2$ §42 et 47. Ar

✠

✠

Verum

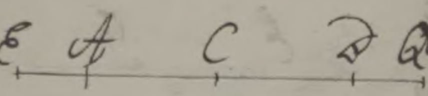
$AH = AQ \times QH$ §175
$= AQ \times QD$ §68.

$AQ \times QD + CD^2 = CQ^2$ §10. Ar

Q. E. D.

138

Aliter.



ex immediata §212 illatione.

Fac $EA = DQ$

Quia $AC = CD$ p. H.

$EC = CQ$ §42

Tota ergo EQ secta est.

1) Aequaliter in C

2) Inaequaliter in D

Ergo

$ED \times DQ + CD^2 = CQ$ §212.

Sed $EA = DQ$ p. C.

$AD = AD$. §40 Ar.

$ED = AQ$. §42. Ar

Ergo

$AQ \times DQ + CD^2 = CQ$. §10. Ar

Q. E. D.

Demonstrationis hujus Auctor Clavio ad L II P. 6 Euclid. p. m. 179. dicitur Mauritius Bressius Gratianopolitanus, Regius Math Professor, vir eruditus et in omni Doctrinarum Genere exercitatissimus.

§219 Scholion.

Esto $AB = 16$. ergo $CB = 8$

$BQ = 12$ $\quad$ $BQ = 12$

$AQ = 28$ $\quad$ $CQ = 20$

Quia.

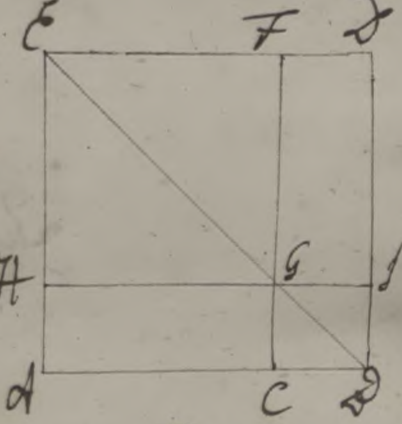
$AQ \times QB + CB^2 = CQ^2$ §218.

$28 \times 12 + 64 = 400.$

$400 = 400.$

§220 Theorema 56.

Si recta Linea secetur utcunq in C quod a Tota AB, quodq, ab uno Segmentorum CB utraq, simul Quadrata, aequalia sunt et illi quod bis sub tota AB et dicto ~~CB~~ segmento comprehenditur Rectangulo, et illi quod a reliquo Segmento AC, fit Quadrato.



Demonstratio.

Descripto Latere AB Quadrato §170.
ductis Diametro EB. §81.
et FC per G ∥la cum DB } §135.
et HI per G ∥la cum AB }

$$AB^2 = AC^2 + 2 \times AC \times CB + CB^2 \text{ §207.}$$

$$CB^2 = CB^2 \text{ §40. Ar.}$$

$$AB^2 + CB^2 = AC^2 + 2 \times AC \times CB + 2 \times CB^2 \text{ §42 Ar}$$

$$= AC^2 + 2 \times (AC \times CB + CB^2) \text{ §31 Ar.}$$

$$\qquad\qquad * \qquad\qquad *$$

p. H dm=dm

$$1)\ AB^2 + BC^2 = 2 \times AB \times BC + AC^2.$$

$$2)\ AB^2 + AC^2 = 2 \times AB \times AC + BC^2.$$

$$* \qquad *$$

$$\text{sed } AC \times CB + CB^2 = AB \times BC \text{ §205}$$

$$AB^2 + CB^2 = AC^2 + 2 \times AB \times BC. \text{ §10 Ar.}$$

Q. E. I.

Simili omnino discursu probatur alterum 2. Est enim

$AD^2 = AC^2 + 2\times AC\times CD + CD^2$. §207.

$\underline{AC^2 = AC^2}$. §40 Ar.

$AD^2 + AC^2 = 2\times AC^2 + 2\times AC\times CD + CD^2$. §42. Ar.

$AD^2 + AC^2 = 2\times(AC^2 + AC\times CD) + CD^2$.

cumque $AC^2 + AC\times CD = AD\times AC$ §205.

Ergo $AD^2 + AC^2 = 2\times AD\times AC + CD^2$. §10 Ar.

Q. E. II. D.

§221. Scholion.

Esto $AD = 11$. $AC = 9$ Ergo $CD = 2$

Quia 1) $AD^2 + DC^2 = 2\times AD\times DC + AC^2$.

2) $AD^2 + AC^2 = 2\times AD\times AC + DC^2$.

§220.

Ergo

1) $121 + 4 = 2\times 11\times 2 + 81$.

$125 = 44 + 81$

$125 = 125$.

2) $121 + 81 = 2\times 11\times 9 + 4$.

$202 = 198 + 4$.

$= 202$.

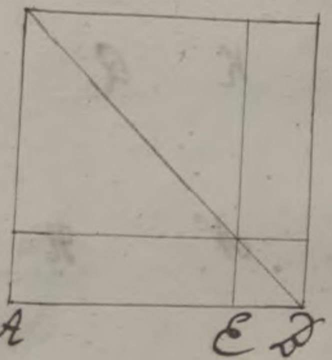
222. Corollarium.

nde quidem liquet, Quadratum diffe-
entiæ duarum Linearum AD et DE
be æquale Quadrato utriusq. duplo
amen Rectangulo sub ipsis demto

Nam:

$AD^2 + ED^2 = 2\times AD \times ED + AE^2$.

$AD^2 + ED^2 - 2\times AD \times ED = AE^2$. §43 Ar.



§223. Scholion.

Ponamus AD = 12. ED = 3.

Ergo AE = 9.

Quia $AD^2 + ED^2 - 2\times AD \times DE = AE^2$. §222

Ergo $144 + 9 - 2\times 12 \times 3 = 81$

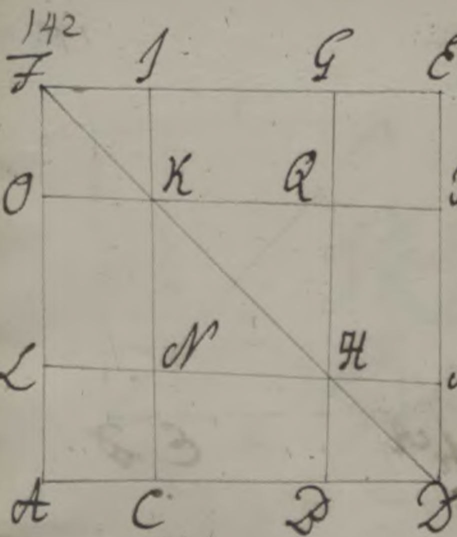
$153 - 72 = 81$

$81 = 81$.

§224. Theorema 57.

Si recta Linea AD secetur utcunq. Rectangulum quater comprehensum sub Tota AD et uno segmentorum AE aut ED cum eo quod a reliquo segmento ED aut AE fit, Quadrato, æquale est ei, quod a tota AD et dicto segmento AE aut ED,

142



P.H. dmdm
1) $4\times AB\times AC+CB^2=(AB+AC)^2$
2) $4\times AB\times BC+AC^2=(AB+BC)^2$

Tanquam ab una Linea $AC+AB$
$BC+AB$ describitur, Quadrato.

Demonstratio.

Produc AB in D. §82.
fac $BD=AC$.
quia $BC=BC$ §40 Ar.
$CD=AB$ §42. Ar.
Super tota AD fac Quadratum. §170
Quia $AD=AB+AC$. p. c.
Ergo $AD^2=(AB+AC)^2$. §44. Ar.
ducta Diagonali FD. §81.
Per pcta divisionum B et C age ≈ GB, IC cum ED
Per pcta H et K age alias ≈ LM. OP cum AD
§t.
Ergo
GL. et BM Quadrata Laterum
LH atq. BD §209.
Verum.
$LH=AB$ §187. et $BD=AC$. p. c.
$AB=AC+CB$ §47 Ar. $BD^2=AC^2$ §175 Ar
$GL=AB^2=AC^2+2\times AC\times CB+CB^2$. §207.
$BD^2=AC^2$ p.d.
$AB^2+BD^2=2\times AC^2+2\times AC\times CB+CB^2$. §42 Ar
$=2\times(AC^2+AC\times CB)+CB^2$. §31

Sed $AC^2 + AC \times CD = AD \times AC$. §205.

Ergo

$AD^2 + Dd^2 = 2 \times AD \times AC + CD^2$ §10. Ar.

cumque

$Dd^2 = \overline{AD + AC}^2$ p. d.

$(AD + AC)^2 = AD^2 + 2 \times AD \times AC + AC^2$. §207.

cumque $AC^2 = Dd^2$ p. d. Ergo

$= AD^2 + 2 \times AD \times AC + Dd^2$. §10. Ar.

Sed $AD^2 + Dd^2 = 2 \times AD \times AC + CD^2$ p. d.

Ergo tandem

$\overline{AD + AC}^2 = 2 \times AD \times AC + 2 \times AD \times AC + CD^2$. §10 Ar. h. e.

$= 4 \times AD \times AC + CD^2$

Q. E. D.

Mutatis mutandis ponendo nempe loco ipſius AC ipſam CD = Dd eadem eſt Caſus ſſdi Demonſtratio quae evincit

$\overline{AD + DC}^2 = 4 \times AD \times DC + AC^2$.

Q. E. D.

☆ ☆

1) $\overline{AD + AC}^2 = 4 \times AD \times AC + CD^2$

2) $(AD + DC)^2 = 4 \times AD \times DC + AC^2$

Ergo.

1) $256 = 4 \times 13 \times 3 + 100.$
$= 156 + 100.$
$= 256.$

2) $529 = 52 \times 10 + 9$
$= 520 + 9$
$= 529$

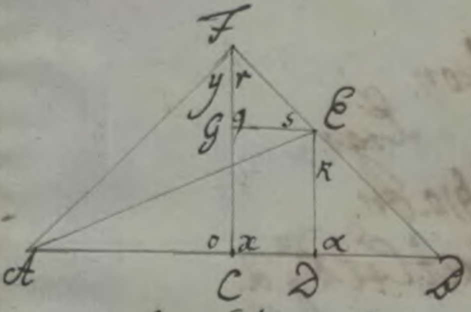
§225. Scholion.

Sit $AD = 13$. $AC = 3$. Ergo $CD = 10$.
Ergo $AD + AC = 16$.
et $AD + DC = 23$.

☆ ☆

§226. Theorema 58.

Si recta Linea AB secetur in æqualia AC et CB et non æqualia AD et DB Quadrata, quæ ab inæqualibus totius Segmentis AD, DB fiunt, muldupplicia sunt, et ejus quod a dimidia AC et ejus quod ab intermedia Sectionum fit CD, Quadrati.



h. e. P. H. dmdm.

$AD^2 + DB^2 = 2 \times AC^2 + 2 \times CD^2$.

Demonstratio.

In C excita ⊥lem $FC = AC$. §120.

duc rectas AF et FB. §81.

quia $\angle o = R$. p. C. §44.

$\triangle$ ACF est Rect∠glum §59.

Ergo $AC^2 + CF^2 = AF^2$. §189.

sed $CF = AC$ p. C.

Ergo $CF^2 = AC^2$. §44. Ar. 175.

Ergo $AC^2 + AC^2 = AF^2$. §10 Ar.

h. e. $2 \times AC^2 = AF^2$.

$AC = CB$ p. H.

$AC = CF$ p. C.

$CB = CF$. §40. Ar.

$\angle B = $ Rllor. §100.

cumq. $\angle$lus° x = R. p. C.
Ergo $\angle B = \angle r = \frac{1}{2}R$. §162.
Per pctm D duc $DE \approx FC$. §135.
Ergo $\angle x = \angle \alpha$ §132.
$\angle x = R$. §44 et C.
$\angle \alpha = R$. §92.
Sed $\angle B = \frac{1}{2}R$.
Ergo $\angle \kappa = \frac{1}{2}R$. §156.
$\angle B = \angle \kappa$ §41 Ar.
Ergo $DB = DE$. §160
adeoq $DB^2 = DE^2$. §44. Ar. et 175. b.
Per pctm E duc $GE \approx$ cum AD §135.
Ergo CE est Pllgm §72.
adeoq $GE = CD$ §167.
atq $\angle x = \angle q = R$. §132. 92.
cumq $\angle r = \frac{1}{2}R$. pd.
Ergo $\angle s = \frac{1}{2}R$. §156.
$\angle s = \angle r$. §41 Ar.
adeoq $FG = GE$. §160.
Sed $GE = CD$. pd.
$FG = GE = CD$. §41 Ar
et $FG^2 = GE^2 = CD^2$ §44. Ar. 175. b.

* *

Cumq FGE Δ R$\angle$lum §59 Ergo
$FE^2 = FG^2 + GE^2$ §189,
$= 2 \times CD^2$ §10. Ar.
Porro $AC = CF$ p. C.
$\angle y = \angle EAF$. §100
$\angle y = \frac{1}{2}R$. §162.
Sed $\angle r = \frac{1}{2}R$. p. d.
$\angle AFE = R$. §42. Ar.
Ergo ducta AE, Triangulum AFE est R$\angle$lum §59.
Quamobrem

$$\left.\begin{array}{l} AF^2 + FE^2 = AE^2 \text{ et} \\ AD^2 + DE^2 = AE^2 \end{array}\right\} \text{§189.}$$

$AF^2 + FE^2 = AD^2 + DE^2$. §41 Ar.

$$\text{Sed } \left.\begin{array}{r} AF^2 = 2 \times AC^2 \\ FE^2 = 2 \times CD^2 \\ DE^2 = DB^2 \end{array}\right\} \text{pd.}$$

Ergo per §10 Ar.
$2 \times AC^2 + 2 \times CD^2 = AD^2 + DB^2$.
Q. E. D.

14b

Aliter

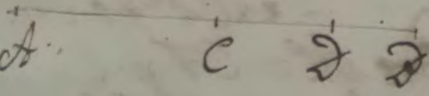
$AD^2 = AD^2 + DD^2 + 2 \times AD \times DD$ §207.

$\underline{AD^2 = 4 \times AC^2}$ §211

$AD^2 + DD^2 + 2 \times AD \times DD = 4 \times AC^2$ §41 (

Sed $AD \times DD + CD^2 = CD^2$ §212.

Ergo $AD \times DD = CD^2 - CD^2$ §43. A



Quare

$AD^2 + DD^2 + 2 \times (CD^2 - CD^2) = 4 \times AC^2$ §100

h. e.

$AD^2 + DD^2 + 2 + CD^2 - 2 \times CD^2 = 4 \times AC$

$AD^2 + DD^2 = 4 \times AC^2 - 2 \times CD^2 + 2 \times CD$

§43. 42. Ar.

cumq $AC = CD$ pr. H.

Ergo $AC^2 = CD^2$ §44 Ar. 175

Ergo

$AD^2 + DD^2 = 4 AC^2 - 2 AC^2 + 2 CD^2$ §100

i. e.

$AD^2 + DD^2 = 2 \times AC^2 + 2 \times CD^2$

Q. E. D.

§227. Scholion.

Esto $AD = 16$. Ergo AC ~~= CD~~ = 8.

et $AC^2 = 64$.

$AD = 13$ Ergo $BD = AB - AD$
$= 16 - 13$
$= 3$
et $CD = CB - BD$
$= 8 - 3$
$= 5.$

Hinc quia.
$AD^2 + BD^2 = 2 \times AC^2 + 2 \times CD^2$. §226.

Ergo

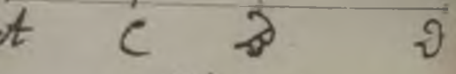
~~$169 + 9 = 2 \times AC^2 + 2 \times CD^2$. §226~~

~~Ergo~~

$169 + 9 = 2 \times 64 + 2 \times 25.$
$178 = 128 + 50$
$= 178.$

§228. Theorema 59.

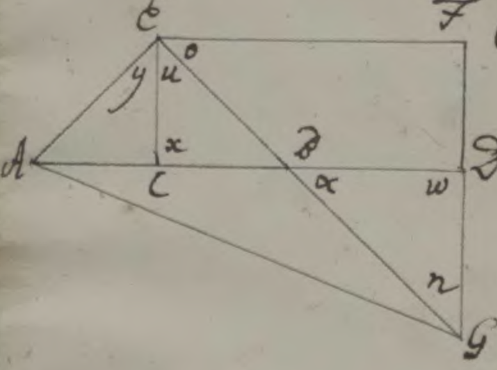
Si recta AB secetur bifariam in C, et adiiciatur in directum quaepiam alia recta BD, quod a Tota AB cum adjuncta BD h.e. AD, et quod ab adjuncta BD, utraque simul Quadrata, duplicia sunt et ejus quod a dimidia AC et ejus quod ~~[illegible]~~ a dimidia CB et adjuncta BD tanquam ab una CD descriptum sit, Quadrati.



p. Hdmdm
$AD^2 + BD^2 = 2 \times AC^2 + 2 \times CD^2.$

Demonstratio.

In C erige ⊥lem CE = AC. §120. duc AE ED §81 itemq; DF ∥ cum EC §135 occu rentem producte ED in G. §82 atq; per EF ∥lam cum AD. §135: tandem AG §



Hinc.

△ ACE est R∠glum æqcr p.C.

Ergo ∠y = ½R. §162.

△ CED est R∠glm æqer. p.C.

Ergo ∠u = ½R. §162.

∠y+u = R §42. Ar.

EC ∥ FG. p.C. Ergo

∠u = ∠n. §133.

∠n = ½R. §41 Ar

cumq ∠F = ∠x §169

et ∠x = ∠R. p.C.

∠F = R. §92.

atq ∠o = ½R. §156.

Ergo △EFG est R∠glum et æquicrurum §

Simili Ratiocinio, quia,

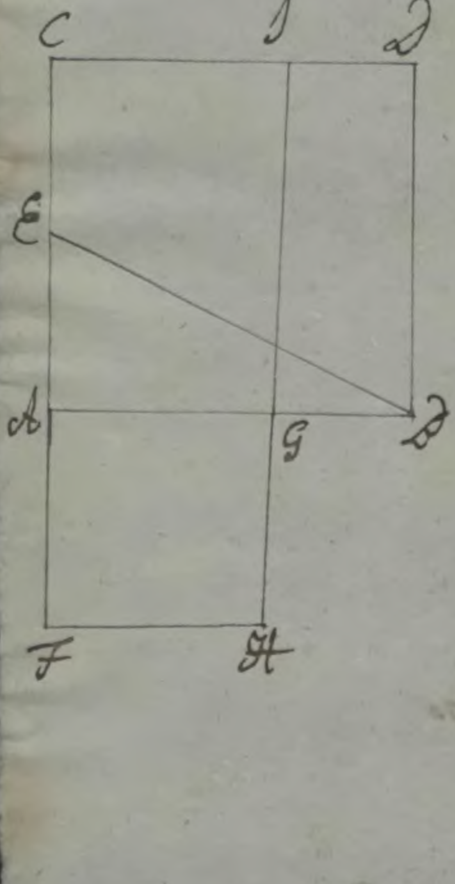
$\angle F = \angle w$. §132.
ergo $\angle w = R$. §92.
cumque $\angle n = \frac{1}{2} R$. p.d.
ergo $\angle \alpha = \frac{1}{2} R$. §156.
adeoque $BD = DG$. §160.
Ergo
$AE^2 = AC^2 + CE^2$ §189.
$= 2 \times AC^2$.
$EG^2 = EF^2 + FG^2$ §c.
cumque $EF = CD$ §167.
$= CD^2 + FG^2$ §44. Ar. 175.
$= 2 \times CD^2$ §10 Ar.
$AE^2 + EG^2 = 2 \times AC^2 + 2 \times CD^2$ §42. Ar.
$AE^2 + EG^2 = AG^2$ §189.
$AG^2 = AD^2 + DG^2$ §c.
$AD^2 + DG^2 = 2 \times AC^2 + 2 \times CD^2$ §41 Ar
sed $DG = BD$ p.d.
ergo $DG^2 = BD^2$ §44 Ar. 175 Ergo
$AD^2 + BD^2 = 2 \times AC^2 + 2 \times CD^2$ §10 Ar.
Q-E-D.

§229. Scholion.

Esto $AB = 12$. Ergo $AC = CB = 6$
$BD = 7$ Ergo $AB + BD = AD = 19$ et $BD + CB = 13 = CD$
Quare cum $AD^2 + BD^2 = 2 \times AC^2 + 2 \times CD^2$.
Ergo $361 + 49 = 2 \times 36 + 2 \times 169$
$410 = 410$.

§230. Problema XXV

Datam rectam AB secare in G, ut comprehensum sub tota AB et altero segmentorum BG rectangulum, aequale sit ei, quod a reliquo segmento AG fit, Quadrato.



Resolutio.

1) Super AB fac Quadratum §170.
2) Diseca CA in E. §112.
3) Duc EB. §18.
4) Produc EA in F. §82 ut $EF = EB$.
5) Lateris AF fac Quadratum §170.

Dico AF^2 s.q.i.e. $AG^2 = AB \times BG$.

Q. E. F.

Demonstratio.

Produc GH in I §82. quia CA est bisecta in E. p.C. et AF in directum adjecta p.C.

$CF \times AF + EA^2 = EF^2$. §218.

$EB^2 = EF^2$. p.C.

$CF \times AF + EA^2 = EB^2$. §41 Ar.

$EA^2 + AB^2 = EB^2$. §189.

$CF \times AF + EA^2 = EA^2 + AB^2$. §41 Ar.

Ergo:

$CF \times AF = AD^2$. §43 Ar ♀

~~Vel etiam hoc modo~~

h. e. $AH + HI = AI + ID$ §41 Ar

$AH = ID$. §43 Ar

h. e. $AG^2 = BD \times GD$ §175.

Sed $BD = AD$. §68.

Ergo $AG^2 = AD \times GD$ §10 Ar

Q. E. D.

§231. Corollarium.

Cum itaque fit per §230.

$CF \times FA = AD^2$

et $AF = AG$. §8.

Ergo

$CF : AD = AD : AG$. §311 Ar.

h. e.

Summa Totius et Segmenti majoris ad Totam, uti Tota ad Segmentum majus.

§232. Scholion.

Problema hoc Numeris accommodari non posse docet Clavius ad LIX P. 14 et 29 Euclid. Dicunt aliqui Rectam hoc modo sectam proportionaliter sectam esse.

♀

Vel etiam hoc modo

$CF \times FA = AG^2 + CA \times AG$ §205

$AD^2 = CA \times AG + CA \times GD$ §208.

$AG^2 + CA \times AG = CA \times AG + CA \times GD$ §41 Ar.

$AG^2 = CA \times GD$ §43 Ar.

Verum $CA = AD$. §68

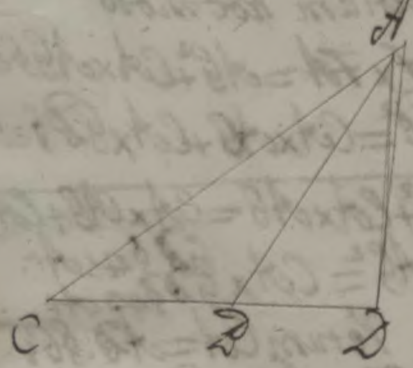
$AG^2 = AD \times GD$ §10 Ar

Q. E. D.

152

§233. Theorema 60.

In obtusis Triangulis Quadratum, quod fit a Latere CA, ∠lum obtusum
subtendente, majus est Quadratis, quae
fiunt a Lateribus AB, BC obtusum ∠
comprehendentibus Rectangulo bis c
prehenso et ab uno Laterum CB, quo p
circa ∠lum obtusum in quodcum p
tractum fuerit, cadit ⊥lis AD et ab
sumta exterius Linea BD sub norma
li, prope Angulum acutum ADB.



P. H. dmdm.
$AC^2 = AB^2 \times CB^2 + 2 \times CB \times BD$.

Demonstratio.

AD est ⊥lis p. H. Ergo
$AC^2 = CD^2 + DA^2$. §189. Sed
$CD^2 = CB^2 + BD^2 + 2 \times CB + BD$. §207.

$AC^2 = CB^2 + BD^2 + 2 \times CB \times BD + DA^2$. §10. Ar.
Verum $BD^2 + DA^2 = BA^2$ §189.

$AC^2 = CB^2 + BA^2 + 2 \times CB \times BD$. §10. Ar.

Q.E.D.

§234. Scholion.

Enimvero cum Euclides assumserit
⊥lem AD cadere in Latus

CB ad partes ∠li obtusi protractum, assumptum istud paucis demonstrabimus. Dicimus ergo: Normalem ex A ductam cadere extra Triangulum in Latus CB productum qualis est AD.

Aut enim intra CB qualis est AE.
extra CB ad partes BC, qualis est AF
extra CB ad partes CB, qualis est AD, cadet

Quare in

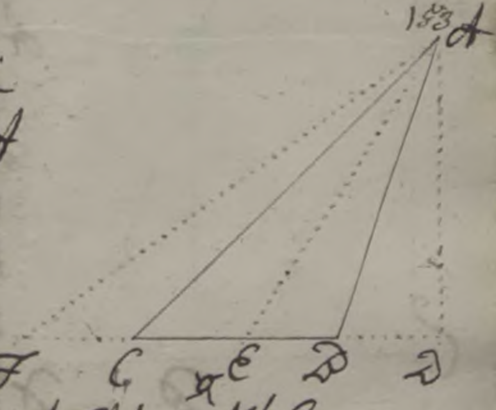
Casu I. Ponamus AE cadere intra CB p. H. ass. Ergo
$\triangle AEB$ est R∠glum ad E. §44.59.
Ergo $\angle AEB = R$. p. H. et C.
Sed $\angle ABE > R$. p. H. Propos.

$\angle AEB + \angle ABE >$ res 2 R. §42. et
I. Q. E. A. per §144.

Casu II. Ponamus AF ⊥lem cadere extra BC ad partes ∠li C.

1) Velut ante demonstrabis, id fieri non posse. Ergo
2) Vel h. tre

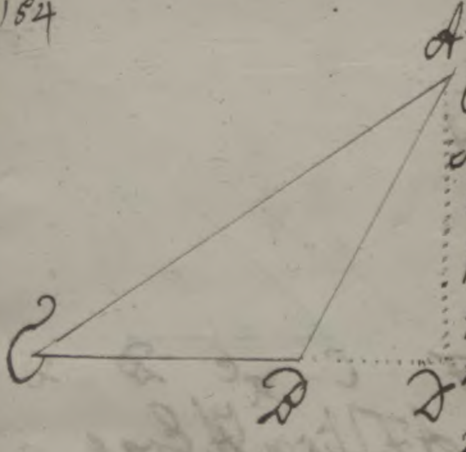
x



AF ⊥lis p. H. Ergo
$\triangle AFB$ R∠glm §44.59.
Ergo $\angle ABF < R$. §141.
Sed $\angle ABF > R$. p. H.
I. Q. E. A. p. §40 etr

Quoniam itaque
Neque Casus I } p. d.
Neque Casus II }
Ergo Casus III.

Q. E. D.



§235. Corollarium.

Cognitis autem Cruribus Trigoni o
sanguli facili omnino negotio inve
es Segmentum BD inter Angulum
tusum atq normalem AD ejusq a
malem AD; Illud a Quadrato Lateris
maximi §153. np. CA^2 auferendo sum
mam Quadratorum 2lum obtusum
complexorum et differentiam per d
plum Lateris CB in quod si produca
tur Alis cadit, dividendo; Hanc Quadra
tum inventi modo Lateris BD cogn
ti BA auferendo et Radicem quadra
ticam extrahendo. Quia enim:

$AC^2 = CB^2 + BA^2 + 2 \times CB \times BD$. §233.

Ergo $AC^2 - CB^2 - BA^2 = 2 \times CB \times BD$. §43. Ar.

Ergo $\frac{AC^2 - CB^2 - BA^2}{2 \times CB} = BD$. §45. Ar.

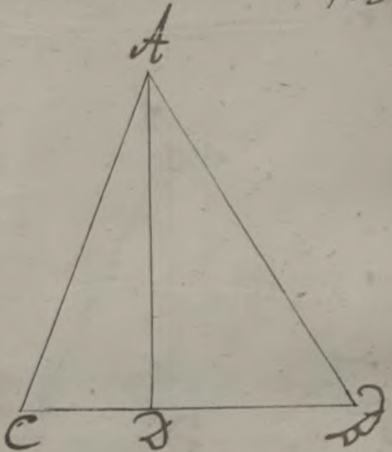
Q. E. I.

Cumque Δlum BDA sit R∠glm §234

Ergo $\sqrt{BA^2 - BD^2} = AD$. §195. Q. E. II D.

§236. Theorema 61.

In Triangulis acutangulis ADC Quadratum a Latere AD, ∠um acutum ACD subtendente, minus est Quadratis, quae fiunt a Lateribus AC et CD acutum ∠um comprehendentibus, rectangulo bis comprehenso et ab uno Laterum DC, quae sunt circa acutum ∠um ACD in quod normalis Ad cadit et ab assumta interius Linea dC sub perpendiculari Ad prope ∠um acutum ACD.



P. H dmda:

$AC^2 + DC^2 = AD^2 + 2 \times DC \times Cd$

Demonstratio.

Ad est ⊥lis p. H.

$AC^2 = Ad^2 + dC^2$ §189.

$DC^2 = DC^2$ §40 Ar.

$AC^2 + DC^2 = Ad^2 + dC^2 + DC^2$ §42 Ar.

$DC^2 = Dd^2 + 2 \times Dd \times dC + dC^2$ §207

$AC^2 + DC^2 = Ad^2 + Dd^2 + 2 \times Dd \times dC + 2 \times dC^2$ §10 Ar.

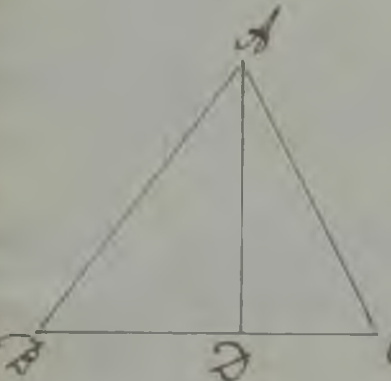
Verum $Ad^2 + Dd^2 = AD^2$ §189

$AC^2 + DC^2 = AD^2 + 2 \times (Dd \times dC + dC^2)$ §81 Ar.

Sed $Dd \times dC + dC^2 = DC \times Cd$. §205.

Ergo $AC^2 + DC^2 = AD^2 + 2 \times DC \times Cd$. §10 Ar.

Q. E. D.



§237. Corollarium.

Simili autem qua §235. usi sumus methodo invenies et Segmentum inter Angulum acutum C atque normalem AD interceptum DC, et ⊥lem AD ipsam, Illud a Summa Quadratorum Angulum acutum incipientium auferendo Quadratum Lateris DA ∠lum dictum subtendentis, Residuumque dividendo per duplum Lateris DC in quod normalis cadit. Hanc autem, Inventi modo Lateris DC Quadratum auferendo ex AC², atque quadraticam radicem extrahendo §256. Ar. Cum enim

1. $AC^2 + DC^2 = AD^2 + 2 \times DC \times CD$ §.236.

$AC^2 + DC^2 - AD^2 = 2 \times DC \times CD$. §48. Ar

$\frac{AC^2 + DC^2 - AD^2}{2 \times DC} = CD$. §45. Ar

Q. E. I.

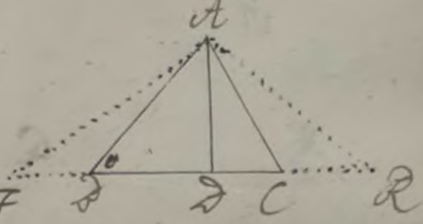
2). Triangulum ADC R.∠glum p. H.

$\sqrt{AC^2 - DC^2} = AD$. §195.

Q. E. II D.

§238. Scholion.

In acutangulis autem Triangulis $\perp$lem AD cadere intra Latus acutis adjacens $\angle$lis B et C ita evincitur.



Aut enim intra cadet
Aut extra et quidem
vel ad partes $\angle$li B crure CB continuato v.c. in F.
vel ad partes $\angle$li C crure BC continuato v.c. in R.
Aut in rectarum utramq. AB vel AC.

Ponamus 1) $\perp$lem AD cadere posse in AB p. H.
Ergo $\angle o = R$. §44.
Sed $\angle o <r R$. p. H.
I. Q. E. A. §40 Ar.
Simili discursu liquet AD cadere non posse in AC.

2) $\perp$lem cadere extra CB in F. p. Hyp.
Ergo $\angle F = R$. §44.
$\angle o >r \angle$lo F. §113.
$\angle o >r R$. §46. Ar.
Verum $\angle o <r R$. p. H.
I. Q. E. A. §40 Ar.

Idem simili modo evincetur, si statuatur $\perp$lem cadere posse ad partes $\angle$li acuti C, qualis est AR.

Quare, cum neque Casus I.
neque Casus II.
Ergo omnino IIIius.

Q. E. D.

§239. Problema XXVI

Dato rectilineo A aequale Quadratu
constituere.

Resolutio.

1) Construe Rectangulum DB = et
2) Majus illius Latus DC produc it
ut sit FC = CB §82. 26
3) Disseca DF in G. §112.
4) Radio DG describe semicirculum DQF. §85.
5) Produc CB §82. ad peripheriam usque in
H. DF.

h. c. $CH^2 = A$.

Demonstratio.

Duc GH. §81. quia

$A = DC \times CB$. p. 1. et 175.

$CB = FC$. p. 2.

$A = DC \times FC$. §19. Arit.

$GC^2 = GC^2$. §40 Ar.

$A + GC^2 = DC \times FC + GC^2$. §42 Ar.

$FG^2 = DC \times FC + GC^2$ §212.

$A + GC^2 = FG^2$. §41. Ar.

Verum

& $FG = GH$. §26.

Ergo $FG^2 = GH^2$. §44. A. 175.

Ergo

$A + GC^2 = GH^2$. §41 Ar

$GC^2 + CH^2 = GH^2$. §189

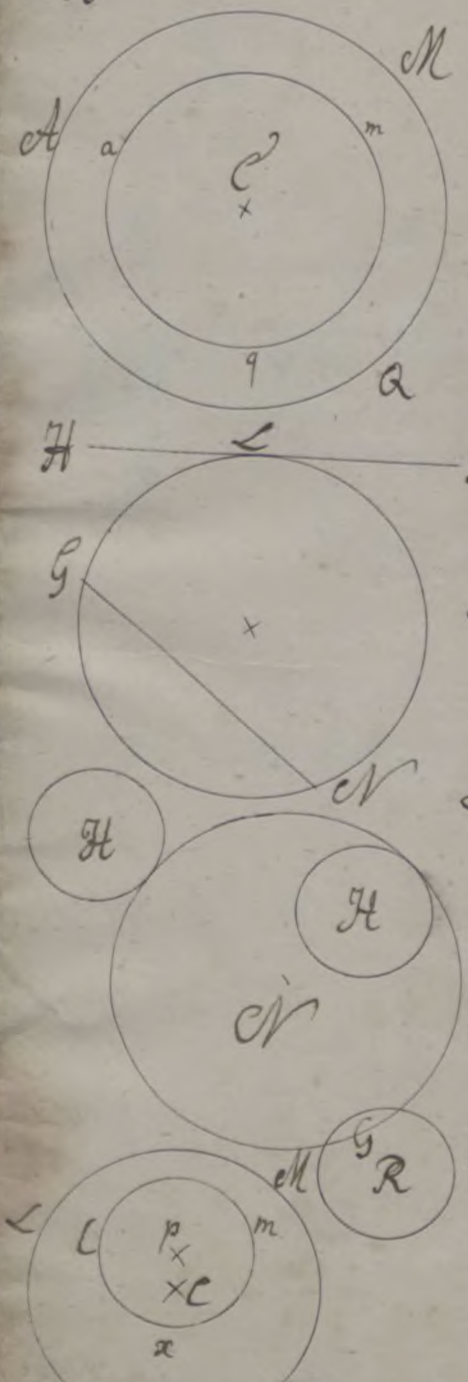
$A + GC^2 = GC^2 + CH^2$. §41 Ar

$A = CH^2$. §43. Ar.

Q. E. D.

Caput III$^{\text{tium}}$

De Circuli affectionibus.



§240. Definitio LIX.

Circuli concentrici AMQA et am -C. sunt qui habent idem Centrum C. Excentrici autem LMXL et lmxl quæ diversa C et p.

§241. Definitio LX.

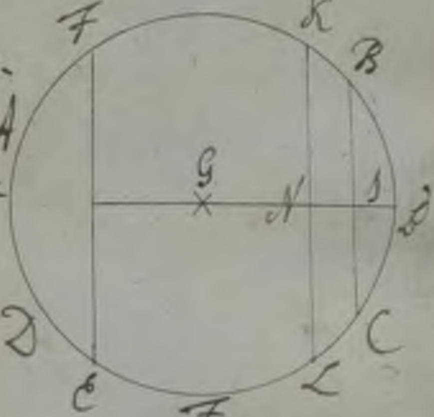
Recta HI circulum tangere dicitur in L si ipsi ita occurrat ut producta, tota extra circulum cadat. Recta autem GN circulum secat, si circulum in partes cis et ultra sitas dirimat.

§242. Definitio LXI.

Circulus N circulum H tangit extus, si huic occurrens, totus extra hunc; intus autem, si totus intra hunc cadat. Secare autem Circulus N circulum R dicitur, si partem communem G habuerint.

§243. Definitio LXII

In circulo GEABD Chordae FE, KL aequaliter distare dicuntur cum perpendicula GH, GN ex centro G ad ipsas ducta fuerint aequalia. Longius autem Chorda BC ab altera KL abesse dicitur in quam majus perpendiculum G.I cadit.

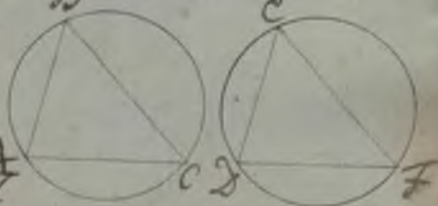


§244 Definitio LXIII.

Segmentum circuli AFBA est pars ipsius arcu AFB et chorda AB comprehensa. Segmentum circuli majus dicitur, quod semicirculo majus; minus autem quod semicirculo minus est

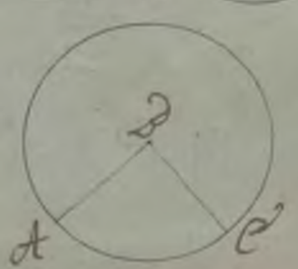
§.245: Definitio LXIV.

Similia circuli segmenta sunt illa, quae ∠los ABC, DEF capiunt aequales; aut in quibus ∠li ABC, DEF sunt aequales.

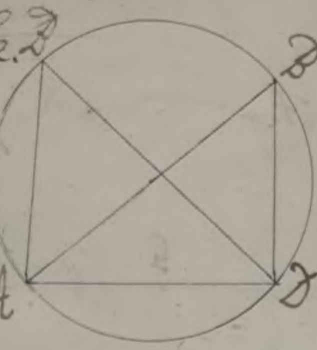


§246. Definitio LXV

Sector circuli ABCA est pars ipsius duobus radiis CB, BA et arcu comprehensa C.A.

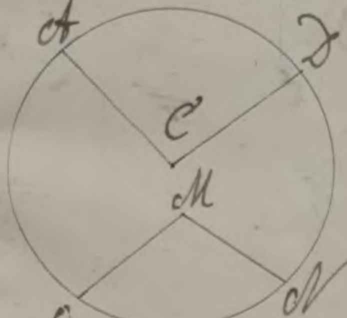


162.

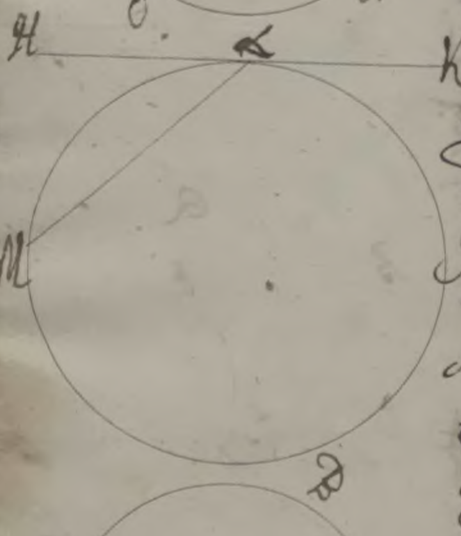


§247. Definitio LXVI.
Angulus in segmento s. q. i. e. Angu
ad Pphiam ABD est, cujus vertex D
crura DA atq; DB in peripheria term
nantur.

§248. Definitio LXVII.
Angulus ad Centrum ACD est cujus ve
tex C in Centro, crura vero CD et CA in
Pphia terminantur. Angulus vero ex
Centrum OMN dicitur, cujus vertex
M extra Centrum crura vero in P
terminantur.



§249. Definitio LXVIII.
Angulus segmenti MLH vel MLK.
dicitur quem chorda ML. cum tan
gente HK ad Contactum efficit. No
autem cum Euclide Def. VI. L. III Lu
segmenti CAB dicemus illum qu
continetur Chorda AC arcuque A
cf. Jo. Barrow et Cph. Clavius ad
Definitionem citatam.



§250. Problema XXVII

Dati Circuli ADC Centrum F invenire.

Reſolutio.

1) Duc in Circulo propoſitam rectam s. chordam quamlibet AC §81.
2) Biſeca illam §112. et duc ⊥ BD. §c.
3) Biſeca et hac BD in F. §c.

Dico in F eſſe centrum.

Demonſtratio.

Aut Centrum non eſt in F, aut eſt in F.

Ponamus in F non eſſe Centrum ergo nullum aliud in recta BD punctum eſſe poterit Centrum §26. Ponatur itaq extra rectam BD punctum G eſſe poſſe Centrum. Educ itaq radios AG, GC ad rectam AC. §81 ut et EG ad eandem AC. §c.

Ergo AG = GC. §26.
AE = EC p. C.
GE = GE §40 Ar:

∠AEG = GEC. §106.
Ergo ∠AEG = R §38. sed et
∠AEF = R. p. C. ad mbr. II.

∠AEG = ∠AEF. §92. Q. E. A. §41 Ar.

§251. Corollarium.

Inde quidem si in Circulo recta q
dam Linea BD aliquam rectam
neam AB bifariam et ad ∠los rect
secet, erit in secante BD centrum.

§252. Theorema 62.

Si in Circuli CAB Pphia duo quo
puncta A et B accepta fuerint, rect
Linea AB, quae ad ipsam puncta
jungitur intra Circulum cadet.

Demonstratio

Accepto in Recta AB quolibet p
to D, duc rectas AC, CD, CB §81.

Quia $CA = CB$ §26.

$\triangle ACB$ est aequicrurum §57.

Ergo $\angle A = \angle B$. §100.

cumq $\angle CDB > \angle$lo A §113

P $\angle CDB > \angle$lo B. §46. Ar.

$CB > CD$. §115.

Cum itaq CB ex Centro ad Pphiam
pertingat, ergo $CD < CB$ ad peri-
pheriam pertingere non pot
Ergo AB intra illam cadit.

Q. E. D.

§283. Corollarium.

Hinc Recta Circulum tangens, ita ut non secet ~~in~~ illud in unico puncto tanget. Si enim tangeret in duobus intra Circulum esset per §252. et ita sane ipsam non tangeret. §271.

§284. Theorema 63.

Si in Circulo CADC recta quaedam DD per Centrum extensa, quandam AC non per Centrum extensam bifariam secet in F secabit ipsam AC ad ∠los Rectos. Et si ad ∠los Rectos secet ipsam, bifariam quoque ipsam secabit.

Demonstratio

Mbr I. Ex Centro C duc CA, CC. §81

Ergo $AF = FC$. p. H.

$EF = EF$ §40 Ax.

$EA = EC$. §26.

$\angle o = \angle y$ §106.

Ergo et ∠lo et $\angle y = R$. §387

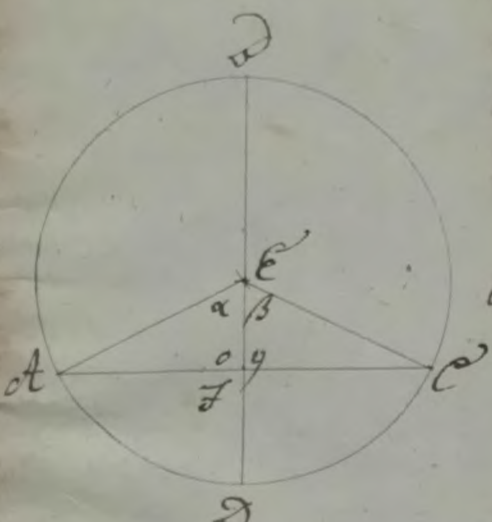
Q. E. I.

Mbr II. $\angle o = \angle y$ p. H.

$EF = EF$ §40 Ax.

$AE = EC$ §26.

$AF = FC$ §129. Q. E. II.



Membrum 2dum aliter ita demon-
strabitur

Quia DF per Centrum E extensa p

Ergo AE = CE. §26

Ergo △ AEC æquicrur: §57.

Ergo ∠A = ∠C. §100

sed ∠o = ∠y p. H.

Cumque AE = EC p. d.

E AF = FC. §114. Q. E. D.

Vel

AE = EC. §26

Ergo ∠A = ∠C. §57. 100

sed ∠o = ∠y p. H.

Ergo ∠α = ∠β. §155.

Cumque EF = EF. §40 Ar

AF = FC. §114. Q. E. D.

§255. Theorema 64.

Si in Circulo ACD duæ Rectæ AB et DC. sese mutuo secent non tamen per Centrum E extensæ fuerint se mutuo bifariam non secant.

Demonstratio

Ponamus AB bifariam secari a C in E. dico CD non secari ab AB bifariam et contra.

Invento enim circuli centro §250. duo radios AF, BF, CF, DF §81. atque ex centro ad mutuam intersectionem FE. §80

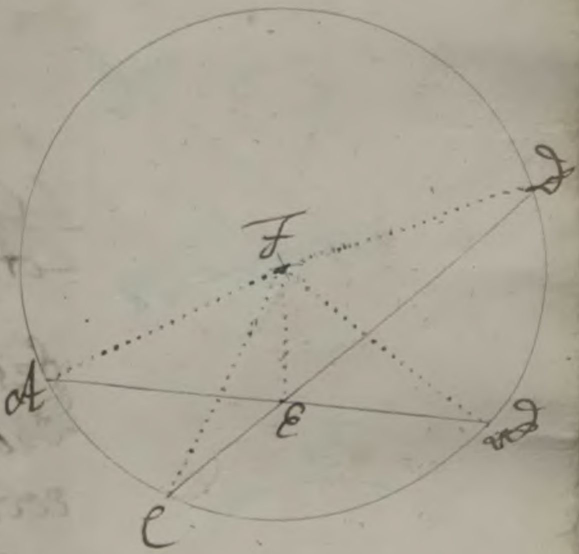
Quia AE = EB. p. Hyp.
AF = FB. §26.
FE = FE. §40 Ar.

∠AFE = EFB §106.
∠CFE <r ∠lo AFE. §47 Ar

∠CFE <r ∠lo EFB. §46 Ar.
∠EFB <r ∠lo EFD. §47. Ar.

∠EFD >r ∠lo CFE.
FD = CF. §26.
FE = FE §40 Ar

ED >r CE. §164. Q. E. D.



Simili modo ostendetur sub hypothesi Theorematis hujus, licet secta sit CD bifariam ab AB in E, neque vicissim bifariam secari AB ab CD.
Q. E. D.

Ponamus autem utramque Rectam et AB et CD in E bifariam sectam esse si fieri possit, neutra earum per centrum extensa.

163.

Invento Circuli Centro §250.
Duc FE. §81.
Quia AE = ED p. Hyp.
et EF per Centrum extensa p. l.
$\angle AEF = R$. §254.

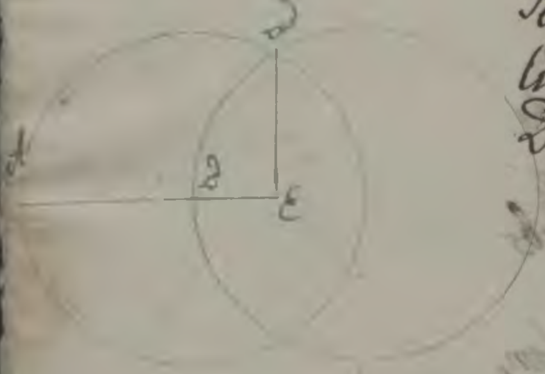
Similiter
CE = ED p. Hyp. et
EF per Centrum extensa. p. l.
$\angle CEF = R$ §c

$\angle AEF = \angle CEF$ §92.
Q. E. A. §47 Ar.

§256 Theorema 65.
Duo Circuli se mutuo secantes sunt eccentrici

Demonstratio

Ponatur si fieri possit utriusque Circuli Centrum E.
Duc radios DE, DE, AE §81
Ergo DE = AE } §26
DE = DE }
AE = DE §41 Ar.
AE = AD + DE. §47. Ar.
AD + DE = DE. §216 } Ar.
J. Q. E. A. §41 }



Aliter

Ductis AB et DB §81.

$DE = EB$ p. H. ass.

Ergo $\angle o = \angle y$. §100.

sed $AE = EB$ p. H.

$\angle A = \angle u + o$ §c. sed

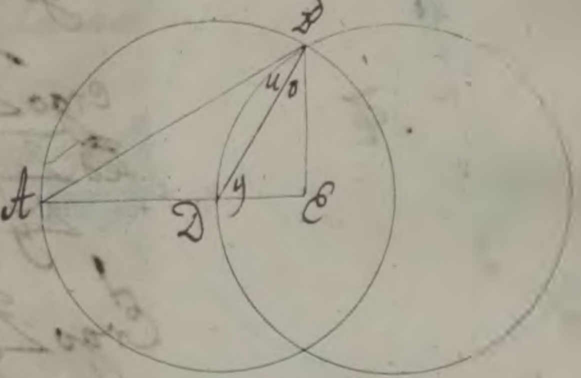
$\angle y >^{r} \angle lo\ A$. §113.

$\angle y >^{r} \angle lis\ u + o$ §46. Ar.

sed $\angle o = \angle y$ p. d.

$\angle y >^{r} \angle lis\ u + y$. §100 Ar.

I. Q. E. A. p. 47 § Ar.



§257. Theorema 66.

Circuli ſeſe mutuo interius tangentes ſunt eccentrici

Demonſtratio.

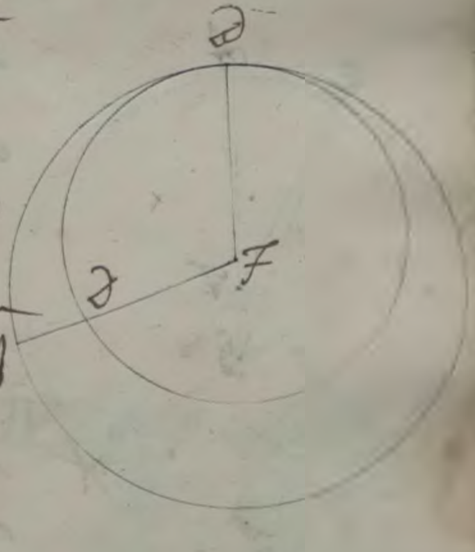
Ponamus utriusq; centrum eſse in F.

Ex contactus B puncto duco radium BF atq; aliam AF §8 A.

Ergo $BF = DF$ } §26
$BF = AF$ }

$DF = AF$. §41 Ar.

I. Q. E. A. §40 Ar.



Aliter

Ductis BD et DA §81 Quia

$BF = FD$ p. H.

ergo $\angle o = \angle y$. §100

Sed et $AF = FD$ p. H.

ergo $\angle A = \angle u+o$. §100.

Sed $\angle y$ >r $\angle A$. §113.

$\angle y$ >r $\angle$lis $u+o$ §46 Ax.

Sed $\angle o = \angle y$ p. D

Ergo $\angle y$ >r $\angle$lis $u+y$ §100 Ax.

I. Q. E. A. §47. Ax.

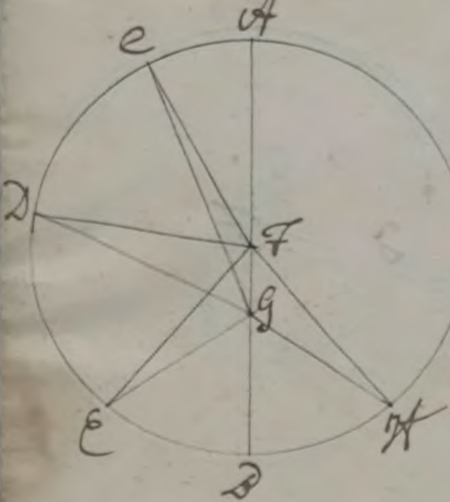
§258. Theorema 67.

Si in Circuli Diametro AB quodam sumatur punctum G quod n sit Circuli centrum, abeoque pun in Circulum quaedam lineae recta GC, GD et GE cadant:

1) Maxima quidem erit ea, GA in qua est centrum F.

2) Minima vero reliqua GB.

3) Aliarum vero illi, quae per centrum ducitur propinquior GC remotiore GD semper major est.



+ Duo autem solum Lineæ rectæ GE et GH æquales ab eodem puncto in Circulum cadunt ad utraque partes minimæ GD vel maximæ GA.

Demonstratio.

Ex Centro F duc rectas FC, FD, FE. §81. fiatque ∠lus DFE = DFH. §107.

Quare.

I $GC < CF + FG$ §116.

Sed $CF = FA$ §26.

cumque $FG = FG$. §40 Ax.

$CF + FG = GF + FA$ §42 Ax.

$GC < GF + FA$. §46 Ax.

$GF + FA = GA$. §47 Ax.

$GC < GA$.

II $EF < EG + FG$. §116.

$EF = FD$. §26.

$FD < EG + FG$ §46 Ax.

$FG = FG$. §40 Ax.

$GD < EG$. §48 Ax. Q. D. II.

III $GF = GF$ §40 Ax.

$EF = CF$ §26.

$\angle GFE < \angle GFC$. §47 Ax.

$EG < CG$. §162.

IV $\angle GFE = \angle GFH$ hyp.

$FG = FG$. §40 Ax.

$EF = FH$. §26

$EG = GH$ §99

Q. E. V. D.

§259. Theorema 68.

Si extra Circulum ſumatur punctum quodpiam A ab eoq; pūcto ad Circulum deducantur quodam Lineæ rectæ AJ, AH, AG, quarum una quidem AJ per Centrum K tranſeat, reliquæ vero ut libet.

I) In cavam Sphiam cadentium rectarum linearum.

1) maxima quidem eſt illa AJ, quæ per Centrum ducetur,

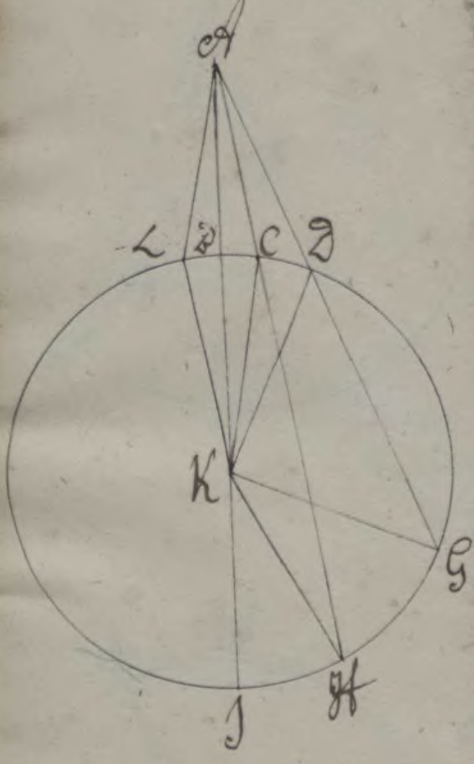
2) aliarum vero ei, quæ per Centrum tranſit propinquior AH remotiore AG ſemper major eſt.

II) In convexam vero Sphiam cadentium rectarum linearum.

1) minima quidem eſt illa AD, quæ inter punctum A et diametrum DJ interponitur,

2) aliarum autem, ea, quæ eſt minimæ propinquior AC remotiore AD ſemper minor eſt.

3) Duæ autem tantum rectæ lineæ AC et AL æquales ab eo puncto in ipſum Circulum cadunt ad utramq; partes minimæ AD vel maximæ AJ.



P.H. dmdm.

I) 1) $AJ > AH$.
2) $AH > AG$.

II) 1) $AB < AC$
2) $AC < AD$.
3) $AL = AC$

Demonstratio

Duc ex Centro, ~~KH~~, KG, KD, KC §81
et fac $\angle$lum $AKL = \angle$lo AKC §107.

Casus 1mi Quare in mbro 1ma.

$AK + KH > AH$. §116.
$KH = KI$ §26
$AK = AK$. §40. Ax
$AK + KH = AI$ §42. Ax
$AI > AH$ §46 Ax

Q.E.I.

Mbro 2do

$AK = AK$. §40. Ax.
$KH = KG$. §26.
$\angle AKH > \angle$lo AKG. §47. Ax.
$AH > AG$. §164.

Q.E.II.

Casus 2di mbro 1.

$AK < AC + CK$. §116.
$DK = CK$. §26.
$AD < AC$. §43. Ax.

Q.E.III.

Mbro 2do

$AK = AK$. §40. Ax
$CK = KD$. §26
$\angle AKC < \angle$lo AKD. §47. Ax
$AC < AD$. §164.

Q.E.IV.

Mbro 3io

$AK = AK$. p. d.
$KL = KC$ §26
$\angle AKL = \angle AKC$. p. C
$AL = AC$. §99

Q.E.V.D.

§280 Theorema 69.

Si in Circulo DCK acceptum fuerit punctum aliquod A et ab eo puncto ad Circulum cadant plures quam duæ rectæ Lineæ æquales DA, CA, AK acceptum punctum A centrum eſt ipſius Circuli

Demonſtratio.

Duc rectas DC et ACK. §81. eaſq; biſeca ~~recti~~ in E. et F. §112. ~~et duc rectas AE et AF.~~ H.

Quare cum DA = AC p. H.

DE = EC p. C.

EA = EA. §40. Ar

$\angle x = \angle y$ §106.

Ergo EA eſt ⊥lis §34. 36 44.

Ergo in EG eſt Centrum §251.

Simili diſcurſu

CA = AK p. H

CF = FK. p. C.

AF = AF. §40. Ar

$\angle o = \angle u$ §106.

Ergo AF eſt ⊥lis §44.

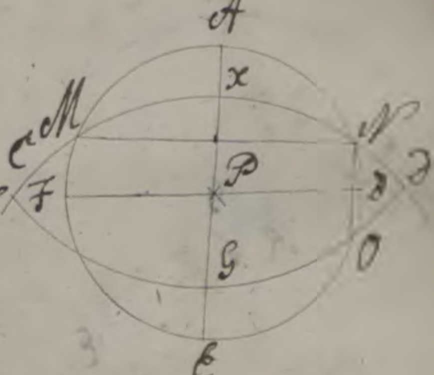
Ergo in FH erit Centrum §251. Quare ~~FH erit~~ ob interſectionem communem ſecantium EG et FH per Centrum tranſeuntium in A §80. erit A Centrum. §23.

Q. E. D.

§261. Theorema 70.

Circulus CDG circulum ADEF in pluribus, quam duobus punctis non secat.

Demonstratio.



Ponamus circulum CDG circulum ADEF secare posse in punctis pluribus v. c. in tribus M. N. et O. Ergo junctis rectis MN et NO. §81.

Diseca et MN et NO. normalibus AE et DF. §112.

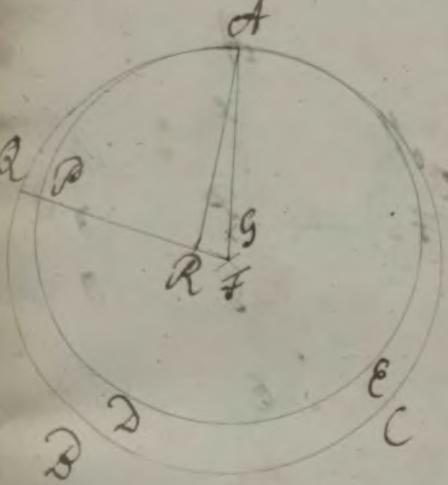
Quia

Puncta M. et N in circulo ADEF p. H.
Ergo illius centrum in lli AE. §251.
et Puncta N atq O in eodem circulo ADEF. p. H.
Ergo centrum illius in lli DF. §251.
Ergo centrum in P. p. d. ad §260.

Similiter quia

Puncta M et N itemq N. et O in circulo CDG p. H
Ergo centrum illius in llibus XG et QF. §251.
adeoq in P. §80. et d. ad §260.
Ergo duo circuli semet secantes idem centrum habent
s. Q. E. A. §256.

Q. E. D.



§26e. Theorema 21

Si duo Circuli GADE et FADC sese intus contingant, et accepta fuerint eorum Centra G et F, ad eorum Centra adjuncta recta Linea FG et producta in Circulorum Contactum cadet,

Demonstratio

Aut recta GF Centra connectens si producatur in Contactus punctum A cadet, aut non cadet.

Si non cadat, secabit utriusque Circuli Peripheriam

Inventis ergo Circulorum ADEG et FADC Centris R atque F. §e50

Duc AR, et AF. §81. et per Centra R et F rectam FRPQ §.c. Ergo

$AR + RF > AF$ §116

sed $AR = RP$ } §26.
$AF = FQ$ }

$RP + RF > FQ$. §10. 47. Ar

$RF = RF$. §40. Ar

$RP > RQ$ §48. Ar

J. Q. E. A. §47. Ar.

§263. Theorema 72.

Si duo Circuli ACD et BCE sese exterius contingant in C linea recta AB, quae ad eorum Centra A et B adjungitur per Contactum C transibit.

Demonstratio.

Sit, si fieri potest alia recta Linea secans Circulos extra Contactum C n. p. ADEB, educta ex A et B. §81. ducitaque ad ipsum Contactum C ex Centris A et B duas alias rectas. AC et CB §81 Ergo

$AC + CB > ADEB$. §116.

Sed $AC = AD$ }
$CB = BE$ } §26

$AC + CB = AD + BE$. §42 Ar.

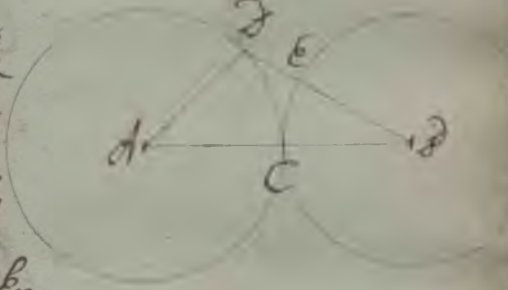
$AD + BE > ADEB$. §46. Ar.

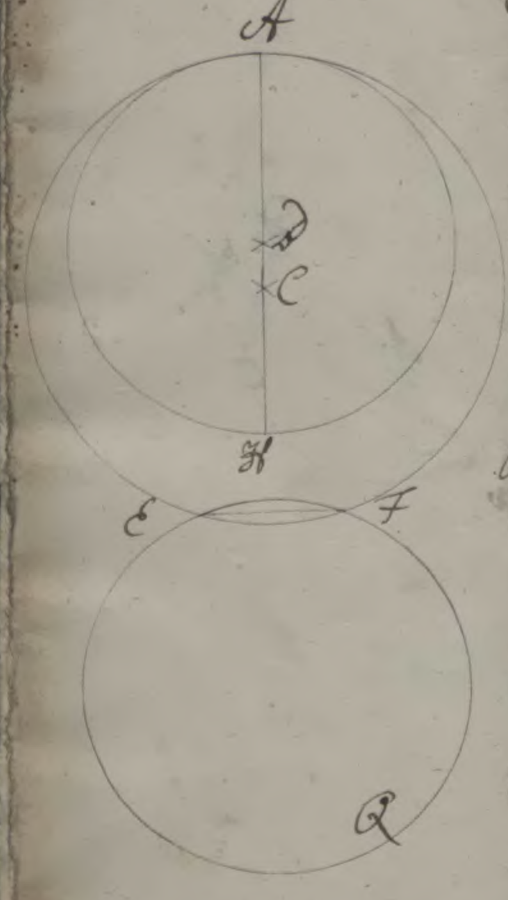
Verum

$ADEB = AD + DE + EB$. §47. Ar.

$AD + BE > AD + DE + EB$. §46. Ar.

q. E. A. per §47 Ar.





§269. Theorema 73.

Circulus CAF Circulum DEAH no
tangit in pluribus punctis quan
uno A, sive intus tangat sive exte

Demonstratio

Casus I. Si Circuli se tangent intus
Pone Circulum DEAH tangere alt
rum CAF in duobus punctis A et
H. Ergo
Recta DC per Centrum utriusque cir-
culi means productaque §82 cadet i
Contactus puncta A et H §262. Ergo
Cum AC = CH §26.
AD = DH §C.
Sed DH > CH §47 Ar
Ergo AD > AC §46 Ar
J. Q. E. A. p. §47 Ar.

Casus IIdus Si Circuli se tangant extus
Ponamus se contingere posse in
punctis E et F. Erit ergo Recta
EF in utroque Circulo CAF et QEF §83 §252.
Ergo se mutuo secant Circuli §222
JQ. C. H. quo Circulos se tantum
contingentes suppon~t Q. E. D.

§265. Theorema 74.

In Circulo EABC aequales Lineae AC et BD aequaliter distant a Centro E. Et Rectae AC et BD aequaliter distantes a Centro, inter se sunt aequales.

Demonstratio

Educ ex Centro E ⊥les EF, EG §119.

Ergo $\left.\begin{array}{l} AF = FC \\ BG = GD \end{array}\right\}$ §254.

Junge AE et EB §51.

Ergo in

Casu I. Quia $AC = BD$ p. H

Ergo $\frac{1}{2}AC = \frac{1}{2}BD$. §45 Ar.

h. e. $AF = BG$.

Ergo $AF^2 = BG^2$ §44. Ar.

cumque $AE = EB$ §26.

Ergo $AE^2 = EB^2$ §44. Ar.

$AE^2 - AF^2 = EB^2 - BG^2$ §43. Ar.

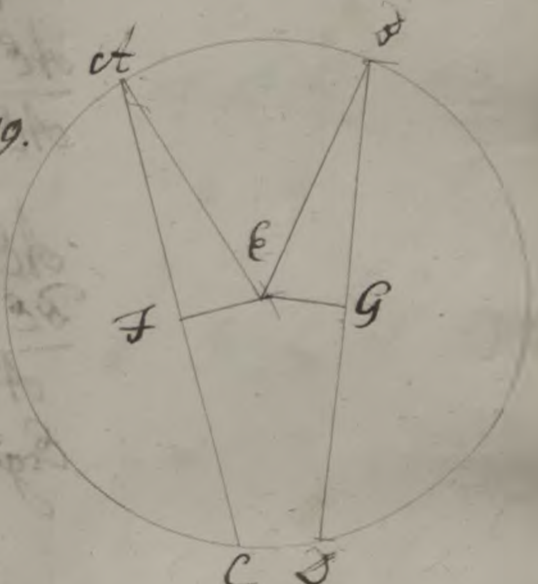
cumque $\angle F = \angle G = R$. p. C. §92.

$\left.\begin{array}{l} AE^2 - AF^2 = FE^2 \text{ et} \\ EB^2 - BG^2 = EG^2 \end{array}\right\}$ §195.

$FE^2 = EG^2$ §41 Ar.

$FE = EG$. §19

Q. E. I.



h. e. dmdm.

1) Si $AC = BD$ erit $FE = EG$

2) Si $EF = EG$ erit $AC = BD$.

Casu 2do:

$\angle F = \angle G = R$. p. C. §92.

et $EF = EG$. p. H. Ergo

$EF^2 = EG^2$. §44 Ar:

$AE = ED$ §26. Ergo:

$AE^2 = ED^2$. §44 Ar.

$AE^2 - ED^2 = ED^2 - EG$. §43 Ar.

Verum

$$\left.\begin{array}{l} AE^2 - EF^2 = AF^2 \text{ et} \\ DE^2 - EG^2 = GD^2 \end{array}\right\} \text{§195}$$

$AF^2 = GD^2$. §41 Ar

Ergo $AF = GD$. §197. sed

$$\left.\begin{array}{l} AF = \frac{1}{2} AC \\ DG = \frac{1}{2} BD \end{array}\right\} \text{p. C.}$$

$\frac{1}{2} AC = \frac{1}{2} BD$. §41 Ar.

Ergo

$AC = BD$. §44 Ar:

Q. E. D.

§266. Theorema 75.

In Circulo GAD(C) maxima quidem Linea est Diameter AD & aliarum autem Centro G propinquior FE remotiore DC semper major est.

Demonstratio.

Membrum 1. Duc DG et GC. §81.

Quia $DG + GC > DC$ §116

et $DG = AG$ } §26.
$GC = GD$ }

$DG + GC = AG + GD$. §42. Ar

$AD > DC$. §46. Ar.

Q. E. 1.

Membrum 2.

Sit Distantia $GH < GI$.

Fac ergo $GN = GH$

per N duc KL $\perp$lem ad GI. §120.

Ergo $FE = KL$. §265.

ductis KG atq. GL §81.

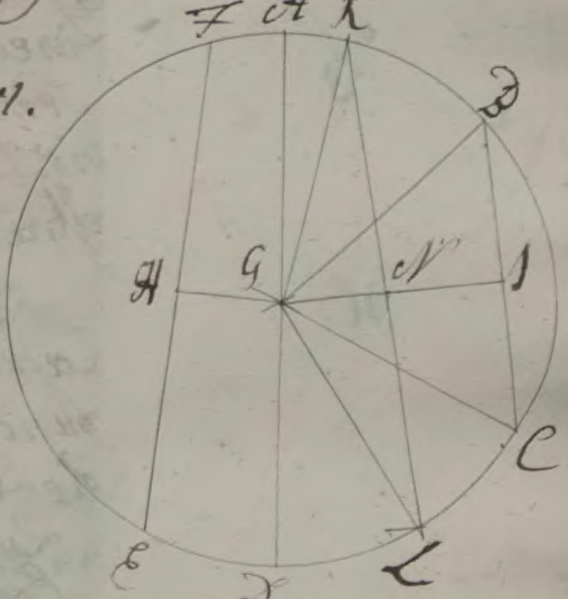
quia $KG = GD$ } §26
$GL = GC$ }

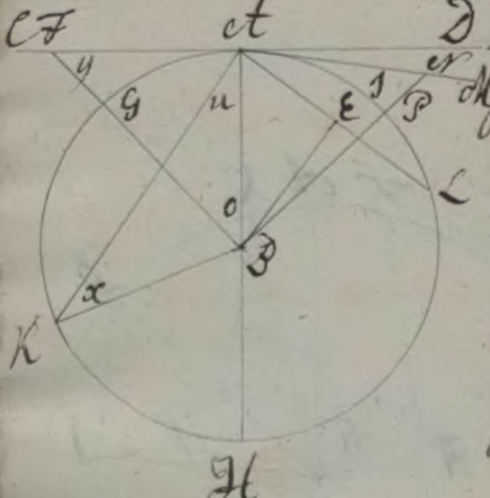
et $\angle KGL > \angle DGC$. §47 Ar

Ergo $KL > DC$. §164

Sed $FE = KL$ p. d.

$FE > DC$. §46. Ar Q. E. D.





§267. Theorema 76.

Recta CD quæ ab extremitate Dia
metri HA cuiusvis Circuli DALH a
∠lo R. ducitur, extra Circulum ea
in Locum inter ipſam Rectam et
Pphiam comprehenſum altera Rec
Linea AL non cadet.
Semicirculi quidem ∠lus HAD
vis ∠lo acuto rectilineo DAL maj
eſt, reliquus autem DAI minor.

Demonſtratio.

Caſus 1. Ex Centro B ad quodvis
punctum F Rectæ CD duc aliam
Rectam BF. §81
Quia CD ⊥lis ad AH p. H
Ergo ∠y < ∠lo FAB. §148.
atq AB < FB. §115.
Cum itaq punctum A ſit in Pphi
Ergo pctm F Lineæ CD extra ea
dem. Idem eodem modo de quibu
aſsumtis in Recta CD punctis de
monſtrabitur. Q. E. 1.

Aliter ex Clavio

Cadat, si fieri possit in A Recta quaepiam alia $\perp$lis ad AH np. KA intra Circulum.

Duc KD. §81. Quia

$AD = KD$ §26.

Ergo $\angle u = \angle x$. §57. 100

Sed $\angle u = R$. pr H. ass.

$\angle x = R$.

J. Q. E. A. §146.

Casus 2dus. Duc DE $\perp$ ad AL §119.

Ergo $\angle DEA = R$. §44.

Ergo $AD > DE$. §149.

Ergo cum pctm A sit in Pphia erit pctm E intra Pphiam adeoque et tota AE intra Circulum cadet.

Q. E. II

Aliter ex Clavio ad XVI. L III. Eucl.

Ponamus si fieri possit Rectam inter spatium AD et AC cadere AM. Duc ergo ex D ad AM $\perp$lem §119.

Ergo $AD > DN$. §149.

$AD = DP$ §26.

$DP > DN$. §46 Ctr

J. Q. E. A. §47 Ctr.

Casus III. Indequidem elucet cum ∠lus DAD = 1 R. p. H.
esse 1. ∠lum DAE > rem ∠lo Contactus DAI. §247. Ar.
2. ∠lum HAL < rem ∠lo semicirculi; eo quod tota AL intra Circulum cadat p. d. ad Cas. II. Q. E. III. D.

§268. Corollarium

Hinc Recta AD ab extremitate A, Diametri Circuli HA, ducta ad angulos rectos, Circulum ipsum tangit.

§269. Scholion.

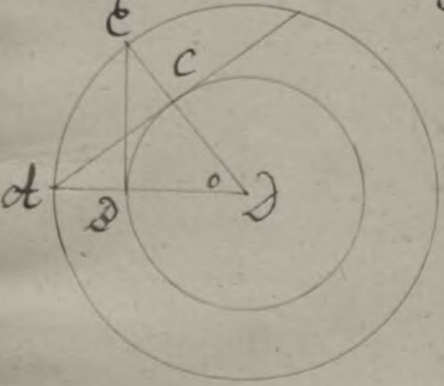
Dicitur autem Angulus semicirculi qui fit a Diametro AH et p AI aut AL.

Caeterum de Contactus Angulo et Paradoxis inde fluentibus notissima est Cxphori Clavii atque Jac. Peletarii Controversia; quorum ille Angulum Contactus rectilineo heterogeneum esse dixit, hic autem pro non quanto declarans ex ∠lo=

rum Numero ſuſtulit, cum hoc fecit Joh Walliſius in Tr. integro, quem de Contactus ∠lo conſcriptum Opp Volum. 2do legimus, ubi illum omni aſsignabili Quantitate minorem h. e. nullius magnitudinis eſse defendit. Aliter adhuc ſentit Andr. Tacquetus ad Propoſ. XVI. L. III Euclidis ∠los non quantos eſse pronuncians, ipſe licet illorum Aequalitatem, Diſectionem, et alias in Quanta cadentes affectiones demonſtraverit, eo ipſo Paradoxa magis cumulans quam ſolvens. Vide præter citatos et plures alios ipſis laudatos Perill: Wolffium Geometr. Lat. §30 b. et in Lex. Mathem: ſub voce Angulus Contactus et Angulus Semicirculi quibus locis Controverſiam et Paradoxa nullius prorſus uſus et momenti recte pronunciat

§270. Problema XXVIII

A dato puncto A rectam lineam AC ducere, quae datum Circulum DBC tangat.



Resolutio.

1) Ex Centro dati Circuli D ad datum punctum A duc rectam AD. §81
2) Radio AD describe Circulum. §83.
3) Ex B super AD excita ⊥lem occurrentem Circulo AE in E. §120.
4) Duc rectam ED secantem Peripheriam BC in C §81.
5) Duc tandem rectam AC &c.

Hanc digo tangere Circulum DBC.

Demonstratio.

$$\left.\begin{array}{l} AD = DE \\ CD = DB \end{array}\right\}\ \S 26.$$

$$\angle o = \angle o\ \S 40\ \text{Ar.}$$

$$\angle ACD = \angle EBD.\ \S 99$$

$$\angle EBD = R\ \text{p. C.}$$

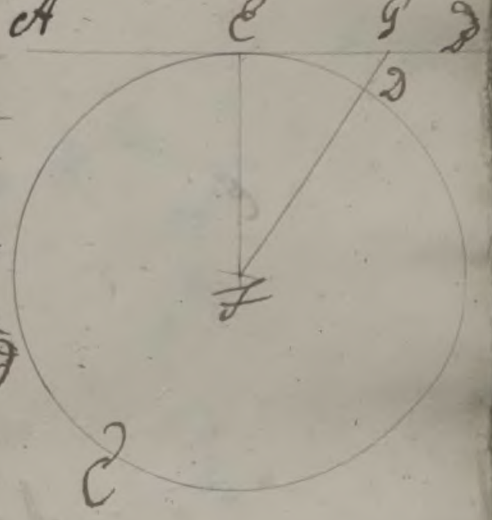
$$\angle ACD = R.\ \S 92.$$

Ergo AC tangit Circulum DBC per §268.

Q. E. D.

§271. Theorema 77.

Si Circulum FEDC tangat recta quopiam Linea AD, a Centro autem ad Contactum E adjungatur recta quopiam Linea FE, quae adjuncta fuerit recta Linea illa FE ad ipsam contingentem normalis est.



Demonstratio.

Ducatur, si fieri possit ex Centro F, alia recta normalis FG ad tangentem AD secans Pphiam in D §81. Quare cum.

$\angle FGE = $ R. p. C. et Hyp.

Ergo $FE > FG$ §149.

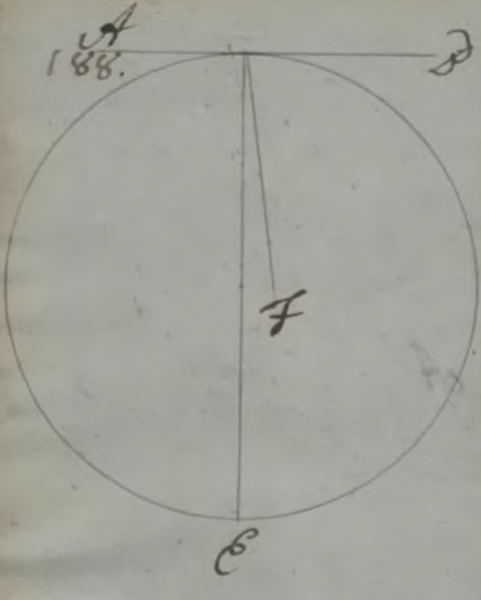
$FG = FD$ §26.

Ergo $FD > FG$. §46. Ar.

Q. E. A. §247. Ar.

§272. Theorema 78.

Si Circulum tetigerit recta quapiam Linea AD, a Contactu autem C recta Linea CE ad ∠los rect. ipsi tangenti excitetur, in excitata CE erit Centrum Circuli.



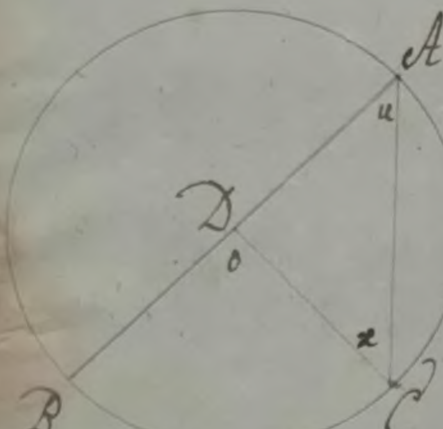
Demonstratio

Aut Centrum erit in CE aut extr
CE. Ponamus in F.
Ducta ergo Recta CF ad Contac
tum §81.
erit ∠lus FCD = R. §271
sed et ∠lus ECD = R. p. H.

∠lus FCD = ∠ECD. §92.
J. Q. E. A. §47

§273. Theorema 79.

In Circulo BADC Angulus ad Centrum duplus est Anguli ad Periphe
BAC cum fuerit idem arcus BC, ba
sis Angulorum BDC et BAC, a. gi
e. cumque uterque Angulus eidem Periph
insistat.

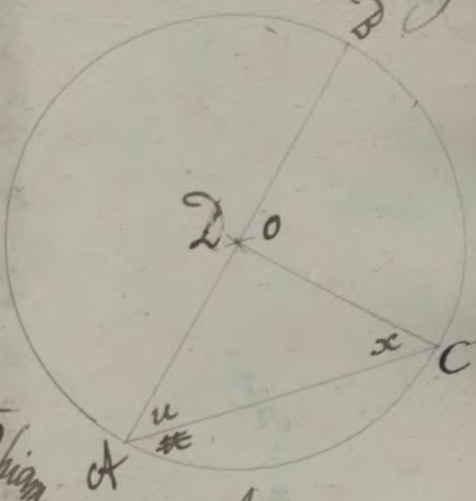


P. H dmdm.
∠o = 2x∠u.

Demonstratio.

Dantur tres Casus; aut enim Anguli ad Peripheriam
1) Crus unum ibit per Centrum.
2) Utrumque Centrum includet.
3) Neutrum Centrum includet.

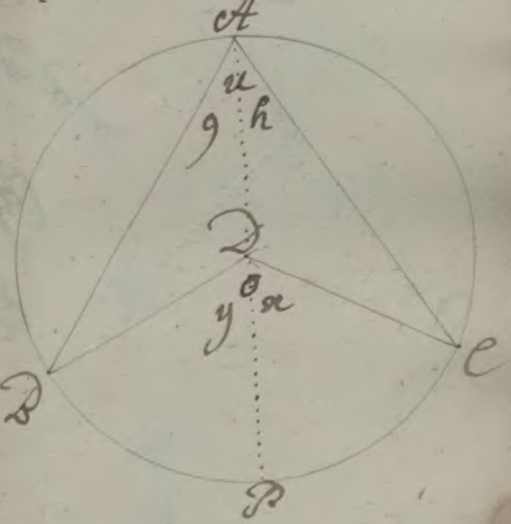
Quare in
Casu 1. $DA = DC$. §26.
Δlum DAC est aeqcr. §57.
Ergo $\angle u = \angle x$. §100.
Sed $\angle o = \angle u+x$. §14.
h.e. $\angle o = \angle u+u$. §10 Ar.
Ergo $\angle o = 2\times\angle u$.



Casu 2do Per Verticem Anguli utriusq et ad Centrum et ad Peripheriam duco rectam ADP. §81. Ergo.

$$\left.\begin{array}{l}\angle x = 2\times h.\\ \text{et } \angle y = 2\times g\end{array}\right\}\text{p Cas. 1.}$$
$$\angle x+y = 2\times h+2\times g \quad \text{§42. Ar.}$$
$$= 2\times\overline{h+g} \quad \text{§.31. Ar.}$$
h.e. $\angle o = 2\times u$ §47. Ar

Q.E.D.

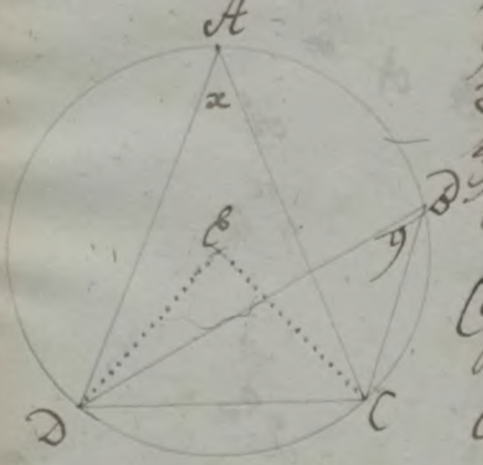


Casu 3tio.
Per A et D duc AP. §81
Ergo $\angle y+o = 2\times\overline{x+u}$ Cas. 1.
$\angle y+o = 2\times x+2\times u$.
Sed $\angle y = 2\times x$. p. Cas. 1.
Ergo $\angle o = 2\times u$. §43. Ar

Q.E.D.

§274. Theorema 80.

In Circulo CADC, qui in eodem segmento sunt Anguli x et y. sunt inter se aequales.



Demonstratio.

Dantur tres Casus, aut enim ∠x et ∠y sunt constituti

1) In segmento majore } §244.
2) In segmento minore }
3) In segmento aequali. h.e. in Semicirculo. Quare in

Casu 1. Si ∠li x et y fuerint in majore segmento.

Ex Centro duc Rectas ED et EC p. §81

Quia $2 \times x = \angle E$ } §273
$2 \times y = \angle E$ }

$2 \times x = 2 \times y$. §41 Ar.

Ergo $\angle x = \angle y$. §45 Ar.

Q.E.I.

Casu 2do Si ∠li dicti in minore segmento fuerint.

Junge ∠lorum Vertices recta A

erunt ergo $\angle$li m et n in majore ſegmento §244. np. ADC et DA.

Quare cum $x+o+m=2R.$

et $u+y+n=2R.$

$x+o+m=u+y+n$. §41 Ar.

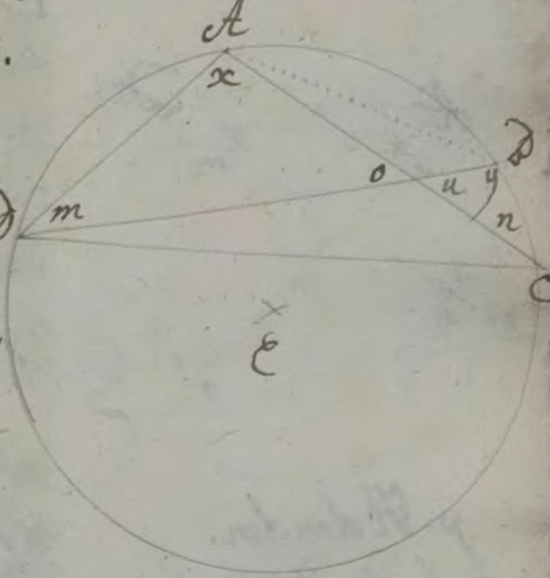
ſed $m=n$. p. l. 1.

et $o=u$. §94.

$\angle o+m=u+n$. §42. Ar.

$\angle x=\angle y$. §43 Ar.

Q. E. II.



Aut brevius.

$\angle o=\angle u$ §94.

$\angle m=\angle n$. p. l. 1.

$\angle x=\angle y$. §154.

Q. E. II.

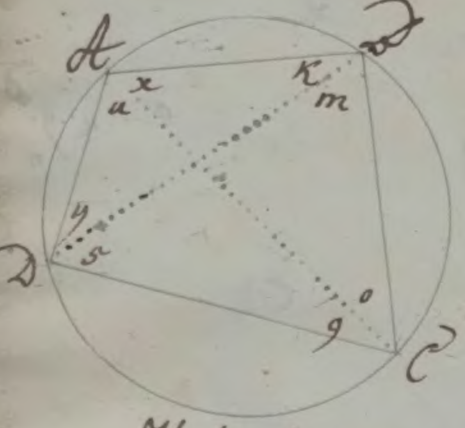
Caſu III.

Si Anguli conſiſtant in ſemicirculo, eadem eſt, quae Caſus 2di Demonſtratio.

Q. E. III. D.

§275. Theorema 81.

Quadrilaterorum Circulo inſcriptorum ABCD $\angle$li ADC et ABC qui ſunt ex adverſo, duobus Rectis ſunt aequales.



p. Hdmdm.

1) $\angle B + D = 2R$.

2) $\angle A + C = 2R$.

Demonstratio

I. Duc AC itemque BD. §81

$\triangle$li $B + o + x = 2R$. §143.

Sed $\left.\begin{array}{l}\angle o = \angle y \\ \angle x = \angle s\end{array}\right\}$ §274.

$\angle o + x = \angle y + s$. §42. Ar.

Ergo

$\triangle$li $B + y + s = 2R$. §10. Ar.

Sed $y + s = \angle D$. §47. Ar.

Ergo.

$\triangle$li $B + D = 2R$. §10. Ar.

Q. E. I.

II $\triangle$li $A + y + k = 2R$. §143

Sed $\left.\begin{array}{l}\angle k = \angle g \\ \angle y = \angle o\end{array}\right\}$ §274.

$\left.\begin{array}{l}\angle y + k = \angle g + o. \text{ §42} \\ \angle g + o = \angle C \text{ §47}\end{array}\right\}$ Ar.

$\angle y + k = \angle C$. §41. Ar.

Proinde

$\triangle$li $A + C = 2R$. §10 Ar

Q. E. II D.

§276. Corollarium.

Quod si ergo Latus quodlibet Quadrilateri Circulo inscripti v. c. AB producatur in E. §82. erit ∠lus externus EBC aequalis ∠lo interno ADC, qui opponitur ei ABC et deinceps positus est externi EBC.

Quia enim $y+s = 2R.$ §90.

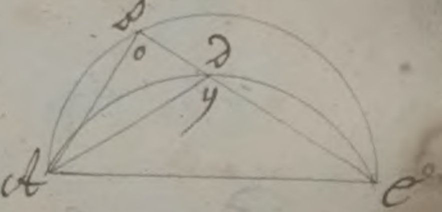
$y+o = 2R.$ §275.

$y+s = y+o$ §41 } Ar.

adeoque $\angle s = \angle o$ §43

§277. Theorema 82.

Super eadem Linea recta AC duo Circulorum Segmenta ABC, ADC similia et inaequalia non constituentur ad easdem Partes.



Demonstratio.

Duc rectam DC secantem Peripherias in B et D. §81.

itemque AB et AD. §80.

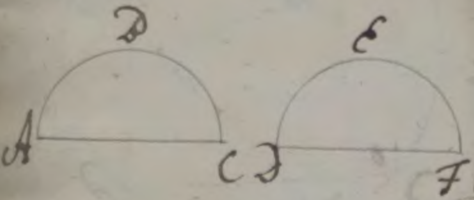
Quia Sgmtm ABC ∾ Sgmto ADC p. H.

ergo $\angle o = \angle y$ §275

Q. E. A. per §113.

§278. Theorema 83.

Super aequalibus rectis lineis A
DF ſimilia Circulorum ſegm
ta ADC, DEF ſunt aequalia.



Demonſtratio.

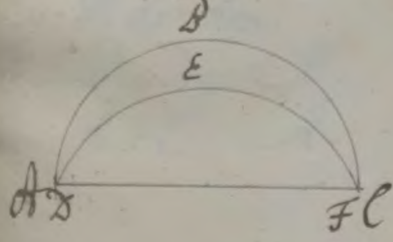
AC = DF. p. H.

Ergo AC congruit DF. §89

Dico ergo et ſegmentum ADC
omnino congruere ſegmento DEF,

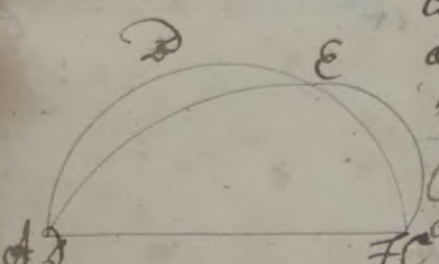
Quodſi non congruat, cadet

1) aut extra
2) aut intra
3) aut partim extra aut par
tim intra



Quare

Caſu I. et II. ſi unum extra alterum
tota cadat ſuper eadem recta DF aut
conſtituentur duo ſegmenta ſimilia et
aequalia quorum unum totum intra
vel extra cadit. J. Q. C. A. §277
Caſu III. Si partim extra partim intr
cadit ſecabunt ſe ſphaerae in pluribus
quam duobus punctis nempe in A, E et
Ergo ADC congruit AEF J. Q. C. A. §
Ergo ADC = AEF. §86. Q. E. D.



§279. Problema XXIX

Dato Circuli Segmento ABC describere Circulum cujus est Segmentum.

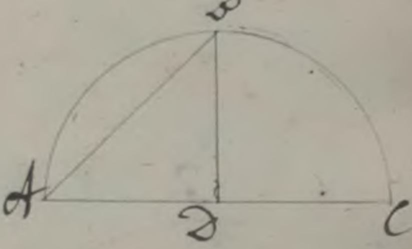
Resolutio.

1) Subtende rectam AC §81.

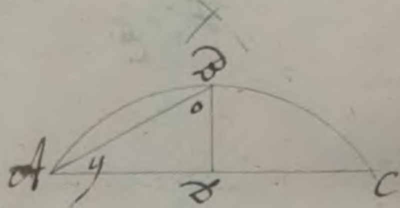
2) Diseca illam excitata illi §112. DB.

3) Junge AB §81. et erit

vel $\angle o >r \angle loy$
vel $\angle o = y$ } §39 etc.
vel $\angle o <r \angle loy$

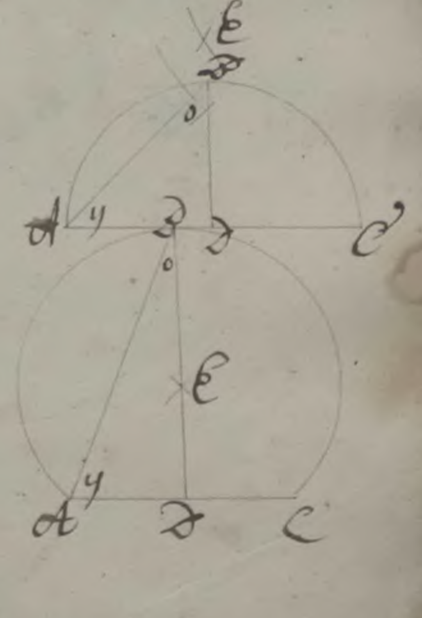


Primum, in Segmento minore in quo BD alias per Centrum transiens E. §251. illud extra se habet per §244.



Secundum in Semicirculo in quo cum AD = DB §26.
ergo $\angle o = \angle y$. §57.100.

Tertium, in Segmento majore in quo BD per Centrum transit per §251. et intra illud constituitur §244. Unde BD > AD. §258.
adeoque $\angle y >r \angle lo\, o$. §115.



Quare in

198

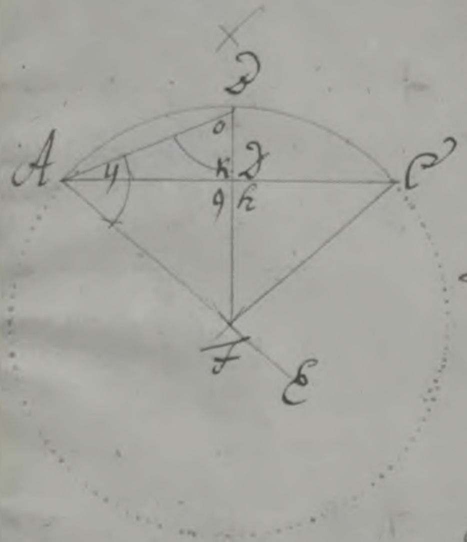
Casu 1.

1) Intra segmentum ad A fac $\angle$lum
$DAE = \angle o$ §107.

2) Produc D ut occurrat Rectae
AE in F. §82.

Dico F esse Centrum Circuli

Demonstratio.



Duc Rectam FC. §81

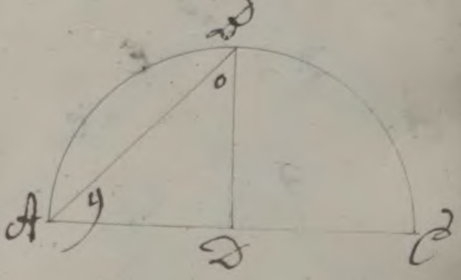
Quia $\angle g = \angle h = R$. p. C. generalem
et $AD = DC$. p. C. eandem
et $DF = DF$. §40. Ar.
$AF = FC$. §99

Porro, quia $\angle o = \angle DAF$. p. C.
Ergo $AF = FD$. §160.
Sed $AF = FC$. p. d.
$AF = FD = FC$. §41 Ar.

Ergo in F est Centrum. §260.
Q. E. I.

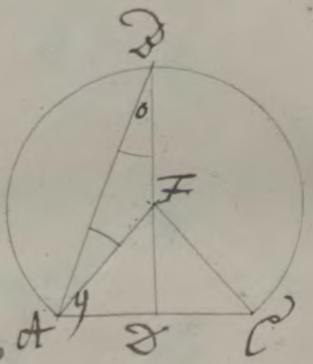
Concursum autem Rectarum
DF et AE ita demonstro:
$\angle k = R$. p. C.
Ergo $\angle o < R$. §147.
Sed $\angle o = \angle DAF$. p. C.
$\angle DAF < R$. §46. Ar.
Ergo $\angle o + DAF < 2R$. §42 Ar.
Proinde AE et DF convergent. §141 Q. E. D.

Casu 2do: Supposita constructione generali quia
$\angle o = \angle y$ p. H.
Ergo DB = AD. §160.
Sed et AD = DC. p. C.
AD = DC = DB. §41 Ax.
Ergo in D est Centrum. §260.
Q. E. II.

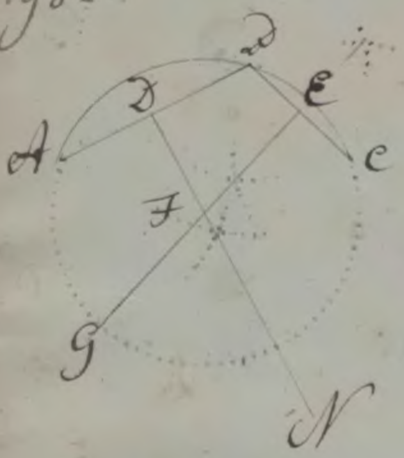


Casu 3io
Fac $\angle$lum FAD = $\angle o$. §107.
D. F.

Demonstratio.
Duc Rectam FC. §81.
quia $\angle$lus k = h = R. p. C. G.
et AD = DC. p. C. eand.
DF = DF. §40 Ax.
AF = FC. §99
cumque $\angle o$ = $\angle$ BAF p. C.
Ergo AF = FB. §160
AF = FC = BF §40 Ax
Ergo in F est Centrum §260
Q. E. III D.



Simili autem ratiocinio, quo in Casu I. usi sumus ostenditur AF occurrere BF.



§280. Scholion

Poterat vero Problemati et alia methodo breviore satisfieri ista

1) subtende quascunq. duas Rectas AD, DC. §81.

2) In bisectis illis excita normales DN, EG intra segmentum occurrentes. §112

Dico Centrum esse in puncto intersectionis F. Nam:

DE = EC. p.C. et
GE ⊥ ad DC. p.C.

Ergo in GE est Centrum §251.

sed et DD = AD }
et DN ⊥ ad AD } p.C.

Ergo et in DN est Centrum &c.

Ergo ob unicum Intersectionis mutuae punctum §80.

Punctum F, Centrum est
Q. E. D.

§281 Theorema 84.

In eodem vel aequalibus Circulis, GADC, GHDEF aequales ∠li aequalibus Pphiis h. e. Arcubus in=

sistunt AC et DF sive ad Centra
G et H sive ad Peripherias D et E con
stituti insistant.

Demonstratio

Circulus ADCG = Circ: DEFH p. H.
Ergo $AG = HD$. §26.
$GC = HF$
$\angle G = \angle H$. p. H.

$AC = DF$. §99.

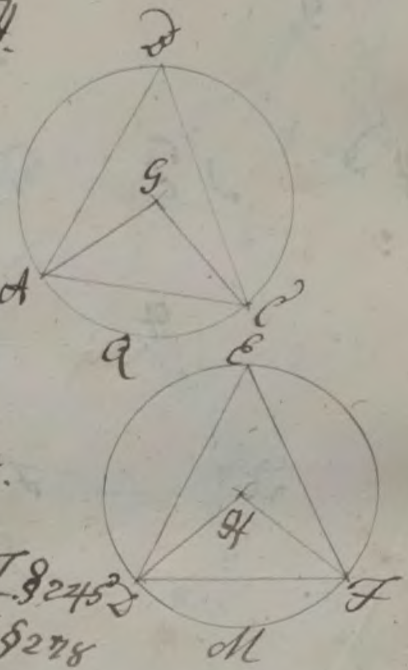
Sed $2 \times \angle D = \angle G$ } §273
et $2 \times \angle E = \angle H$ }
cumq. $\angle G = \angle H$. p. H.

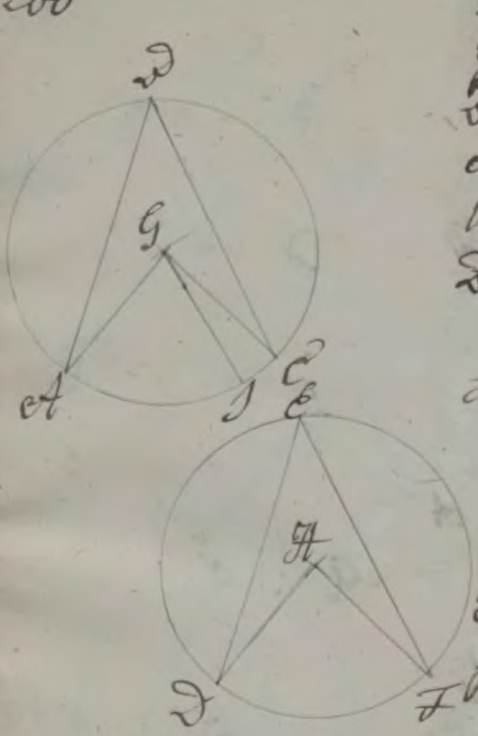
Ergo $2 \times \angle D = 2 \times \angle E$. §41 Ar.
et $\angle D = \angle E$. §45 Ar.
Ergo segmentum ADC ~ Sgto DEF. §245
Ergo Segtm ADC = Sgto DEF. §278
Congruet ergo ADC ipsi Sgto DEF. §88.
Ergo et Arcus ADC ipsi arcui DEF. §4.
Proinde Arcus ADC = Arcui DEF. §86.

Enimvero et
Circulus HDEF = HDEF p. H. cumq.
Circulus HGCD ~ Circ. HDEF. §29. 23.

Ergo congruet AGCD Circulo HDEF. §88.
Ergo et Peripheria ADCQ Peripheria DEFM. §4.
Proinde Periphia ADCQ = Periphia DEFM. §86
Verum Arcus ADC = Arcui DEF p. d.

Arcus AQC = Arcui DEMF §43 Ar.
Q. E. D.





§ 282 Theorema 88

In eodem vel aequalibus Circulis ut D C et HDEF, ∠li qui aequalibus Periph. AC et DF insistunt, sunt inter se aequales, sive ad Centra G et H, sive ad Periph. D et E constituti insistant.

Demonstratio

Salva Arcuum aequalitate aut

$\angle G = \angle H$

aut $\angle G$ <r $\angle$lo H } § 39 Ar.

aut $\angle G$ >r $\angle$lo H

Ponamus $\angle G$ >r $\angle$lo H p. Hyps.

fac ergo $\angle AGI = \angle$lo H. § 10 n.

Proinde Arcus AI = Arc. DF. § 281

Verum DF = AC. p. Hy.

Arc. AI = AC. § 41 Ar.

I. Q. E. A. p. § 47 Ar.

Similiomnino discursu evincitur neq. $\angle G$ <rem esse $\angle$lo H.

Ergo $\angle G = \angle H$.

Q. E1.

Cumq. $2 \times D = \angle G$. § 273.

et $2 \times E = \angle H$. § c.

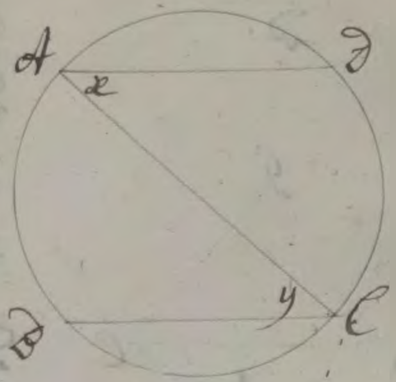
$2 \times D = 2 \times E$. § 41 Ar.

Ergo et $\angle D = \angle E$. § 45 Ar.

Q. E. ll. d.

§283 Scholion 1.

Hinc, si in circulo ABCD Arcus AB = arc. DC, ducto AD et BC §81 erunt ≈lae.

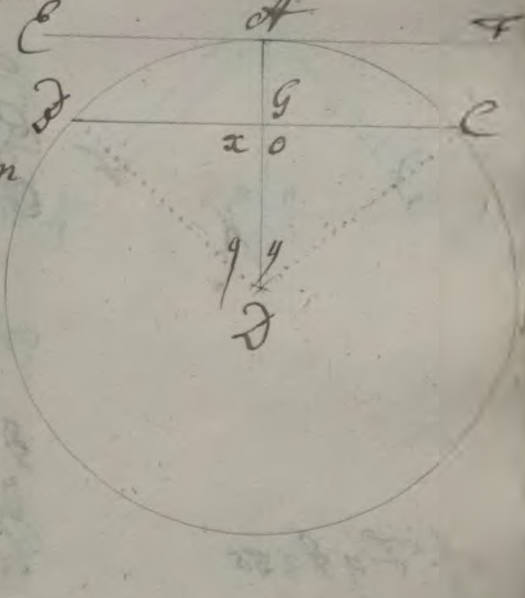


Ducta enim AC. &c.
Quia DC = AB p. H.
Ergo $\angle x = \angle y$. §282.
Ergo AD ≈ BC. §133.

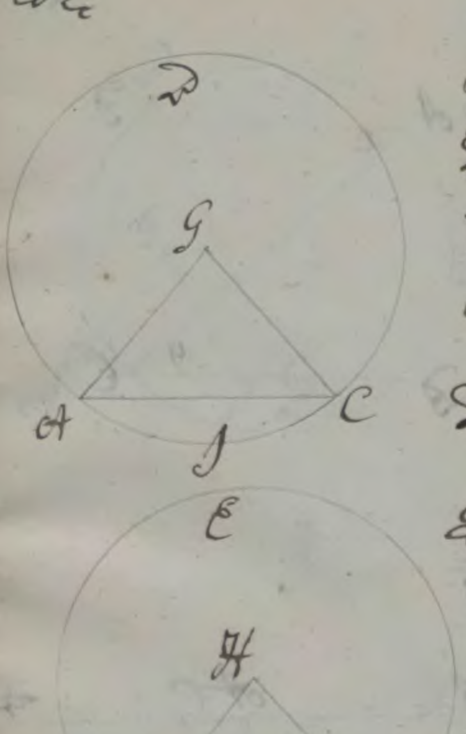
§284. Scholion 2

Linea recta EF quae ducta ex medio puncto A Arcus alicujus BC Circulum tangit ≈la est rectae Lineae BC quae Arcum illum subtendit.

Nam.



Duc ex Centro D ad Contactum rectam DA, junge DB, DC. §81.
Quia Arcus BA = AC. p. H.
$\angle q = \angle y$ §282.
sed DB = DC. §26.
et GD = GD. §40. Ax.
$\angle x = \angle o$. §99
Ergo $\angle o$ = R. §30.
sed et $\angle$FAG = R. p. H. et 271
$\angle o = \angle$FAG. §92.
Quare BC ≈ EF. §133.



cf. Fig §285

§285. Theorema 86.

In eodem vel æqualibus Circulis GADC et HDEF æquales Rectæ AC, DF æquales Pphias h. e. Arcus auferunt, majorem quidem ADC majori DEF, minorem autem AJC minori DKF.

Demonstratio

Duc Radios AG et GC, } §81.
item DH et HF }
Quia Circ: GADC = Circ: HDEF p. H.
Ergo AG = DH } §26.
GC = HF }
Sed et AC = DF. p. H.

∠G = ∠H. §106.
Ergo Arc AJC = Arc. KDF. §281.
Cumq. Pphia ADCA = Pphia DEFD p. H. ad §c.
Ergo et
Arcus ADC = Arc DEF. §43 Ax. 2 El. et 112.

§286. Theorema 87.

In eodem vel æqualibus Circulis GADC et HDEF æquales Arcus ADC et DEF æquales Lineas Rectas subtendunt AC, et DF.

Demonſtratio.

Duc radios GA, GC.
itemq HD et HF } §26. 81.

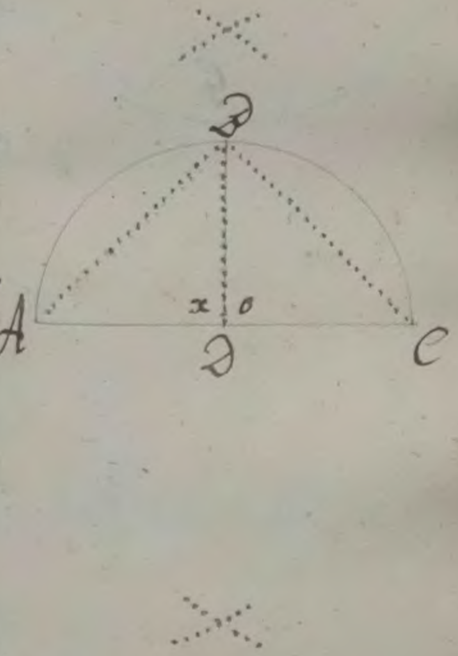
Arcus AIC = Arc. DKF. p. H.
∠ G = ∠ H. §282.
ſed GA = DH }
GC = HF } §26.
Ergo AC = DF. §99.
Q. E. D.

§287. Problema XXX.

Datum Arcum ABC biſecare.

Reſolutio.

1) Duc AC. §81.
2) Biſeca in D. §112.
3) Excitatumq §e. Perpendiculum produc ad Concurſum usq Arcus dati in B. §82. Q. F.



Demonſtratio.

Ductis AB et BC. §81.
Quia AD = DC. p. C.
∠ x = ∠ o. p. C.
et BD = BD §40. Ar.
AB = BC. §99.
Ergo Arc. AB = Arc. BC. §285.
Q. E. D.

§288. Theorema 88.

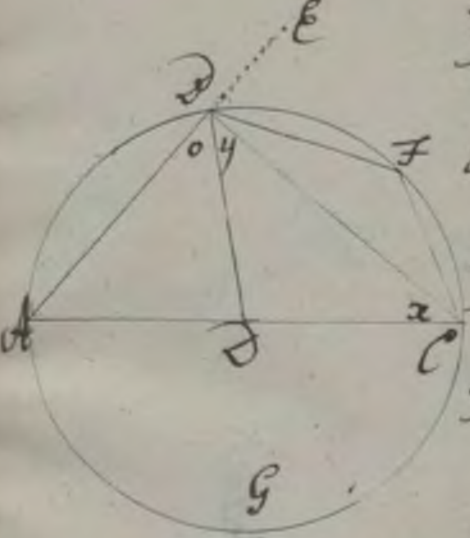
In Circulo 1) Angulus ABC, qui est in Semicirculo, Rectus est.

2) Qui autem in majore Segmento est BAC minor recto est

3) Qui vero in minore Segmento est BFC major Recto est. et insuper.

4) Angulus majoris Segmenti major Recto.

5) Angulus minoris Segmenti minor erit Recto.



Demonstratio.

Produc AB in E. §42.

Ergo $\angle EBC = \angle A + \angle x$. §142.

Duc BD. §81.

Quia $BD = DA$. §26.

Ergo $\angle o = \angle A$. §100.

et Quia $BD = DC$. §26.

$\angle y = \angle x$. §100.

Ergo $\angle o + \angle y = \angle A + x$ §42. Ar.

Sed $\angle o + \angle y = \angle ABC$. §47. Ar.

et $\angle x + A = \angle EBC$. p.d.

$\angle ABC = \angle EBC$ §41 Ar.

$\angle ABC = R$. §38. Q. E. 1.

& $\angle ADC = R$. p.d.
Ergo $\angle A$ $<$r R. §147.
Q. E. II.

ADFC est Quadrilaterum Circulo inscriptum
Ergo $\angle A + F = 2R$. §275
Verū $\angle A$ $<$r R. p. Cas. II.

Ergo $\angle F$ $>$r R. §43. Ar.
Q. E. III.

Arcus ADFC = Arc CGA. §84
Arcus ADFC $<$ CGA + AD. §47. Ar.
$<$ CGAD.

Angulus h. e. majoris segmenti $>$r R.
Q. E. IV.

Simili Discursu
Arc. DFC $<$ ADFC. §47. Ar.

Angulus h. e. minoris segmenti $<$ R.
Q. E. V. et D.

§289. Theorema 89.
Si in Circulum tetigerit aliqua recta Linea AB, a Contacta autem C producatur quodam recta Linea CE Circulum secans:

Anguli ECB et ECA, quos ad contingentem facit, æquales ſunt iis, qui alternis circuli ſegmentis conſiſtunt angulis: EDC, EFC.

Demonſtratio

Dantur duo Caſus aut
1) CE tranſit per Centrum
2) CE non tranſit per Centrum.

Quare in Caſu 1mo

$\angle ACE = \angle BCE = R.$ §271.
Sed $EDC = EFC$ §84.
$\angle o = \angle x = R.$ §288.

$\left.\begin{array}{l}\angle ACE = \angle x \\ \angle BCE = \angle o\end{array}\right\}$ §92.

Q. E. I.

Caſu 2do I. Ducta Diametro CD et DE recta §81.
Ergo $DC \perp$ ad AB. §271
et $\angle y = R.$ §288.
Ergo $\angle DCB = y.$ §92.
Sed $\angle o + p. = y.$ §147. Ar

$\angle DCB = \angle o + p.$ §41 Ar
$\angle p = \angle p.$

$\angle ECB = \angle o.$ §43. Ar. Q. E. D.

cf. Fig I. hujus §phi.

dem similiter demonstrabis de quo-
unq; alio Angulo ejusdem v. c. n.

Quia enim $\angle n = \angle o$. §274.

atq; $\angle o = \angle ECB$ p. d.

B $\angle n = \angle ECB$. §41 Ar.

□DCFE est Quadrilaterum inscriptum Circulo.

Ergo $\angle o + x = 2R$. §275.

sed $\angle ECA + ECB = 2R$. §93.

$\angle o + x = \angle ECA + ECB$. §41 Ar.

$\angle o = \angle ECB$ p. Cas. II. m. 1.

$\angle x = \angle ECA$. §43. Ar.

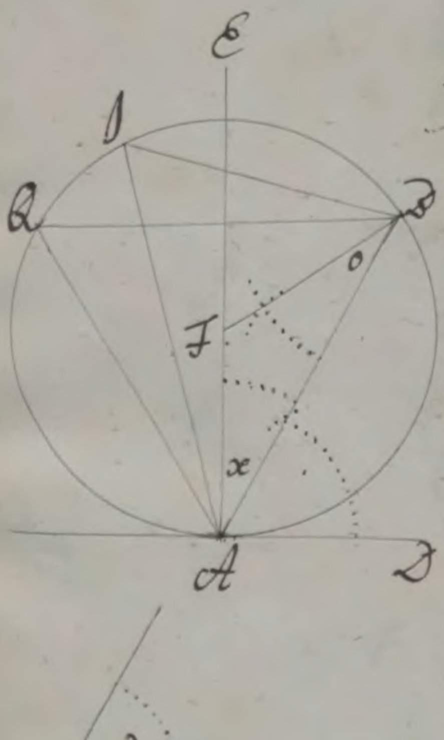
Q. E. III D.

Ergo. Problema XXXI

Super data recta Linea AB describere Circuli segmentum AQB quod capiat ∠lum ADB, æqualem ∠lo rectilineo C.

Resolutio.

) Ad datæ AB punctum extremum v. c. A constitue ∠lum DAB = ∠lo C. §107.



2) Ex A duc AE ⊥ ad AD. §158

3) Ad alterum punctum D fac ∠o = x, §107 cujus alterum Latus secet AE

4) Intervallo DF aut AF æquali per § describe Circulum. §83.

Dico Sgmtm AQIB esse quæs tum et quemlibet ∠um AQB, AI in ipso descriptum esse ∠o C, dato, æqualem

Demonstratio.

AD ⊥lis ad AE. p. C.
Ergo AD est Tangens Circuli §268.
et AB secans ejusdem. §241.

Ergo ∠BAD = ∠AIB. §289.
∠BAD = ∠C.

∠AIB = ∠C. §41. Ar.
2 El.

∠AIB = ∠BQA. §274.

∠BQA = ∠C. §41. Ar.
2. Ell. D

Rectarum autem concursus ita demonstrabitur.

✠ ✠

✠ ✠

∠EAD = R. §268.

∠x ∠r ∠to R. §40 Ar.
sed ∠x = ∠lo o p. C.

∠x + o less 2 R. §42. Ar.
Ergo AF et FD convergunt §141.

§291. Problema XXXII

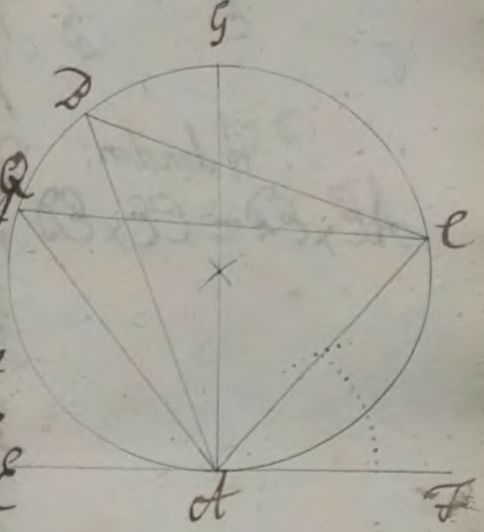
A dato Circulo ADC segmentum AQDC abscindere, quod capiat Angulum D aut ∠lum Q aequalem dato rectilineo D.

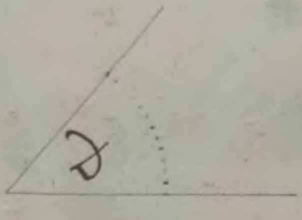
Resolutio.

1) Duco Tangentem EF. §270.

2) In contactus puncto A fac ∠lum CAF = ∠lo D. §107. cujus latus CA secet Circulum datum.

3) Dico Rectam CA auferre sgmtm imperatum, cujus quilibet ∠lus AQC, ADC = ∠lo D.





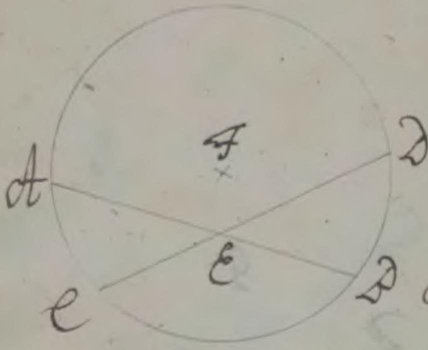
Demonstratio.

Quia EF = Tangenti p. C.
AC = secanti p. C.
Ergo ∠CAF = ∠ADC. §289.
∠CAF = ∠D. p. C.
∠ADC = ∠D. §41 Ar. Q-E-1.

∠ADC = ∠AQC. §289.
∠AQC = ∠lo D. Q-E-||D.

§292. Theorema 90.

Si in Circulo ABCD duo Recto
AB, DC ſeſe mutuo ſecuerint, A
angulum comprehenſum ſub ſegm
tis AE, ED unius, æquale eſt ei, quo
ſub ſegmentis CE, ED alterius com
prehenditur Rectangulo.

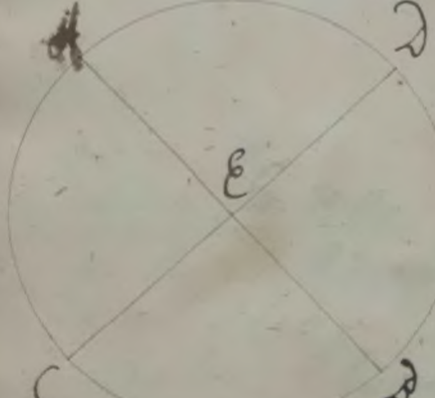


P. Hdmdm.
$AE \times EB = CE \times ED$

Demonſtratio.

Dantur IV Caſus autenim
1) Vtraq et AB et CD per Centrum tr
ſit.
2) Altera per Centrum tranſit et al
teram bifariam ſecat.
3) Altera per Centrum tranſit et a
ram inæqualiter ſecat.
4) Neutra per Centrum tranſit.

Quare in



Caſu I. Demonſtratio per ſe man
feſta eſt utraq enim Recta per
Centrum tranſeunte p. H.

erit: $\left.\begin{aligned} AE &= EC \\ BE &= ED \end{aligned}\right\}$ §26.

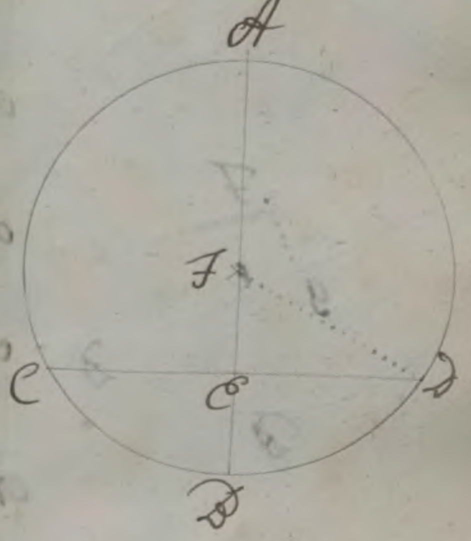
$AE \times BE = EC \times ED$. §175.
et §44. Ar. Q. E. I.

Casu 2do. Tranſeat AB per Centrum F, ſecetqꝫ alteram CD bifariam in E.

Duco ergo Rectam FD. §81.

Quia AB per Centrum tranſit p. H.

erit AB in F ſecta aequaliter §250.
eademqꝫ in E ſecta inaequaliter
atqꝫ ad ∠los R. §254.



Quare

$AE \times EB + EF^2 = FB^2$. §212.

ſed $FB = FD$. §26.

$FB^2 = FD^2$. §44. Ar: 175.

$AE \times EB + EF^2 = FD^2$. §41 Ar.

$FD^2 = EF^2 + ED^2$. §189.

$AE \times EB + EF^2 = EF^2 + ED^2$. §41 Ar.

$EF^2 = EF^2$. §40. Ar.

$AE \times EB = ED^2$. §43. Ar.

Verum $ED^2 = ED \times ED$

et $ED = CE$ p. H.

Ergo $ED^2 = CE \times ED$. §10 Ar.

Ergo tandem

$AE \times EB = CE \times ED$. §41 Ar.

Q.E.D.

Casu 3io Transeat ut ante AB per
Centrum F, secetq; alteram inaequ
liter in E. Ergo
ducta ex Centro Hi FG ad CD §119.

$CG = GD$. §254.

Ergo

$AE \times EB + EF^2 = FB^2$ §212

$FB^2 = FD^2$. §26. G. 44 Ar

$AE \times EB + EF^2 = FD^2$. §41 Ar.

$FD^2 = FG^2 + GD^2$. §189.

$AE \times EB + EF^2 = FG^2 + GD^2$. §41 Ar

$GD^2 = CE \times ED + GE^2$. §212.

$AE \times EB + EF^2 = FG^2 + CE \times ED + GE^2$ §100

$EF^2 = FG^2 + GE^2$. §189.

$AE \times EB = CE \times ED$. §43 Ar

Q.E.III.

Casu IVto Si neutra per Centrum tran
seat.
Per Centrum §250. inventum et in
tersectionem mutuam E duc rect
GH. §81, 82. Ergo

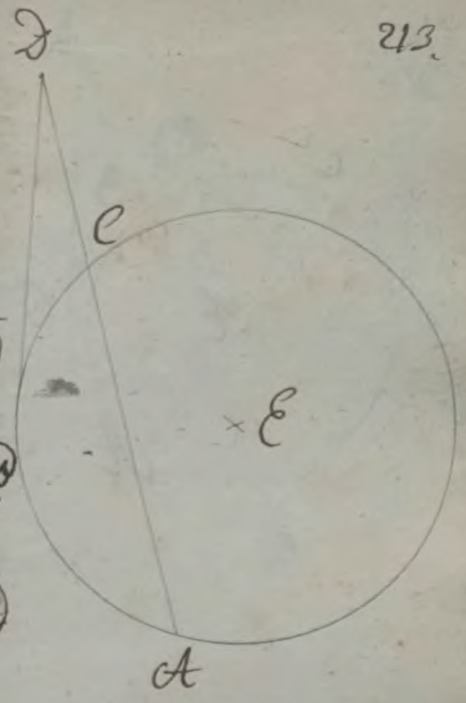
$\left.\begin{array}{l} GE \times EH = AE \times EB \\ GE \times EH = CE \times ED \end{array}\right\}$ p. Cas. III.

$AE \times EB = CE \times ED$. §41 Ar

Q.E.IV.D.

§293. Theorema 91

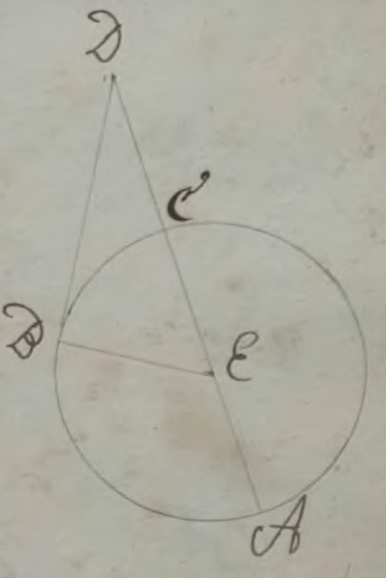
Si extra Circulum EBC sumatur punctum aliquod D, ab eoq. pcto ad Circulum cadant duo rectae Lineae DA et DB, quarum altera DA Circulum secet, altera vero DB tangat, quod sub tota Secante DA et exterius inter pctm. D. et convexam Pphiam assumta Recta DC comprehenditur Rectangulum aequale ē ei, quod a Tangente DB describitur, Quadrato.

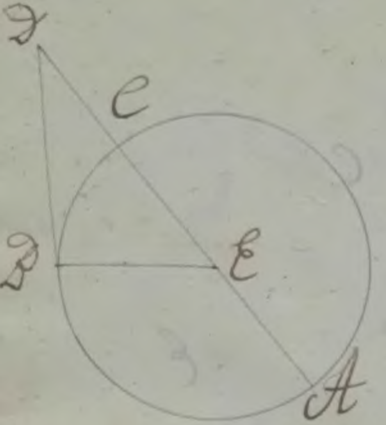


Demonstratio

Dantur duo Casus, aut enim
1) Secans DA transit aut
2) Eadem DA non transit per Centrum E.

Ponamus ergo in Casu I DA transire per Centrum E. Ergo CE = EA §25. 26
Ducat. Contactus pctm B radium EB. §81. Ergo
DB ⊥ ad EB. §271





Ergo
$DE^2 = BE^2 + DB^2$ §189.
Cumq AC sit bisecta in E p. d
atq CD adjecta p. H.

$DE^2 = AD \times DC + EC^2$ §218.

$AD \times DC + EC^2 = BE^2 + DB^2$ §41 Ar.
sed $EC^2 = BE^2$ §26 G. 44 Ar.

$AD \times DC = DB^2$ §42 Ar.
Q. E. I.

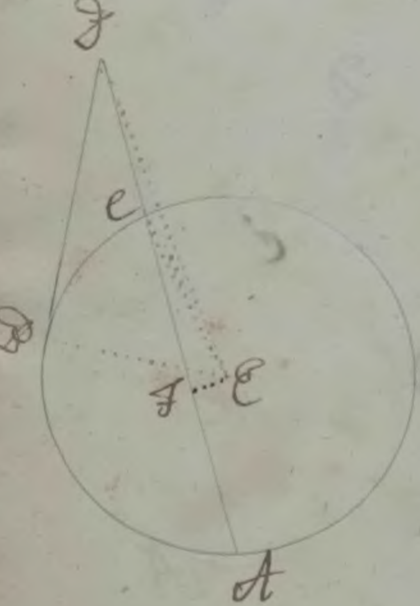
Casu IIdo DA non transire per E
Ductis EB, EC, ED §81. itemq
$DB \perp$li ad BE §119.
atq $EF \perp$li ad AD §c. erit
$CF = FA$ §211.

Quare
$DB^2 + BE^2 = DE^2$ §189
$DE^2 = DF^2 + FE^2$ §c.

$DB^2 + BE^2 = DF^2 + FE^2$ §41 Ar.
sed $DF^2 = AD \times DC + CF^2$ §218.

$DB^2 + BE^2 = AD \times DC + CF^2 + FE^2$ §10 Ar
sed $CF^2 + FE^2 = CE^2$ §189
et $CE^2 = BE^2$ §26 G. 44 Ar.

$DB^2 + BE^2 = AD \times DC + BE^2$ §41 Ar.
Ergo $DB^2 = AD \times DC$. §43 Ar
Q. E. D.



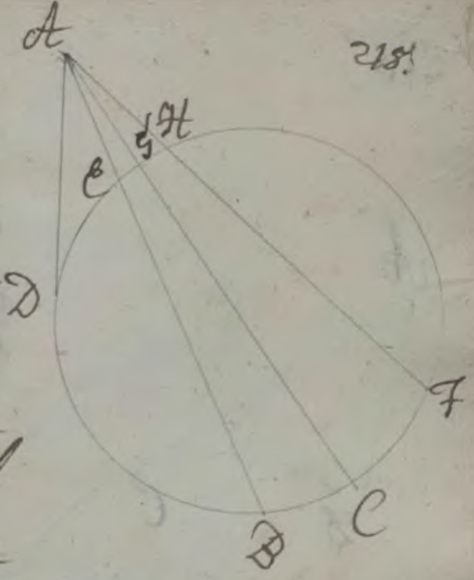
§294. Corollarium 1.

Hinc si a puncto quovis A extra Circulum assumto plures Lineae rectae AB, AC, AF circulum secantes, ducantur Rectangula sub totis Lineis AB, AC, AF et partibus externis AE, AG, AH comprehensa sunt inter se aequalia. Duc enim Tangentem AD §270.

$$\left.\begin{array}{l} \text{Ergo } AD^2 = AB \times AE. \\ AD^2 = AC \times AG. \\ AD^2 = AF \times AH \end{array}\right\} \S 293.$$

$$AB \times AE = AC \times AG = AF \times AH.$$

§41 Ar.



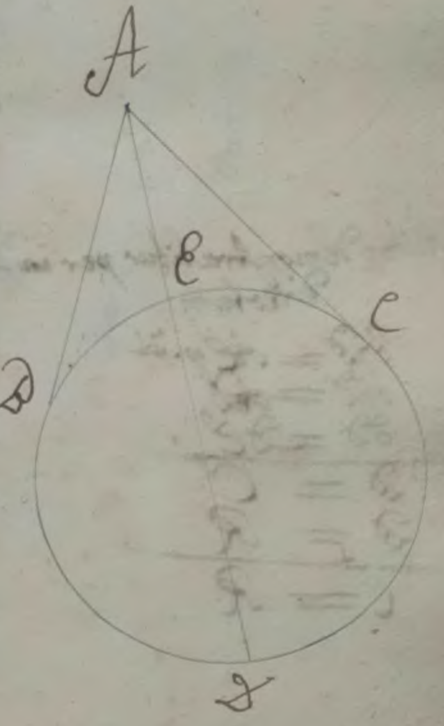
§295. Corollarium 2

Duae Tangentes AD et AC, ab eodem extra Circulum pto A, ductae sunt inter se aequales: Nam ducta secante AD. §241. erit

$$\left.\begin{array}{l} AD^2 = AD \times AE \\ AC^2 = AD \times AE \end{array}\right\} \S 293.$$

$$AD^2 = AC^2 \ \S 41 \text{ Ar}$$

$$AD = AC \ \S 197.$$



§296. Corollarium 3.

Sic etiam patet ab eodem extra Circulum assumto puncto A duas solummodo duci posse Tangentes AB et AC. Ponamus enim plures AB: et AD. Ergo

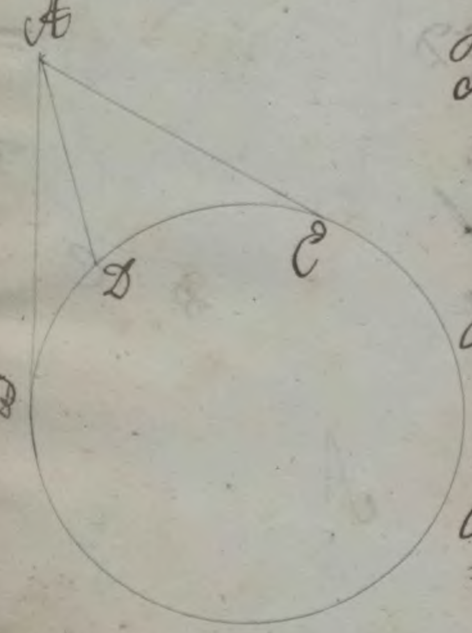
AD = AB
& AB = AC } §295.

Ergo AD = AB = AC. §41 Ar.
I. Q. E. A. per §259.

§297. Corollarium 4.

Et tandem liquet, si duae rectae Lineae aequales AB, AC ex puncto quopiam A extra Circulum assumto in Convexitatem Circuli incidant, et earum altera AB ipsum tangat, et alteram eundem tangere. Quod si enim fieri possit, non AC sed alia quaedam AD Tangens esto. Ergo

AD = AC. §295.
Sed AB = AC. p. H.
AD = AC = AB. §41 Ar.
I. Q. E. A. §259

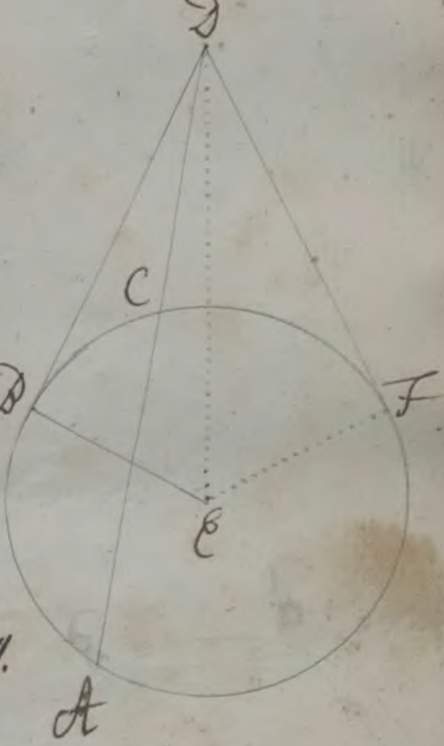


Aliter demonstrabitur per ~~in~~ directum

AB = AC p. H
AD = AD
BD = DC
B = C
B = R
C = R

§298. Theorema 92.

Si extra Circulum CBF sumatur pctm D ab eoq; in Circulum cadant duæ Rectæ Lineæ DA, DB, quarum altera DA Circulum secet, altera DB in eum incidat, sit autem quod sub tota secante DA et exterius inter punctum D et convexam Pphiam assumta Recta DC comprehenditur Rectangulum, æquale ei, quod ab incidente DB describitur Quadrato, incidens ipsa DB Circulum tangit.



Demonstratio.

Duc Tangentem DF. §270.
atq; ex Centro rectas DE, EB, EF. §81.
Ergo $DF^2 = DA \times DC$. §293.
sed $DB^2 = DA \times DC$. h. H

$DF^2 = DB^2$ §41 Ar
$DF = DB$. §197.
$EF = EB$. §26.
$ED = ED$. §40 Ar

$\angle F = \angle B$. §106.
$\angle F = R$. §271

$\angle B = R$. §92. Ergo
DB Tangens Circuli CBF. §26 &c.
Q. E. D.

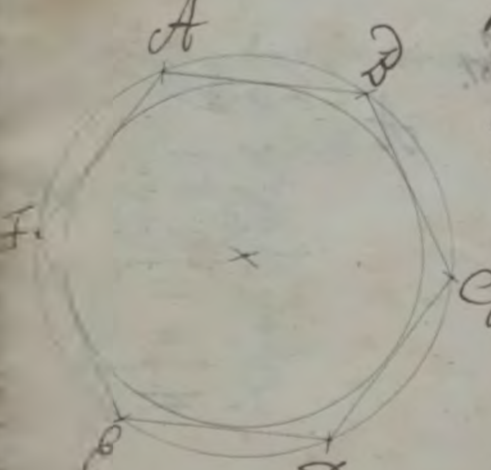
Caput IV

De Figurarum regularium ſ. ordinatarum deſcriptione.

§299. Definitio LXIX.

Figura regularis ſ. ordinata eſt figura æquilatera et æquiangula. Irregularis quæ non ſimul æquilatera et æquiangula.

Polygona dicitur cujus Perimeter pluribus, quam quatuor Rectis terminatur. In ſpecie Pentagonum ſi quinq; Hexagonum ſi ſex, Heptagonum ſi ſeptem, Latera adfuerint quæ et ipſæ Figuræ Regulares vel irregulares ſunt.



§300. Definitio LXX.

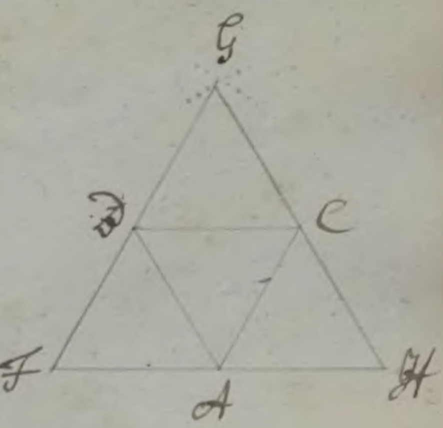
Figura ABCDEF dicitur Circulo inſcripta, ſi Pphia per vertices ſingulorum Angulorum ipſius tranſit tumq; Circulus dicitur Figuræ circumſcriptus.

§301. Definitio LXXI.

Figura ABCDEF dicitz Circulo circumſcripta, ſi ſingula ipſius latera Pphiam tangant. Et tunc Circulus Figuræ dicitz inſcriptus.

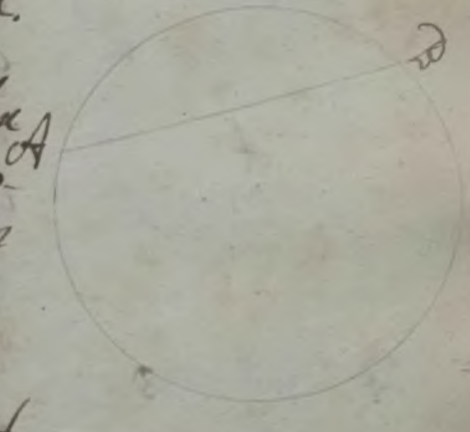
§ 302. Definitio LXXII.

Figura rectilinea alteri Figuræ rectilineæ inſcribi dicitur, ſi ſinguli ejus figuræ, quæ inſcribitur, Anguli, ſingula Latera ejus in qua inſcribitur tangunt; ſic Triangulum ADC dicitur inſcriptum alteri. Similiter Figura circum Figuram deſcribi dicitur, cum ſingula ejus, quæ circumſcribitur, Latera, ſingulos ejus Figuræ ∠os tetigerint, circa quam illa deſcribitur, ſic Triangulum FHG erit circumſcriptum alteri ADC.



§ 303. Definitio LXXIII.

Recta Linea circulo inſcribi ſeu in Circulo accommodari vel coaptari dicitur, cum ejus extremam Circuli Pphia fuerint, uti AD.



§ 304. Definitio. LXXIV.

Figuræ inter ſe æquilateræ ſunt ſi ſingula Latera unius fuerint ſigillatim æqualia ſingulis lateribus homologis alterius figuræ.

§305. Definitio LXXV.

Figurae inter se aequiangulae sunt, si singuli ∠li unius figurae singulis ∠lis homologis alterius figurae fuerint aequales.

§306. Definitio LXXVI.

Sunt autem Anguli et Latera homologa, si eundem ordinem a primo simili sc. vel aequali:) in utraque figura servent. Euclides def. XII. V. Homologas s. similes ratione magnitudines dicit, Antecedentes quidem Antecedentibus, Consequentes Consequentibus, ita si A.B. C.D, tam A et C, quam B et D dicuntur homologae cf. Barrow ad Eucl. l.c. idem de Lineis atque Angulis valet.

§307. Problema XXVIII

In dato Circulo ABC rectam lineam accommodare aequalem datae D, quae Circuli Diametro AC non sit major.

Resolutio et Demonstratio

1) Centro A in Pphia arbitrario assumto spatio AE = D. describe Circulum EAD. §83. dato Circulo occurrentem in D. 2) duc AD quae erit = AE §26 cumque AE = D p. C. erit ipsa D in circulo accomodata per §303

§308. Problema XXVII. E. F et D. In dato Circulo ABC Triangulum ABC describere dato Triangulo EDF aequiangulum

Resolutio

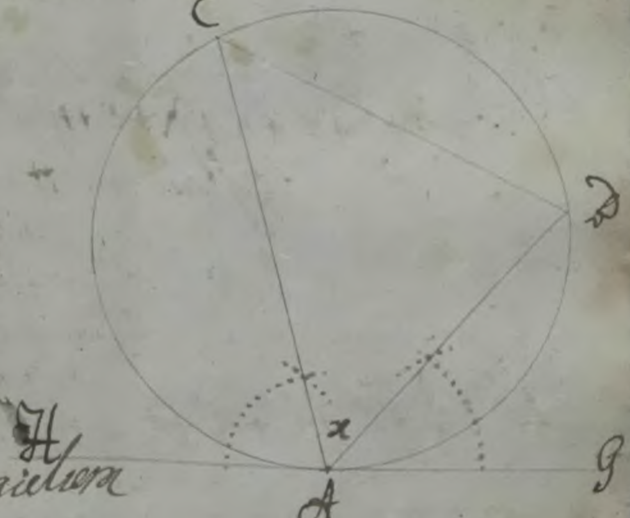
1) Duc Tangentem Circuli GH. §270
2) Ad Contactus pctm A fac angulum HAC = ∠lo E. §107
3) Ad idem pctm A fac ∠lum GAB = ∠lo F. &c.
4) Junge rectam BC. §81. Q. F.

Demonstratio

∠HAC = ∠lo B. §289.
∠HAC = ∠lo E. p. C.
∠B = ∠E. §41 Ar
∠GAB = ∠C. §289
∠GAB = ∠F. p. C.
∠C = ∠F. §41 Ar
∠x = ∠D. §155.

Ergo

Triangulum ABC aequiangulum Triangulo EDF. §305. Q. E. D.



§309. Theorema 93.

Omnes simul ∠li cujuscunque Figurae rectilineae conficiant bis tot Rectos, quot sunt Latera, demtis quatuor.

Demonstratio.

Assumto intra Figuram puncto quovis F, duc Rectas AF, BF, CF, DF, EF. §81.

Ergo.

$$\left.\begin{aligned} \angle li\ o+\pi+\varphi &= 2R.\\ \mu+\xi+r &= 2R.\\ \iota+\kappa+\lambda &= 2R.\\ \varepsilon+\eta+\zeta &= 2R.\\ \alpha+\beta+\gamma &= 2R. \end{aligned}\right\}\ \S 143.$$

$$\angle li\ o+\pi+\varphi+\mu+\xi+r+\iota+\kappa+\lambda+\varepsilon+\eta+\zeta+\alpha+\beta+\gamma = 10R.\ \S 42.\ Ar.$$

$$sed\ o+\mu+\iota+\zeta+\gamma = 4R.\ \S 95.$$

$$\angle li\ \pi+\varphi+\xi+r+\kappa+\lambda+\varepsilon+\eta+\alpha+\beta = 6R.\ \S 43\ Ar.$$

$$Est\ autem\ \left.\begin{aligned} \pi+\alpha &= E.\\ \varphi+\xi &= D.\\ r+\lambda &= C.\\ \kappa+\eta &= B.\\ \varepsilon+\beta &= A. \end{aligned}\right\}\ \S 47.\ Ar.$$

Ergo.

$$\angle li\ A+B+C+D+E = 6R.\ \S 42.\ 41\ Ar.$$

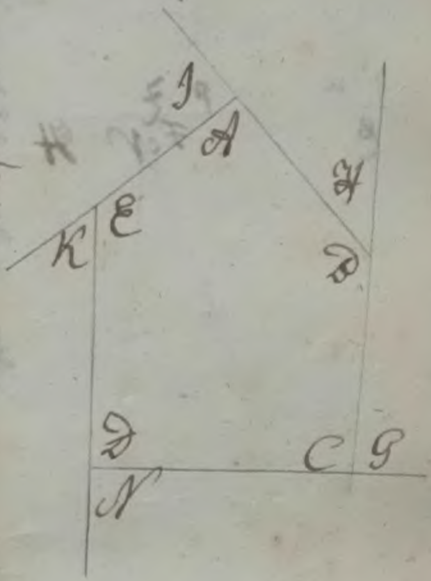
Q.E.D.

§310. Corollarium.

Numerant ergo ejusdem speciei figuræ rectilineæ omnes æquales angulorum summas.

§311. Theorema 94.

Omnes simul anguli externi cujuslibet figuræ rectilineæ sunt æquales quatuor Rectis.



Demonstratio.

∠li $D + N = 2R.$

$C + G = 2R.$

$B + H = 2R.$

$A + I = 2R.$

$E + K = 2R.$

$D + N + C + G + B + H + A + I + E + K = 10R.$ §42. Ar. sed.

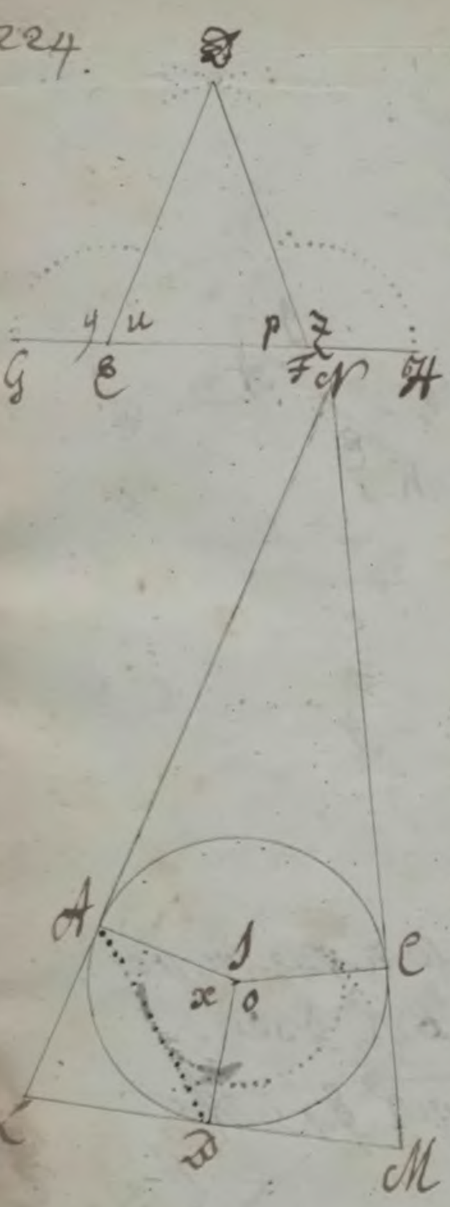
$D + C + B + A + E = 6R.$ §309.

$N + G + H + I + K = 4R.$ §43. Ar.

Q.E.D.

§312. Problema XXXV.

Circa datum Circulum ABCd Triangulum LMN describere dato Triangulo DEF æquiangulum.



Resolutio.

1) Quaere Circuli Centrum §250. et duc utcunq; radium AI. §81.

2) Productoq; Trianguli DEF latere quocunq; EF utrinq; in G et H §82.

3) Fac ∠lum ad Centrum x = ∠y externo sc. ∆li DEF. §107.

4) Itemq; ∠lum ad Centrum o = ∠z externo np. ∆li DEF &c.

5) Excita in radiorum AI, CI, BI extremitatibus A, C et B, normales circulum tangentes §268. et §158.

6) Productis utrinq; in L, M et N ad concursum usq; §82. D. F.

Demonstratio.

Demonstranda autem duo sunt momenta,

1) Tangentes NL, LM et MN coituras in L, M et N.

2) Triangulum coëuntium Tangentium NML esse aequiangulum dato DEF.

Mbr 1. Duc AD. §81.

$\angle \angle AJ = R$ } p.C.
$\angle \angle DJ = R$ }

$\angle$i $\angle AJ + \angle DJ = 2R$. §42. } Ar
Ergo $\angle cAD + \angle cDA <$ rec 2R. §47 }
Ergo $\angle$ D et A $\angle$ coëunt in $\angle$. §141.

Simili Discursu
Rectarum AD, et C in e, iterumque
CM, et M in M convergentia probatur. Q.E.1.

Mbr. 2.

$\angle$i $\angle AJ + x + JDL + L = 4R$. §309.
$\angle$i $\angle AJ + JDL = 2R$. p.d.

$\angle$i $x + L = 2R$. §43. Ar
Sed $\angle$i $y + u = 2R$. §98.

$\angle x + L = \angle y + u$ §41. Ar
Sed $\angle x = \angle y$. p.C.

$\angle L =$ llo u. §48. Ar
Simili modo demonstratur
$\angle cM =$ llo p. Ergo
$\angle cN = \angle D$. §159.

Ergo Δlum $\angle cM$, N aequiangulum Δlo DEF. §330 cumque
$\angle cM$, cM, N sint tangentes p.C.
Ergo Δlum $\angle cMN$ circulo circumscriptum est §526 Q.E.D.

§813. Problema $\underline{\boxed{\text{XXXVI}}}$

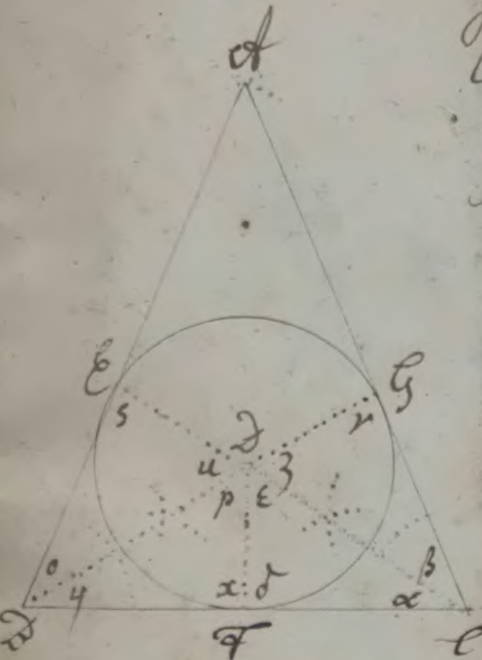
In dato Triangulo ADC Circulum EGF describere.

Resolutio

1) Diseca quoscunque duos ∠los D et C §108. rectis in D coëuntibus; patet autem in D coituras rectas, quod duo quilibet ∠li cujuslibet Δli minores 2R §144. proinde bisecti multo minores 2R. ut inde locus sit §141

2) Ex D duc ⊥les DE, DF, DG, §119. (123) ad AD, DC, CA.

3) Centro D radio DE vel DF vel DG describe Circulum §83. D.F.



Demonstratio

$$\angle s = \angle x = R.\ p.C.$$
$$\underline{\angle o = \angle y\ p.C.}$$
$$\angle u = \angle p.\ \S 153\ (\S 150)$$
$$\underline{DD = DD.\ \S 30.\ Ax.}$$
$$ED = DF.\ \S 114.\ \boxed{\S 119}$$

Sic et

$$\left.\begin{array}{l}\angle \delta = \angle V\\ \underline{\angle \alpha = \angle \beta}\end{array}\right\}p.C.$$
$$\angle \varepsilon = \angle 3.\ \S 155.\ A\ \boxed{159}$$
$$\underline{DC = DC\ \S 40\ Ax.}$$
$$\underline{DF = DG.\ \S 114.}$$
$$ED = DF = DG.\ \S 41\ Ax.$$

Ergo in D Centrum Circuli §260. 2.El. (181)

DE, DF, DG sunt normales p.C. Tangentes ergo singula Trianguli Latera §268.

Ergo Circulus Δlo inscriptus §326 301

2 Coll. D.

§314. Scholion.

Quamobrem cognitis Lateribus cujusvis Trianguli rectilinei Segmenta illorum, quæ fiunt a Contactibus Circuli inscripti simul innotescunt. Quia enim. cf Fig §313.

$$\left.\begin{array}{l}FC = CG\\ EA = AG\end{array}\right\}\ §\cancel{29.}\ §319$$

$$\overline{FC + EA = AC}.\ §42.\ Ar.\ \text{Cumque}$$

$$AD + DC = AD + DC.\ §40\ Ar.$$

$$\overline{AD + DC - FC - EA = AD + DC - AC}.\ §43\ Ar.$$

$$\text{Sed}\ AD - EA = ED$$

$$\overline{DC - FC = FD}.\ \text{Ergo}$$

$$\overline{AD - EA + DC - FC = ED + FD}.\ §42\ Ar.\ \text{Ergo}.$$

$$AD + DC - AC = ED + FD.\ §41.\ Ar.$$

$$\overline{\text{Sed}\ ED = FD}.\ \cancel{\text{Ergo}}\ (§319)$$

$$AD + DC - AC = 2 \times ED\ \text{aut}$$

$$= 2 \times FD.\ §10\ Ar.$$

$$\text{Ergo}.$$

$$\frac{AD + DC - AC}{2} = \left.\begin{array}{l}ED\\ FD\end{array}\right\}\ §45\ Ar.$$

Hinc

$$.\ \left.\begin{array}{l}EA\\ AG\end{array}\right\} = AD - ED.$$

$$.\ \left.\begin{array}{l}FC\\ CG\end{array}\right\} = DC - DF.$$

¤ ¤

¤ ¤

Esto in C. S.

$AD = 12.\ AC = 18.\ DC = 16$

$AD + DC - AC = 10$

Ergo DF aut DE = 5. Hinc

1) EA vel AG $= 12 - 5 = 7$.

2) FC vel CG $= 16 - 5 = 11$.

§315. Problema XXXVIII

Circa datum Triangulum ABC
Circulum deſcribere.

Reſolutio

1) Latera quaevis duo AB, AC biſeca
normalibus erectis D.F. et FE. co
turis in F. §112.

(2) Centro F radio FB aut FC deſc
be Circulum §83.

D. F.

Demonſtratio

Duc DF, AF, FC. §81.

BD = DA. p.C.

$\angle o = \angle y$. §44. 38.

DF = DF. §40 Ar

BF = AF. §99.

Porro

AE = EC. p. C

$\angle n = \angle x$. §44. 38

EF = EF. §40 Ar.

AF = FC. §99.

BF = AF = FC. §41 Ar.

Ergo in F Centrum Circuli. §280

Q. E. D.

§316. Corollarium.

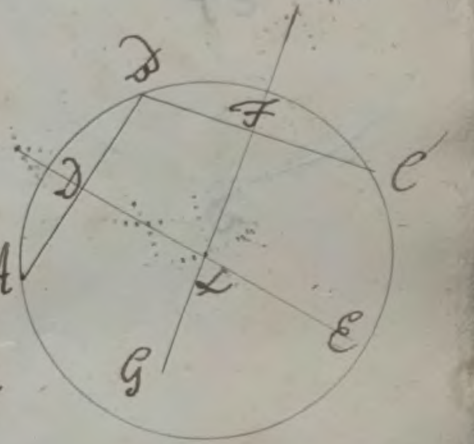
Inde quidem in Triangulo obtusangulo h. e. in minore segmento descripto Centrum extra in Triangulo acutangulo, Centrum intra in Triangulo rectangulo, in Latus recto oppositum cadit cf. §§279. 288.

§317. Scholion

Simili omnino operatione describetur Circulus per data quovis tria puncta, non in directum h. e. in eadem recta linea jacentia, A, B, et C.

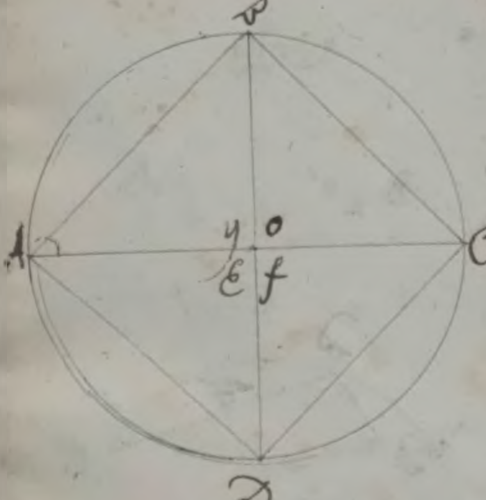
Junctis enim AB et BC §81. utramque bisecca normalibus DE et FG. §112. Centro mutuo Intersectionis puncto L. radio AL vel LB vel LC describe Circulum §83.

Q. F.



Demonstratio

Coincidit cum §280. Demonstratione.



§318. Problema XXXVIII.

In dato Circulo EABCD Quadratum ABCD inscribere.

Resolutio.

1) Quære Circuli Centrum §250.
2) Duc Diametrum AC. §81. 82.
3. Ex E. excita ⊥lem BE eamq; produc in D. §120. 82.
4. Junge Rectas AB, BC, CD, DA §81.

Q. F.

Demonstratio.

AE = AE. §40 Ar.
ED = EB. §26.
$\angle y = \angle E$. §92. p. C.
AD = AB. §99.
ED = ED §40. Ar
AE = EC. §26
$\angle E = \angle f$. §92. C
AD = DC. §99
EC = EC
EB = ED
$\angle o = \angle f$. §92. Ar.
BC = DC. §99

$AB = AD \overset{\text{Ergo}}{=} BC = DC$ §41 Ar

h. e.

Figura descripta ABCD est quadrilatera et aequilatera §64. 56.

Porro:

Est $ADC = \frac{1}{2}$ Circulo §84.

$\angle^{us} D = R.$ §288.

$\angle^{i} B + D = 2R.$ §275.

$\angle^{us} B = R.$ §43. Ar

Simili Discursu.

Est $BAD = \frac{1}{2}$ Circulo §84.

$\angle^{us} A = R.$ §288.

$\angle^{i} A + C = 2R.$ §43. Ar.

Ergo Figura descripta est quadrilatera, aequilatera atque rectangula

Ergo Quadratum §68.

Idque Circulo inscriptum §300

Q. E. D.

§319. Probloma XXXIX

Circa datum Circulum EABCD Quadratum describere HFGI.

Resolutio

1) Ad invento Centro §250
2) Ducentur Diametrum AC §81
3) Et aliam BD secantem AC ad ∠∠ Rectos in E. §129
4) In A, B, C, D excita ⊥les utrinque ad Concursum continuandas FG, FH, HI, IG. §158. 82.

Q. F.

Demonstratio

FA ⊥ ad AC }
HC ⊥ ad AC } p. C.

FG ∥ HI. §138.

GD ⊥ ad BD }
FB ⊥ ad BD } p. C.

GI ∥ FH. §c.

FGIH est Pllgm §72.

∠∠ GAC + ACI = 2R. p. C. §42 Arit.

ACIG est Pllgm §133. 72.

AC = GI. §167.

∠∠ FBD + BDG = 2R. p. C. 42 Arit.

FBDG est Pllgm §133. 72.

BD = FG. §167.

BD = AC. sunt enim diametri

GI = FG. §41 Ar. sed
GI = FH }
FG = HI. } §167.

HI = HF = FG = GI §41 Ar.

Proinde

Pllgm. FGIH est æquilaterum §56.

Porro quia in Pllgo. DF
∠lus D = R. p. C. et
∠li G + D = 2R. §169.
∠lus G = R. §43. Ar.
cumq; F + G = 2R. §169.
∠lus F = R. §43. Ar

Est autem et FI Pllgm p. d.

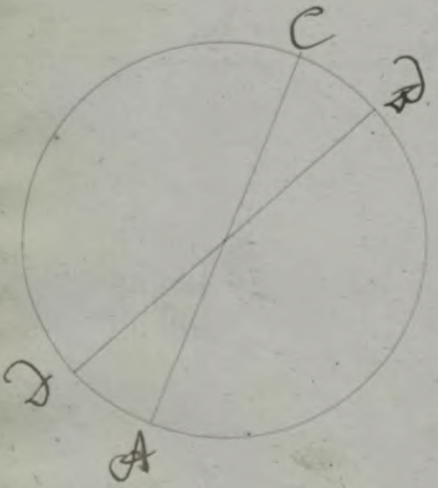
Proinde
∠lus F = ∠ I }
∠lus G = ∠ H } §169.

Ergo Pllgm FI est et æquilaterum et rectangulum p. d.

Ergo Quadratum. §68.

Cumq; singula illius Latera circulum tangant. p. C. et §268.

Quadratum FGIH Circulo est circumscriptum §301 Q. E. D.



§320. Scholion 1.

Cum aſsumſerimus ejusdem Circuli Diametros inter ſe æquales eſse paucis aſsumtum demonſtravimus.

$$\frac{AC \text{ eſt Diameter p. H.}}{ABC = \frac{1}{2} \text{ Pphia Circuli §84}}$$

$$\frac{DB \text{ eſt Diameter p. H}}{DCB = \frac{1}{2} \text{ Pphia Circuli §84.}}$$

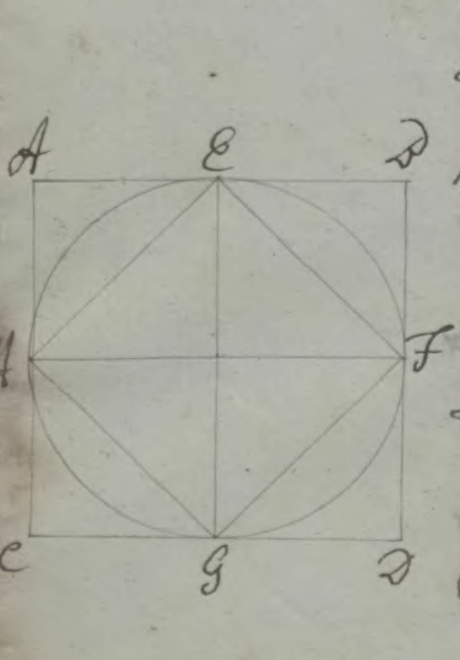
Arc. ABC = Arc. DCB. §41 Ar.

Ergo AC = DB. §281.

Q. E. D.

§321 Scholion 2.

Quadratum Circulo circumſcriptum ABCD duplum eſt Quadrati Circulo inſcripti HEFG.



Nam

Pllgm HB = 2×ΔHEF. §181.

Pllgm HD = 2×ΔHGF. §c.

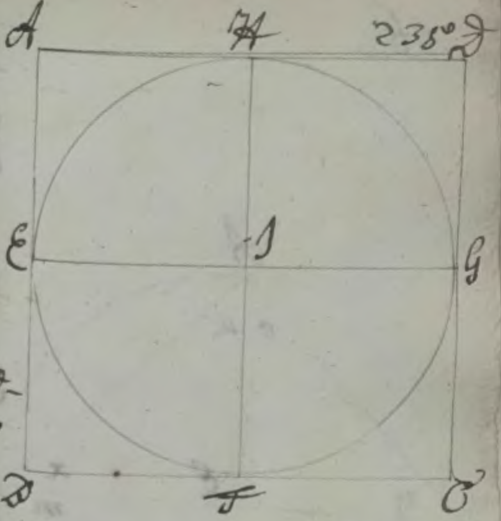
HB + HD = 2×ΔHEF + 2×ΔHGF. §42. Ar.

h. e. ABCD = 2×HEFG. §47 et 31. Ar.

Q. E. D.

§322. Problema XI.

In dato Quadrato ABCD Circulum IEFGH inscribere.



Resolutio.

1) Diseca Quadrati Latera in E, F, G, H. §112.

2) Junge Rectas HF et EG ex bisectionum punctis sese mutuo secantes in I. §81.

3) Centro I radio IE aut HI describe Circulum §83.

Q. F.

Demonstratio.

$AH = BF$ p.C. $\quad$ et $\quad HD = FC$ p.C.
$AH \cong BF$ §168. $\quad$ $HD \cong FC$ §168.
$AB \cong HF$ } 139. $\quad$ $HF \cong DC$ } 139.
$AB = HF$ $\quad$ $HF = DC$

$DG = AE$ p.C. $\quad$ et $\quad GC = EB$ p.C.
$DG \cong AE$ §168 $\quad$ $GC \cong EB$ 168.
$AD \cong EG$ } §139 $\quad$ $EG \cong BC$ } §139.
$AD = EG$ $\quad$ $EG = BC$

AI, BI, IC, ID Ergo sunt Pllgma §72.

$AH = AE$ p.C. $\quad$ Ergo $\quad EI = BF$ §167.
$AH = EI$ } §167. et $\quad BF = EB$ p.C
$AE = HI$ $\quad$ $EB = IF$ §167.
$EI = IH$ §41 Ar. $\quad$ $EI = IF$ §41 Ar.

*

Ergo $EI =$ * $IH = IF$ §41 Ar.
Ergo Centrum Circuli in I §260.
Cumq in Pllgmo AI
∠li A + E = 2R. §169.
et ∠A = R. §68.

∠E = R. §43 Ar.

Ergo AB tanget Circulum §268.

Atque cum simili ratione de Lateribus AD, DC, CB evincatur Circulus IEFGH Quadrato inscriptus est §301.

Q. E. D.

238

§323. Problema $\boxed{XLI}$

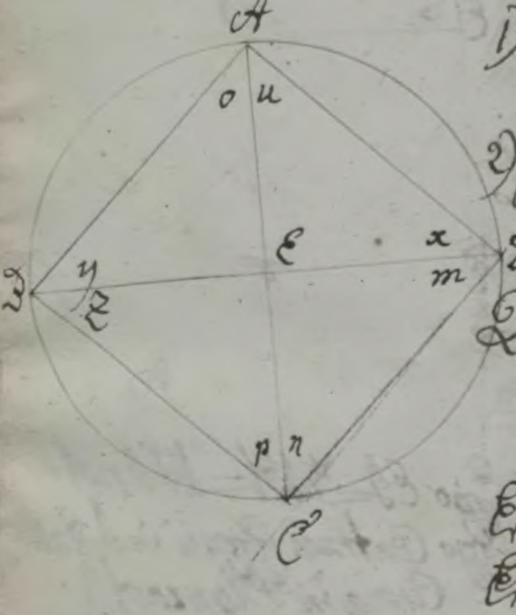
Circa datum Quadratum ABCD Circulum EABCD deſcribere.

Reſolutio.

1) Duc Diagonales AC, BD ſemet ſecantes in E. §81.

2) Centro E radio EA, EB, deſcribe Circulum §83.

D. F.



Demonſtratio.

Quia $\angle A = R.$ p. H. §68.

$\angle B = R.$ p. c.

$\angle A+B = 2R.$ §42. Ar.

Ergo $\angle o+y$ $\angle$res $2R.$ §47. Ar.

Ergo BE et AE coëunt §121.

Porro $BA = AD$ p. H. §68.

Ergo $\angle y = \angle x.$ §100.

$= \frac{1}{2}R.$ §162.

ſimili omnino Ratiocinio

$BA = BC.$ p. H. §68.

$\angle o = \angle p.$ §100.

$= \frac{1}{2}R.$

Quare cum ſimili diſcurſu evin
catur:

$\angle z = m = \frac{1}{2}R.$

$\angle u = n = \frac{1}{2}R$ adeoque

$\angle u = \angle x$ §41 Ar.

Ergo

$AE = ED$. §160 sed et

$\angle o = \angle y$ §41 Ar.

Ergo

$AE = DE$. §160.

$AE = ED = DE$ §41 Ar.

E
Ergo
in E est centrum circuli §260.

Circulus atq E ABCD Quadrato circumscriptus est. §300.

§324. Problema XLII

Isosceles Triangulum ABD constituere quod habeat utrumque eorum quae ad Basin sunt Angulorum B et ADB duplum reliqui A.

Resolutio.

1) Accipe quamcunque rectam AB

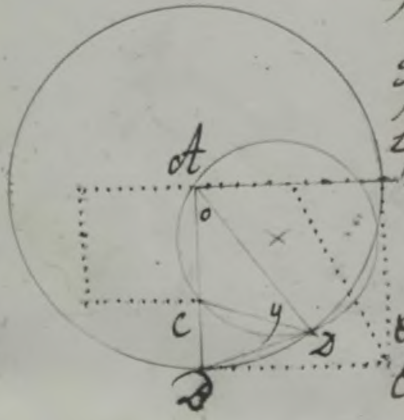
eam seca in C proportionaliter §230.

2) Centro A per B describe Circulum ABD. §83.

3) In hoc accommoda $BD = AC$ §307.

4) Junge AD §81.

D. F.



Demonstratio.

Duc CD. §81.

et per ACD describe Circulum §315.

Ergo CD Circulum secat §241.

Ergo quia

$AB \times CB = AC^2$ p.c.

et $AC = BD$ p.c.

$AB \times CB = BD^2$. §10 et 44 Ar.

Ergo BD Circulum tangit §298.

Ergo $\angle BDC = \angle o$ §289.

cumq $\angle y = \angle y$. §40. Ar.

$\angle BDA = \angle o + y$. §42. 47. Ar.

$\angle BCD = \angle o + y$ §142.

$\angle BCD = \angle BDA$. §41 Ar.

cumq $AB = AD$. §26.

$\angle BDA = \angle B$. §100.

$\angle B = \angle BCD$. §41 Ar.

Quare:

✕

✕

$CD = BD$. §160.

sed $BD = CA$. p.c.

$CD = CA$. §41 Ar.

Ergo $\angle y = \angle o$. §100.

cumq $\angle BDA = \angle o + y$ p.d.

Ergo $\angle BDA = 2 \times o$. §10 Ar.

cumq $\angle BDA = BCD$ p.d.

et $BCD = \angle B$ p.d.

$\angle B = 2 \times o$ §41 Ar.

Q. E. D.

§325. Corollarium.

Hinc Angulus ad Verticem A Trianguli ABD aequalis est duabus quintis unius Recti.

Nam.

$\angle A + B + BDA = 2R.$ §143.

Sed $B = 2\times o$ }
et $BDA = 2\times o$ } §324.

$\angle B + BDA = 4\times o.$ §42. Ar.

$B = 4\times A.$ §40 Ar.

Ergo

$\angle A + 4\times A = 2R.$ §10 Ar.

$5\times\angle A = 2R.$ §47 }
$\angle A = \frac{2R}{5}$ §45 } Ar.

§326. Problema XLIII.

In dato Circulo ABCDE Pentagonum aequilaterum et aequiangulum describere.

Resolutio.

1) Describe Triangulum Isosceles GFH habens utrumque ad Basin ∠um G et H duplum ejus, qui est ad Verticem F. §324.

2) Huic aequiangulum ACD inscribe in Circulo dato §308.

3) ∠los ad basin ACD et CDA biseca chordis CE, DB. §108.

4) Junge rectas CB, BA, AE, ED §81.
D.7

Demonstratio.

$\angle o + x = 2 \times \alpha$ p.C.

$\angle o = \angle x$ p.C.

$2 \times \angle x = 2 \times \alpha$ §100 Ar.

$\angle x = \angle \alpha$. §45 Ar.

Quarecum.

$\angle o = \angle x = \angle y = \angle z = \angle \alpha$ p.C.

Ergo

Arcus AE = ED = DC = CB = BA. §280.

Ergo et

Chorda AE = ED = DC = CB = BA. §280.

Ergo

Pentagonum ABCDE est aequilaterum §56.

Porro.

cum Arcus BC = Arc. BC. §40 Ar.

CD = CD §c.

BA = DE p.d.

Arcus ABCD = BCDE. §42. Ar.

∠lus AED = ∠BAE §282.

Id quod cum simili discursu probetur de ∠lis D, C, B, aequalibus ipsis A vel E. §41 Ar.

Ergo.

Pentagonum descriptum est et aequiangulum. §70.

Idque circulo inscriptum §300.

Q.E.D.

§327. Corollarium:

Inde quidem ∠lus Pentagoni regularis quilibet v.c. D. aequalis est sex quintis unius Recti. Nam

$\angle x + \beta + D = 2R$. §143.

Sed $\angle \beta = \angle x$ §282.

$2 \times \angle x + D = 2R$. §10 Ar.

Sed $\angle x = \frac{2R}{5}$ §325.

$\frac{2 \times 2R}{5} + D = 2R$. §10 Ar.

$\frac{4}{5} R + D = \frac{10}{5} R$.

$\angle D = \frac{10}{5} R - \frac{4}{5} R$ §43 Ar

$\angle D = \frac{6}{5} R$.

Sed

$\angle A + B + C + D + E = 6R$. §309.

Sed $A = B = C = D = E$. p. d. ad §326.

Ergo $5 \times D = 6R$. §10. Ar.

$\angle D = \frac{6}{5} R$. §45 Ar.

§328. Problema XLIV

Circa datum Circulum ABCDE Pentagonum æquilaterum et æquiangulum HIKLG describere.

Resolutio.

1) Dato Circulo inscribe Pentagonum regulare §326.

2) Ex centro F duc rectas ad omnes Angulos Pentagoni inscripti FA, FB, FD, FC, FE, §81.

3) In illarum extremis A, B, C, D, E excitata ⊥les HA, IB, KC, LD, GE §152. producendas ad Concursum usque in G, H, I, K, L. §82. D. F.

Demonstratio.

$\angle FAH = R$ }
$\angle FBH = R$ } p.C.

$\angle FAH + FBH = 2R.$ §42. } Ax.
$\angle HAB + HBA < 2R.$ §47 }

Ergo GH et IH convergunt §141.

Simili discursu Convergentia reliquarum Tangentium evincitur

GA et GE ⊥ler ad AF et EF Perp: p.C.

Ergo GA et GE sunt Tangentes §268.

Ergo

$GA = GE$. §295

sed $AF = FE$ §26

et $GF = GF$. §40 Ar

$\angle GFA = \angle GFE$ §106.

$\angle GFA + GFE = \angle AFE$. §47. Ar.

$2 \times GFA = \angle AFE$

simili discursu demonstratur

$2 \times \angle AFH = \angle AFD$. sed

$\angle AFD = \angle AFE$. §282.

$2 \times \angle GFA = 2 \times \angle AFH$. §41 Ar.

Ergo $\angle GFA = \angle AFH$. §45. A.

sed et $\angle FAG = \angle FAH$. §92.

& $AF = AF$. §40. Ar.

$AG = AH$. §114.

simili ratiocinatione erit:

$EG = EL = LD = DK = KC = CI =$
$ID = DH$. §41 A.

ita ut

$HG = GL = LK = KI = IH$. §42 Ar:

Proinde.

$GHIKL$ est Pentagonum aequilaterum. §56.

Tandem.

Vel ſic:

$AG = AH$ p.d.

$AG = GE$ p. def. §295

$AH = HD$ p. cit.

$GE = HD$ §41 etc.

$AE = AD$ p. C.

$\angle G = \angle H$ §106.

atque ſimiliter in reliquis

$\angle FAG + FEG + AFE + G = 4R$. §30

$\angle FAG + FEG = 2R$ p. C.

$\angle AFE + \angle G = 2R$ §43 etc.

ſic etiam demonſtrabo

$\angle AFD + H = 2R$. Ergo

$\angle AFE + G = \angle AFD + H$ §41 etc.

$\angle AFE = \angle AFD$ p.d.

$\angle G = \angle H$ §43 etc.

Idque cum ſimili omnino diſcurſu probetur de ∠is $I = K = L = G$, Ergo $GHIKL$ eſt etiam æquiangulum ſi Idque Circulo circumſcriptum §30 Q.E.D.

§329. Corollarium.

Eadem prorſus methodo, ſi in Circ
lo quæcunque Figura ordinata et ad
extrema ſemidiametrorum ex
Centro ad ∠los ductarum ⊥les ſta
hæ normales productæ adcohen
ſum aliam figuram totidem Late
rum et ∠lorum æqualium Circ
lo circumſcriptam conſtituent.

§330. Problema XLV

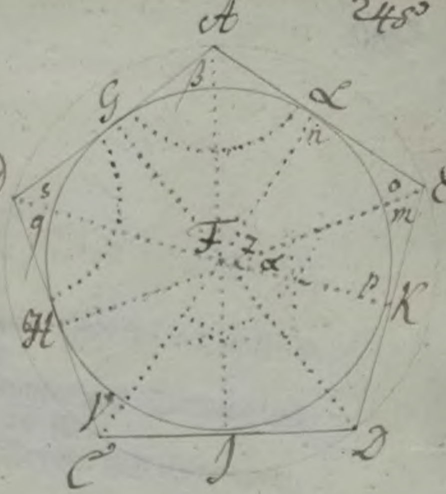
In dato Pentagono regulari ABCDE Circulum inscribere.

Resolutio.

1) Duos Pentagoni Angulos A et B biseca rectis AF, BF coituris in F §108. 141.

2) Ex F duc ⊥les FG, FH, FI, FK, FL.

3) Centro F radio FG vel FH &c describe Circulum §83. D.F.



Demonstratio.

Duc FC, FD, FE. §81.

BA = BC p.H.

BF = BF §40 Ax.

$\angle s = \angle q$. p.C.

AF = FC

et $\angle \beta = \angle \gamma$ §99

sed $\angle \beta = \frac{1}{2} \angle BAE$ p.C.

cumque $\angle BAE = \angle BCD$ p.H.

$\frac{1}{2}$ BAE $= \frac{1}{2} \angle BCD$. §45. Ax.

$\angle \beta = \frac{1}{2} \angle BCD$. §41 Ax.

sed $\angle \beta = \angle \gamma$. p.d.

$\angle \gamma = \frac{1}{2} \angle$li BCD. §41 Ax.

×

× Simili modo demonstratur illos datis bisectos esse; Quare cum

$\angle n = \angle p$. §92.

$\angle o = \angle m$ p.d.

et FE = FE §40 Ax.

FL = FK. §114.

Simili quoque discursu ostendetur FL = FG = FH = FI = FK. §41 Ax.

Ergo in F Centrum §23. 260.

Cumque FG, FH &c ⊥les sint ad AB, BC &c p.C.

Ergo

Pentagono regulari inscriptus est Circulus §301. Q.E.D.

§331. Corollarium

Quare si ∠li duo proximi cujuscu
Figuræ ordinatæ bisecentur atq; a pun
cto Disectionis ad Vertices reliquorum
Angulorum rectæ Lineæ ducantur o
nes Anguli erunt bisecti.

§332. Scholion.

Simili omnino methodo omnibus Fi
ris regularibus polygonis Circulus i
scribitur.

§333. Problema XLVI

Circa datum Pentagonum regular
ABCDE Circulum F et ABCDE describ

Resolutio.

1) Duos Pentagoni ∠los C et D bisec
§108. rectis lineis CF, DF, coitur
in F. §141.

2) Centro F radio CF aut DF describ
Circulum §83. Q. F.

Demonstratio.

36. Eine Wald-Rappe von Silberfarbenem Landtuch, mit halbſeidenen Schnüren, ſehr mottenfreßig, 20. ſgr.

37. Eine grau-tuchene Schabrack, nebſt Kappen, vor den Kutſcher, 12. ſgr.

38. Ein Zaum, Trenſe, Vorder- und Hinter-Gezeug von grünem Leder, mit ſtarck vergaldetem Carlsbader-Beſchlag, 6. Rthlr.

39. Ein alter pohlniſcher Zaum, Trenſe, Vorder- und Hinter-Gezeug, von grünem Leder, mit weiſſem Beſchlag, 2. Rthlr. 20. ſgr.

40. Zwey Fioqves auf die Pferde, acht Stück einfache Qvaſten, zwey einflechte Schnüre, zwey Ziegel und Lengſeile, von Seladon-Seide mit Crepinel, 7. Rthlr.

Wobey etwas Seide und Crepinel zum ausbeſſern.

41. Zwey Fioqves, nebſt vier einflecht Qvaſten, Ziegeln und Lengſeilen, von gelber Seide und Rheiniſch, 3. Rthlr.

42. Zwey alte gelbe ſeidene Fioqven, 10. ſgr.

43. Eine orangen-farben ſeidene mit Silber gewürckte Trenſe, 24. ſgr.

44. Eine roth-ſeidene mit Silber gewürckte Trenſe, 20. ſgr.

45. Eine Trenſe von weiſſem Zwirn, 8. ſgr.

46. Eine ſchwartz lederne Trenſe, 6. ſgr.

47. Ein ſchwartz-lederner Kappzaum, 12. ſgr.

48. Ein paar Pferde-Kompter, 16. ſgr.

49. Ein eintzeles Kompt, 6. ſgr.

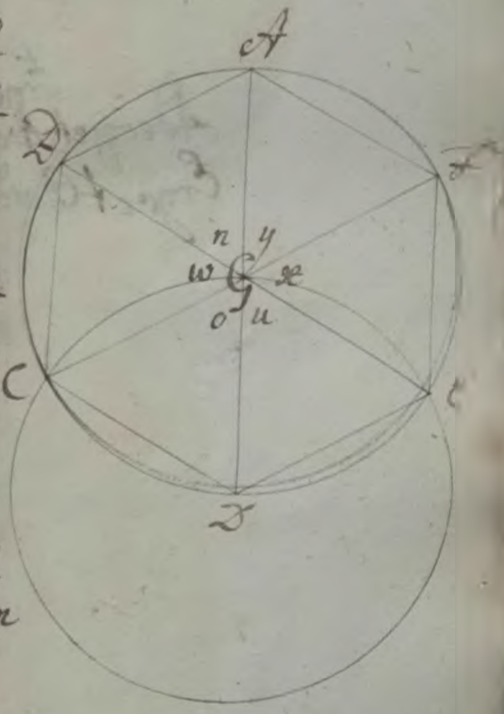
50. Ein Reit-Zeug, mit Vorder- und Hinterzeug, No. 1. 18. ſgr
51. Ein ſchlechter Reitzaum mit Vorder- und Hinterzeug, No. 2. 10 ſgr.
52. Ein Reitzaum mit Vorder- und Hinterzeug, No. 3. 15. ſgr.
53 Ein dergleichen Zaum und Zeug, No. 4. 15. ſgr.
54. Ein dergleichen Zaum und Zeug, etwas ſchlechter, No. 5. 12. ſgr.
55 Ein alter rother Zaum ohne Gebiß, No. 6. 5. ſgr.
56. Ein paar meßingene Steige-Biegel, ohne Riemen, No. 7. 20. ſgr.
57. Vier Stück weiſſe Zäume, 20. ſgr.
58. Ein Wagen-Heber, 1. Rthlr.
59. Ein Sattel, das Gefäß mit weiſſem Leder beſchlagen, 1. Rthlr. 10. ſgr.
60 Ein ſchwartzer Sattel vor den Reit-Knecht, 1. Rthlr 6. ſgr.
61. Ein ſchlechter Sattel, 12. ſgr.
62. Ein Acker-Sattel, worinnen der Baum zerbrochen, 10. ſgr.
63. Ein paar Kompter mit Strang-Scheiden und Schwantz-Riemen, 1. Rthlr 10. ſgr.
64. Ein Juchtener Zaum, Hinter- und Vorder-Zeug, 20. ſgr.
65. Ein alt rother Zaum, Vorder- und Hinterzeug, nebſt weiß-zwirnerner Trenſe, 15. ſgr.
66. Ein ſchwartz-lederner Zaum, Vorder und Hinter-Zeug vor den Reitknecht, 10. ſgr.

67. Vier

Duc FE, FA, FD. §81.
Ergo $\angle o = \angle n$. §331.
Ergo $FD = FE$. §160.
Sed et $\angle x = \angle y$. §331.
$FD = FC$. §160
$FD = FE = FC$. §40 ctr.
Ergo in F Centrum est. §260. atq Circulus Pentagono circumscriptus. §300
Q.E.D.

§334. Scholion.
Artificio eodem Circulus quibusvis figuris regularibus circumscribetur.

§335. Problema XLVII.
In dato Circulo GABCDEF Hexagonum regulare ABCDEF describere.



Resolutio
1) Quaere Circuli Centrum. §250.
2) Duc Diametrum AD. §81.
3) Centro D radio DG describe Circulum §83, secantem priorem Circulum in C et E §261
4) Duc Diametros CF et EB. §84
5) Junge rectas AB, BC, CD, DE, EF, FA. §80.

Q.E.F.

Demonstratio.

$CG = GD = CD$. §26.

$\angle o = \frac{2}{3} R$. §145. Sic et

$\angle u = \frac{2}{3} R$. §c.

$\therefore \angle o + u = \frac{4}{3} R$. §42. Ar:

Cumque CF diameter p. C.

$\therefore \angle o + u + x = 2R. = \frac{6}{3} R$. §93.

$\angle x = \frac{2}{3} R$. §40. Ar.

cumque $\left.\begin{array}{l} \angle x = \angle w \\ \angle u = \angle n \\ \angle o = \angle y \end{array}\right\}$ §94.

Ergo $\angle o = \angle u = \angle x = \angle y = \angle n = \angle w$. §41 Ar

Adeoque et arcus $CD = DE = EF = FA = AB = BC$. §281.

Ergo et Chorda $CD = DE = EF = FA = AB = BC$. §286.

Ergo

Hexagonum est aequilaterum §56.

Porro:

Arcus $\left.\begin{array}{l} CD = EF \text{ p.d.} \\ BC = BC \\ AB = AB \\ AF = AF \end{array}\right\}$ §40 Ar:

Arc. $FABCD =$ Arc. $EFABC$ §41 Ar.

$\angle$lus $E = \angle D$. §282.

Id quod cum simili omnino discursu de ∠lis
F = A = B = C = D §41 Ar. evincatur.
Erit quoque Hexagonum æquiangulum §177.
Ergo ordinatum §299.
Eidq; Circulo inscriptum §300.
Q.E.D.

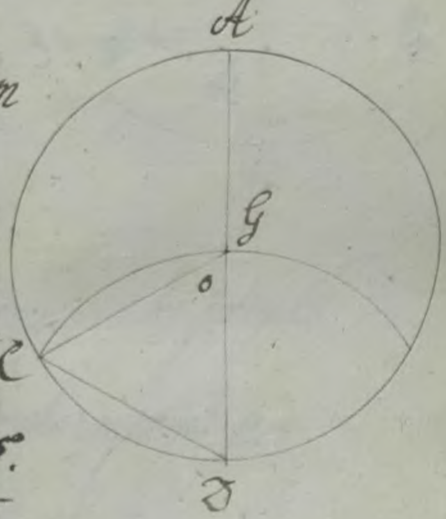
§336. Corollarium I.

Inde quidem Radius Circuli GD est æqualis Lateri Hexagoni eidem inscripti CD. Nam.

$CG = GD$. §26.
$\angle GCD = \angle D$. §100.
$\angle GCD + \angle D + o = \frac{6}{3} R$. §143.
Ergo $2 \times GCD + o = \frac{6}{3} R$. §10 Ar.
$\angle o = \frac{2}{3} R$. p. d. ad §335.

$2 \times GCD = \frac{4}{3} R$. §43. Ar.
Ergo $\angle GCD = \frac{2}{3} R$. §46. Ar.

$\angle GCD = \angle o$. §41. Ar.
Ergo $DG = CD$. §160.



§337. Corollarium 2

Hinc facile Circulo inscriber Trig
num aequilaterum ACD. Facti
enim omnibus ad M. 1—4. §. 33
duc AC, CE, EA eritq; Factum.

Est enim.

$\angle CGE = \angle EGA = \angle AGC = \frac{4}{3}R$ 7. ad
§33

Ergo et

Arc. CDE = EFA = ADC §281

Ergo Trigonum aequilaterum §55.
Idq; Circulo inscriptum §300.

§338. Scholion.

Patet etiam quomodo super data recta Linea AD Hexagonum regulare describatur. Nam.

1) Describe super data Recta AD aequilaterum Triangulum §9
2) Centro C radio CD fac Circulum §83 is capiet Hexagonum Lateris AD. §336.

§339. Problema XLIIX

In dato Circulo ADCA Quindecagonum regulare describere.

Resolutio

1) Dato Circulo inscribe Pentagonum regulare §326.

2) Eidem Circulo ad punctum A inscribe Triangulum æquilaterum §337.

3) Duc DF. §81.

Dico DF esse Latus Quindecagoni regularis.

Demonstratio.

AD est Latus Trianguli æquilateri Circulo inscripti p. C.

Ergo AD subtendit $\frac{1}{3}$ Pphiæ §285.

AE est Latus Pentagoni ordinati Circulo inscripti p. C.

Ergo AE subtendit $\frac{1}{5}$ Pphiæ §c.

similiter EF subtendit $\frac{1}{5}$ Pphiæ

Ergo AE + EF subtendunt $\frac{2}{5}$ Pphiæ §42. Ar

Ergo Arcus AEDF − Arc AD = $\frac{2}{5} - \frac{1}{3}$ Pphiæ §43. Ar

= $\frac{6}{15} - \frac{5}{15}$ Pphiæ §210. Ar.

h. e. Arcus DF = $\frac{1}{15}$ Pphiæ

Ergo Chorda DF subtendit $\frac{1}{15}$ Pphiæ

Ergo
Quindecagonum eſt æquilaterum
ſed et æquiangulum.
Duc enim FQ = DF. §307.
Ergo ∠DFQ inſiſtit $\frac{13}{15}$ Pphiæ §282.
Id quod ſimili modo de omnibus
∠lis demonſtretur,
Quindecagonum ordinatum eſt §29
idque inſcriptum circulo §300
Q. E. D.

§340. Scholion.
Ex hactenus demonſtratis liquet
circulum geometrice dividi

I. 2. §84.
II. 4. 8. 16. 32. 64 pp. §108.
III. 3. 6. 12. 24. 48 p. &c
IV. 5. 10. 20. 40. 80 p. &c.
V. 15. 30. 60. 120 pp. &c.

} æquales partes

Deſideratur autem univerſale
Pphiam in quascunque partes æquales ſubdividendi artificium; quod

enim cum orbe erudito Carolus Renaldinus in Tr. quem Libris II. de Resolutione & compositione Mathematica Patav. 1668. edidit, Libro II fol. 367, communicavit, fallit, uti quidem perudite ostendit Rud. Christ. Wagnerus Math. Prof. Helmst. in Diss. Examen Methodi Renaldinianæ inscripta Helmst. 1700.

Cæterum quæ de Methodis atq; symptomatibus figurarum ordinatarum Circulo in et circumscriptarum dici poterant, vid. in Clavii Commentario ad Eucl. L. IV. pm. 336—350.

Caput. V^{tum}

De Proportionibus Figurarum.

§341. Definitio LXXVII.

Similes figuræ rectilineæ ij dicuntur sunt, quæ et ∠los singulos singulis æquales habent atque etiam Latera quæ circum æquales ∠los disponuntur proportionalia h.e.

$\triangle ABC \sim \triangle DEF$ ſi

1) $\angle A \doteq \angle E$ et $BA : AC = DE : EF$.

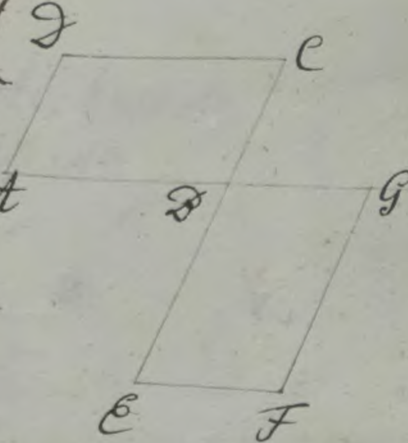
2) $\angle B = \angle D$ et $AB : BC = ED : DF$.

§342. Definitio LXXVIII.

Figuræ autem seu Rectilinea super Lineas rectas descripta dicuntur esse similia similiterque posita, quando ∠li æquales constituuntur super ipsas rectas Lineas et tam reliqui æquales anguli, quam latera proportionalia semper ordine |: eodem :| sese consequuntur.

§343 Definitio LXXIX.

Figurae dicuntur reciprocae DB BF cum in utraque figura antecedentes et consequentes rationum termini fuerint. h. e. si
AB : BG = EB : BC.
dicentur figurae DB et BF reciprocae.



§344. Definitio LXXX.

Secundum extremam et mediam rationem recta linea AB secta esse dicitur, cum ut tota AB ad majus segmentum AC, ita majus AC ad minus CB sese habuerit. h. e. si
AB : AC = AC : CB dicetur AB media atque extrema ratione secta.

§345. Scholion.

Alii rectam h. m. divisam dicunt sectam esse proportionaliter alii sectionem divinam appellant.

§346. Definitio LXXXI.

Parallelogrammum secundum

aliquam Lineam applicatam deficere dicitur Parallelogrammo, quando non occupat totam Lineam.

Excedere vero, quando occupat majorem Lineam, quam sit ea secundum quam applicatur: ita tamen, ut Parallelogrammum deficiens aut excedens eandem habeat Altitudinem cum Pllgo applicato, constituatq; cum eo totum unum Parallelogrammum.

§347. Theorema 95.

Triangula ABC et ACD ut et Parallelogramma BA et AD, quorum eadem vel aequalis fuerit altitudo, se habent inter se ut bases BC et CD.

Demonstratio.

Area Δli ABC $= \frac{1}{2}$ BC × AE $\}$ §182.

Area Δli ADC $= \frac{1}{2}$ CD × AE.

ΔABC : ΔADC $= \frac{1}{2}$ BC × AE : $\frac{1}{2}$ CD × AE. §148

$= \frac{1}{2}$ BC : $\frac{1}{2}$ CD. §160 $\}$ Ar.

ΔABC : ΔADC $=$ BC : CD. §109 $\}$

Q. E. I.

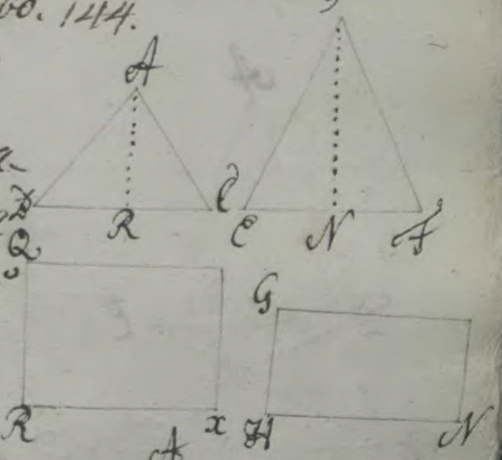
Pllgm EC = BC × AC }
Pllgm AB = CD × AC. } §175.

Pllgm EC : Pllgm AB = BC × AC : CD × AC. §145. Ar.
Pllgm EC : Pllgm AB = BC : CD. §160. 144.

Q.E.ll.D.

§348. Scholion.

Simili omnino discursu demonstrabitur Δla ABC et GEF, ut et Pllgma QX et GN sub iisdem vel aequalibus Basibus esse inter se ut Altitudines AR et GN, itemq[ue] QR et GH.



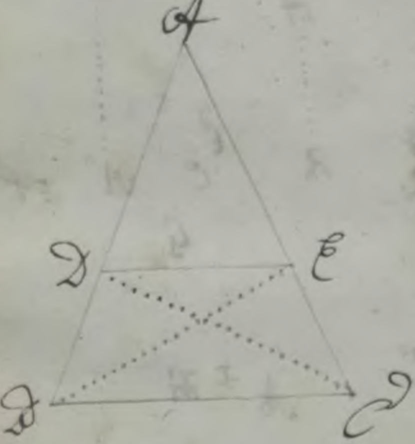
§349. Theorema 96.

Si ad unum Trianguli ABC Latus BC parallela ducta fuerit recta quaepiam Linea DE haec proportionaliter secabit ipsius Trianguli Latera. Et contra:

Si Trianguli ABC latera proportionaliter secta fuerint in D et E, quae ad sectiones dictas adjuncta fuerit Linea recta DE erit ad reliquum ipsius Trianguli Latus parallela.

h.e.
1) Sin in Triangulo DE ≅ BC erit AD : DB = AE : EC.
2) Si AD : DB = AE : EC erit DE ≅ BC.

258.



Demonstratio

Mbr 1. Duc Rectas DC, BE, §81.

Quia BC ≃ DE. p. H.

et DE = DE. §40 Ax.

$\triangle DEC = \triangle DEB$. §177.

Ergo

$\triangle ADE : \triangle DEB = \triangle ADE : \triangle DEC$.

§145 Ax.

Cumq Δlorum ADE et DEB eadem sit Altitudo scil. ⊥lis ex Vertice communi E in AB demissa §

Ergo:

$\triangle ADE : \triangle DEB = AD : DB$. §347.

simili quoq Discursu erit

$\triangle ADE : \triangle ECD = AE : EC$. §c.

$AD . DB = AE : EC$. §144 Ax.

Q. E. 1.

Mbr 2.

Ducti paullo ante DC et BE §8 quare etiam uti ante Δla ADE et DBE habent eandem Altitudin §76. Sic et Δla ADE et DEC eandem Alti

Quare

△ ADE : △ DEB = AD : DB } §347 sed

△ ADE : △ DEC = AE : EC }

AD : DB = AE : EC. p. H.

△ ADE : △ DEB = △ ADE : △ DEC §144 Ar.

Ergo

△ DEB = △ DEC. §152 } Ar.

Sed et DE = DE. §40 }

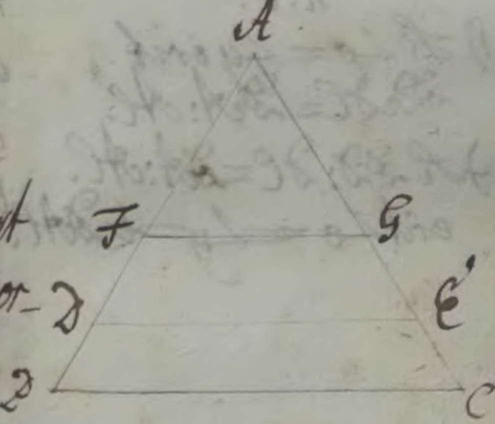
atq; △ DEB et △ DEC ad easdem partes

DE ≃ BC. §179.

Q. E. D.

§350. Scholion

Quodsi et plures DE, FG ad unum Latus BC parallelae fuerint, erunt omnia Laterum segmenta proportionalia



Nam:

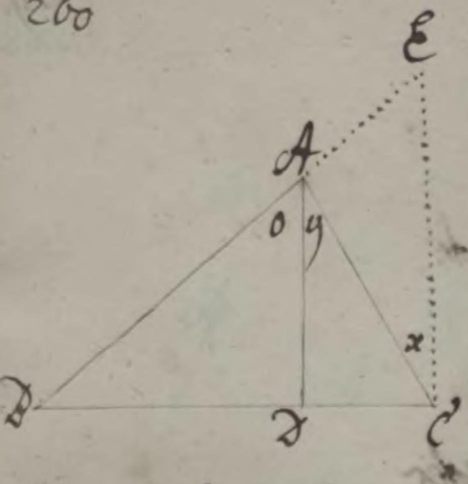
AD : DB = AE : EC. §349.

et AF : DF = AG : GE &c. Ergo:

AF + DF : DF = AG + GE : GE. §168 Ar. h. e.

AD : DF = AE : GE. §47.

DB : DF = EC : GE. §174 Ar.



h. e.

1) Si $\angle o = \angle y$ erit
$BD : DC = BA : AC$.

2) Si $BD : DC = BA : AC$.
erit $\angle o = \angle y = \frac{1}{2} BAC$.

§351. Theorema 97.

Si Trianguli BAC ∠lus BAC bifa
am ſectus fuerit, ſecans autem ∠lu
recta, AD, ſecuerit quoq; baſin BC,
ſeos ſegmenta BD, DC eandem hab
bunt rationem, quam reliqua ipſi
Trianguli Latera BA et AC. Et con
Si Baſeos ſegmenta eandem habea
rationem, quam reliqua ipſius Tr
anguli Latera, recta Linea AD qu
a Vertice A ad ſectionem D duc
bifariam ſecat Trianguli ipſius A
gulum BAC.

Demonſtratio.

Produc BA in E. §82.
fac $AE = AC$, ducq; EC §81
Ergo $\angle E = \angle x$ §100.
ſed $\angle o + y = \angle E + x$. §142.
cumq; $\angle o = \angle y$ p. H.
$2 \times \angle y = 2 \times \angle x$. §10 Ar.
et $\angle y = \angle x$. §45. Ar.
Ergo EC ≈ AD. §133.
Ergo $BD : DC = AD : AE$. §349.
ſed $AE = AC$ p. C.
Ergo $BD : DC = AD : AC$ §10 Ar. Q. E.

Membrum 2.

Manente constructione eadem quia

$BD : DC = DA : AC$ p. H.

et $AC = AE$. p. C.

$BD : DC = DA : AE$. §10 Ar

Ergo $CE \approx AD$. §349 M. 11

Ergo $\angle E = \angle o$ §132.

et $\angle x = \angle y$. §c.

Sed $\angle x = \angle E$. §100

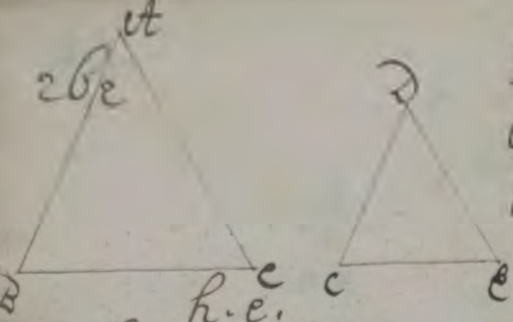
$\angle o = \angle y$ §41 Ar

Enimvero $\angle BAC = \angle o+y$. §47 Ar.

$\angle BAC = 2 \times \angle y$. §10 Ar.

$\frac{\angle BAC}{2} = \angle y$. §45 Ar

Q. E. D.



h. e.

Si $\angle B = \angle DCE$

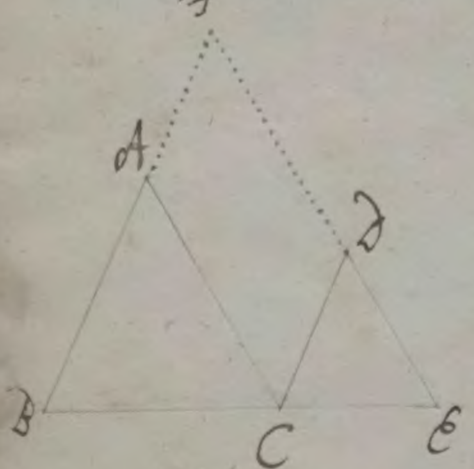
$\angle ACB = \angle E$

$\angle A = \angle D$ erit

1) $AB : BC = CD : CE$.

2) $BC : CA = CE : ED$. et

3) $CA : AB = ED : CD$.



§352. Theorema 98.

Aequiangulorum Triangulorum ABC, DCE proportionalia sunt Latera, quae circum aequales ∠los constituuntur, atque homologa sunt Latera quae aequalibus ∠lis subtenduntur

Demonstratio.

Ordina Latus BC alteri CE in directum §82. 83 atque Triangula ita describe ut

$\left.\begin{array}{l}\angle B = \angle DCE\\ \text{atque } \angle ACB = \angle E\end{array}\right\}$ §98. 107.

Produc Latera AB et DE usque ad concursum in F §82 sunt autem coitura; nam

$\angle B + ACB$ ∠res 2R. §144.

sed $ACB = \angle E$. p. C.

$\angle B + E$ ∠res sunt 2R §100 Ar.

Ergo

BF atque FE coëunt §141

Quare cum

$\angle D = \angle DCE$. p. H. et H.

$DF \approx CD$. §133.

$\angle ACD = \angle E$. p. H. et H.

$FE \approx AC$. §c.

AD est Pllgm §72.

Ergo $AF = CD$, $AC = FD$ } §161.

Ergo cum $FE \approx AC$ p.d.

$DA : AF = DC : CE$. §349.

sed $AF = CD$ p.d.

$DA : CD = DC : CE$. §10 Ar.

$DA : DC = CD : CE$. §150 Ar.

Q.E. I.

$DF \approx CD$. p.d.

$EC : CD = ED : DF$. §349.

$CD : EC = DF : ED$ §146 Ar

$CD : DF = EC : ED$. §150 Ar.

sed $DF = AC$. p.d.

$CD : AC = EC : ED$ §10. Ar.

Q.E. II.D.

cumque $DA : DC = CD : CE$ p.d. ad Cas. I.

$AC : DA = ED : CD$. §175. Ar.

atque latera in omni casu homologa §306.

Q.E. III.D.

§353. Corollarium.

Quare cum sit.

$BA : BC = CD : CE$ §352 C. I.

$BA : CD = BC : CE$ §150. Ar.

$BC : AC = EC : ED$ §352 C. II.

$BC : EC = AC : ED$. §150 Ar.

$AC : BA = ED : DC$. §352 Cas III.

$AC : ED = BA : DC$ §150 Ar.

Ergo

$BA : CD = AC : ED = BC : CE$ §144 Ar.

§354. Scholion.

Quare si in Triangulo BFE ducatur Lateri $BF \approx CD$ §135.

erit $\Delta DCE \sim \Delta FBE$.

Nam $\angle DCE = \angle B$. §132.

$\angle E = \angle E$. §40 Ar

$\angle CDE = \angle F$. §155.

Ergo Latera homologa sunt proportionalia §352. adeoque

$\Delta DCE \sim \Delta FBE$. §341.

§355. Theorema 99.

Si duo Triangula ABC, DEF latera aequalia habuerint aequiangula erunt Δla aequalesque habebunt ∠los eos sub quibus homologa Latera subtenduntur.

Demonstratio.

Fac super EF ad pctm F $\angle x = \angle C$ } §107.
et ad pctm E $\angle y = \angle B$ }
Ergo $\angle G = \angle A$. §155.

Ergo

$\triangle$ ABC aequiangulum $\triangle$lo EFG. §305.

Ergo

AB : BC = GE : EF. §352. sed
AB : BC = DE : EF. p. Hyp.

GE : EF = DE : EF. §144 } Ar.
adeoque GE = DE. §152 }

Porro: AC : BC = GF : FE. §352.
sed. AC : BC = DF : FE. p Hyp.

GF : FE = DF : FE. §144 } Ar.
ergo GF = DF. §152 }

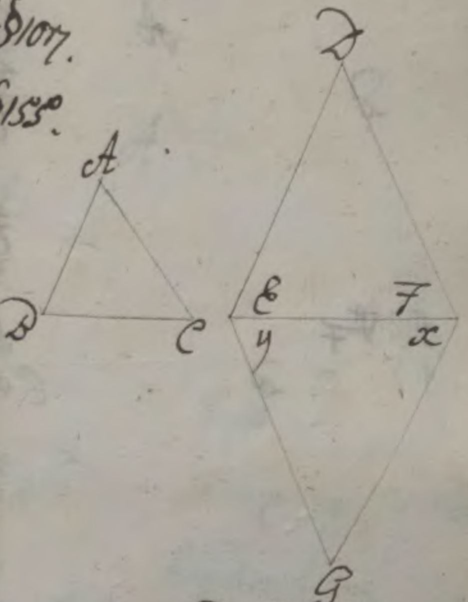
sed GE = DE p d.
EF = EF §40 Ar.

$\angle y = \angle E$ }
$\angle x = \angle F$ } §106.
$\angle G = \angle D$ }

cumque

$\angle G = \angle A$. $\angle x = \angle C$. $\angle y = \angle B$ p. C. et d.
Ergo $\angle A = \angle D$ }
$\angle C = \angle F$ } §41 Ar
$\angle B = \angle E$ }

$\triangle$ ABC aequiangulum $\triangle$lo EDF. §305. QED



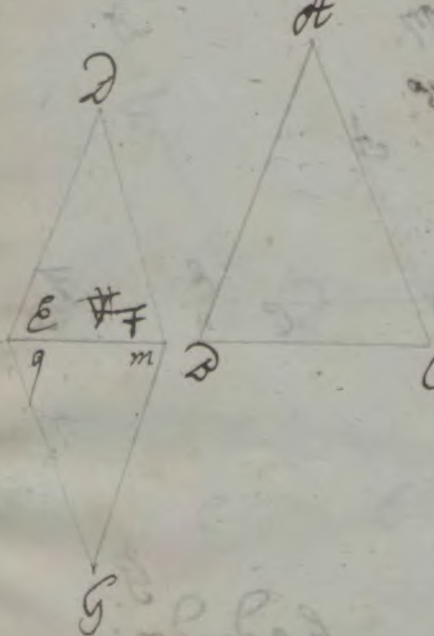
h. e dmdm:

Si AB : BC = DE : EF fuerit
et AC : BC = DF : EF

) $\angle A = \angle D$
$\angle C = \angle F$
$\angle B = \angle E$.

§356. Theorema 100.

Si duo Triangula ABC et DEF unum ∠lum D uni ∠lo E æqualem et circum dictos ∠los æquales D et E latera ppdlia habuerint, æquiangula erunt Triangula ABC et DEF, æqualesque habebunt ∠los sub quibus homologa Latera subtenduntur.



Demonstratio.

Ad Latus EF fac $\angle g = \angle B$ } §107
et $\angle m = \angle C$ }

Ergo et $\angle G = \angle A$ §155.

Ergo △ gmG æqui∠lum △lo ABC §305

Ergo AB : BC = GE : EF. §352. sed
AB : BC = DE : EF p. H.

GE : EF = DE : EF. §144. } Ar.
et GE = DE - §152. }
EF = EF. §40. Ar

cumque $\angle g = \angle B$ p. C.
& $\angle B = \angle E$. p. H.

$\angle g = \angle E$. §41 Ar

& $\angle m = \angle F$ } §99
$\angle G = \angle D$ }

sed et $\angle m = \angle C$. p. C.
& $\angle G = \angle A$. p. d.

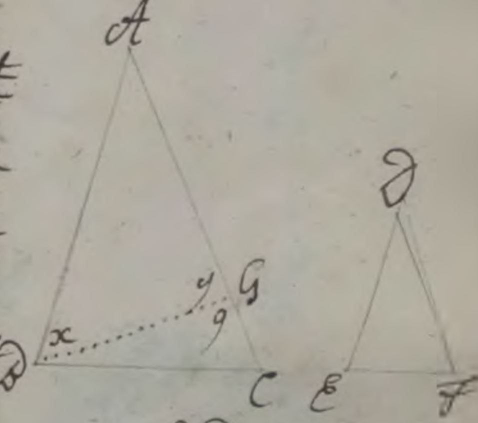
$\angle C = \angle F$ } §41 Ar.
et $\angle D = \angle A$ }

Ergo
△ ABC æqui∠l. △lo DEF. §305.

Q. E. D.

§ 357. Theorema 101.

Si duo Triangula ABC et DEF unum ∠lum A uni ∠lo D aequalem, circa autem alios ∠los B et E latera proportionalia habeant; reliquorum autem simul ~~ut~~ utrumq. C, F aut minorem aut majorem Recto, aequiangula erunt Triangula ABC, DEF et aequales habebunt illos ∠los circum quos sunt Latera proportionalia.



Demonstratio.

Et, si fieri possit ∠B > ∠lo E. p. H. ass:
fiat ergo ∠x = ∠lo E. §107.
cumq. ∠A = ∠D. p. H.
∠y = ∠F. §155.
Ergo.
ΔABG aequiangl. ΔDEF. §305.
Ergo.
AB : BG = DE : EF §352 sed
AB : BC = DE : EF p. H. e
AB : BG = AB : BC. §144 Ar
Ergo BG = BC. §152
Ergo ∠g = ∠C. §100
cumq. ∠C <r ∠lo R. p. H.
Ergo ∠g <r ∠lo R. §46. Ar.

h. e d m d m.
si I ∠A = ∠D atq.
∠C <r R.
et F <r R
et AB : BC = DE : EF.
esse: ∠B = ∠E.
∠C = ∠F.
II si ∠A = ∠D atq.
∠C >r R.
∠F >r R
et AB : BC = DE : EF.
esse: ∠B = ∠E.
∠C = ∠F.

Enimvero.

$\angle l + q + cDG = 2R.$ §143.

$\angle l \quad < R.$ p. H.

$\angle q + cDG >$ res. R. §43 Ar.

cumq. $\angle l = \angle q$ p. d.

$\angle q + cDG = \angle y.$ §142.

$\angle y > r$ R. §46. Ar.

Sed $\angle y = F.$ p. d.

$\angle F > r$ R. §46. Ar.

I. Q. C. H. quo

et $\angle$lum l et $\angle$lum F ponit minorem Recto. Q. E. I.

Membrum 2.

Manente eadem Constructione qu

$\angle q = \angle l.$ p. d. ad Cas. I.

et $\angle l > r$ R. p. H.

Ergo $\angle q > r$ R. §46 Ar.

I. Q. E. A. §150.

Q. E. II D.

§358. Theorema 102.

Si in Triangulo rectangulo ADC ab $\angle$lo R. A in basin DC normalis ducta fuerit, quo ad perpendiculum Triangula ABD et ABC fiunt tum toti

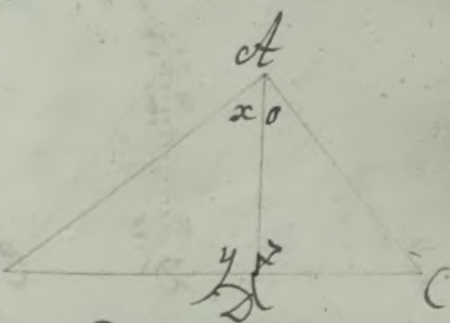
Triangulo ABC tum ipsa inter se sunt similia.

Demonstratio.

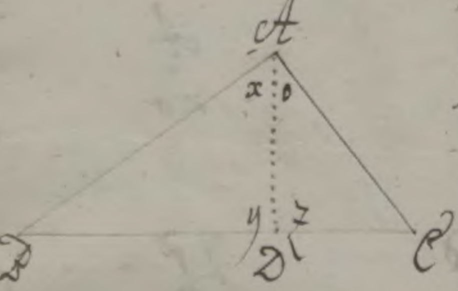
AD ⊥lis ad BC p. H.
Ergo $\angle y = \angle A$. §92.
cumque $\angle B = \angle B$. §400 Ar
$\angle x = \angle C$. §155.
Ergo △ ADB aeq∠gl. △ ABC §305
Ergo Latera circum aequales ∠los ∂ sunt ppalia §352.
Ergo △ ADB ~ △ ABC §341. Q.E.I.

AD ⊥lis ad BC. p. H.
Ergo $\angle z = \angle A$. §92.
cumque $\angle C = \angle C$. §400 Ar.
$\angle o = \angle B$. §155.
Ergo △ ADC aeq∠gl. △lo ABC. §305
Ergo Latera homologa ppalia §352.
Ergo △ ADC ~ △ ABC. §341. Q.E.II.

$$\left.\begin{array}{l}\angle B = \angle o\\ \angle C = \angle x\end{array}\right\}\text{p.d.}$$
$\angle y = \angle z$ §38.
△ ADB aeq∠lum △lo ADC. §305.
Ergo Latera homologa ppalia §352.
Proinde △ ADB ~ △ ADC. §341. Q.E.II. D.



P. H. dmdm:
1) ABC ~ △ ADB
2) ABC ~ △ ADC
3) ADB ~ △ ADC.



§359. Corollarium.

Quare cum Δ BDA $\sim$ Δ CDA p.d.

I) BD : DA = DA : DC. §341.
cum Δ ADC $\sim$ Δ ABC p.d.

II) BC : AC = AC : DC. §341
cumq Δ ADB $\sim$ Δ ABC p.d.

III) BD : AB = AB : BC. §c.

§360. Problema XLIX

A data recta linea AB imperatam partem AG v.c. tertiam auferre.

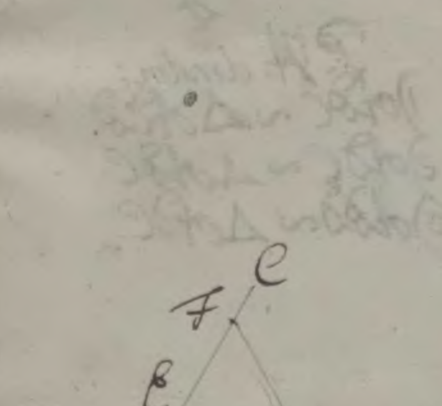
Resolutio.

1) Ex puncto A datae Rectae AB duc infinitam AC §81. utcunque $\angle$lum rectilineum cum data constituentem.

2) Abscinde in illa tres partes aequales AD, DE, EF.

3) Junge FB. §81.

4) Cumque hac age $\parallel$lam DG per D §135. D. F.



Demonstratio

FD ≈ DG. p. C.

Ergo $AD : DF = AG : GD$. §349.

$AD + DF : AD = AG + GD : AG$. §168 } Ar.

h. e. $AF : AD = AD : AG$. §47 } Ar.

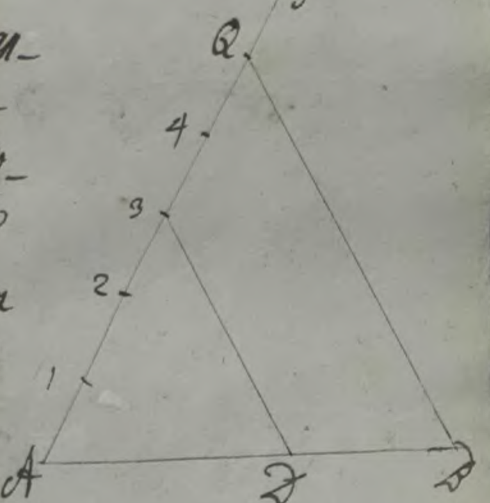
Sed $AF : AD = 3 : 1$ p. C.

$AD : AG = 3 : 1$ §144 Ar.

Q. E. D.

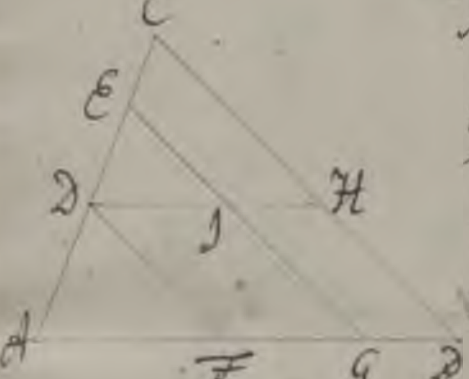
§361. Corollarium.

Ponamus non aliquotam, sed aliquantam Totius partem v. c. $\frac{3}{5}$ auferendam esse ex AD, eadem et Operatione et Demonstratione conficies Negotium. Divide enim Rectam AG utcunque ductam in 5 æquales partes, junge QD, inque in puncto trium quintarum duc =lam 3D, §135. factumque erit.



§362. Problema I.

Datam rectam Lineam AD insectam similiter secare in F et G, ut data Recta altera AC secta fuerit in D et E.



Resolutio.

1) Junge datas Rectas in A sub quolibet ∠lo rectilineo.

2) Extrema illarum connecte Recta CB §81.

3) Cum CB duc per puncta D et E ≈las EG et DF §135. D. F.

Demonstratio.

Per D duc DH ≈ AB §135.

quia EG et DF ≈ cum CB p. C.

Ergo DG item GH sunt Pllga §72.

Ergo DI = FG }
IH = GB } §167. Ergo

AD : DE = AF : FG §349 et

DE : EC = DI : IH §c. Ergo

DE : EC = FG : GB §10 Ar.

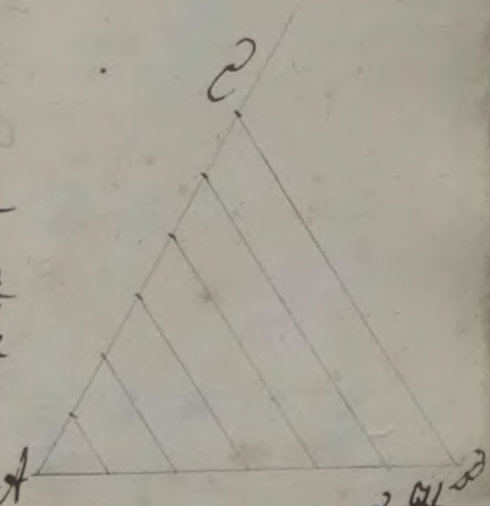
§363. Scholion 1.

Poterat Problema hoc expeditius demonstrari per §350.

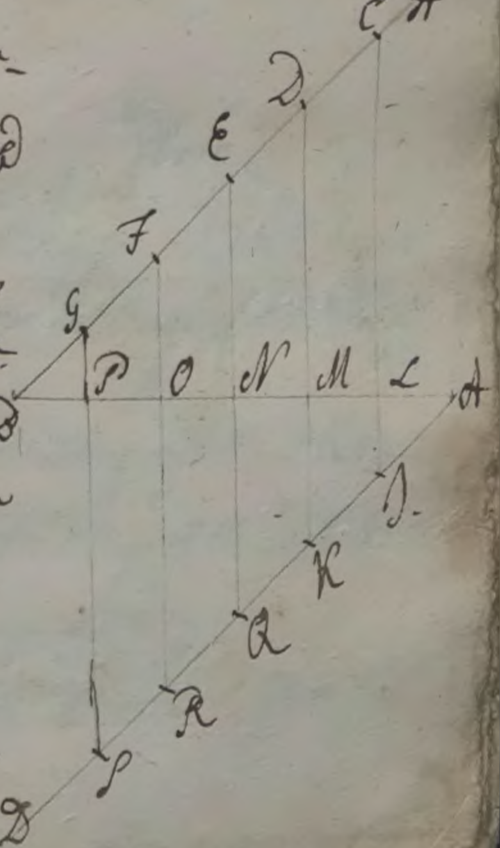
§364. Scholion 2.

Inde quidem facile elucet, quo modo Recta quaevis in imperatas

æquales ſubdividatur, partes aut
I per §382. quamlibet rectam Al
ſubdividendo utcunq; in partes
imperatas ſed æquales, ipſiq; AB,
propoſitam np. ſecandam ſub quo-
vis ∠lo rectilineo adiiciendo, per-
q; ſingula ſubdiviſionum puncta
=las agendo, junctis primo pctis
C et B recta lB. §81. 135.



II hoc modo: Ponatur AB in 6 par-
tes ſubdividenda
1) Sub ∠lo quovis adjunge ipſi AB
aliam AB §81 infinitam.
2) Et huic AB duc =lam aliam
infinitam per B np. BH. §135.
3) In utraq; ſc. AC et BH ſumes
partes æquales una pauciores
quam deſiderantur in AB.
4) Tum duc rectas GS, FR, EQ.
DK. CI. §81.
Dico has transverſas in 6 æqua-
les partes ſubdividere ipſam AB. D



Nam

$CD \simeq$ et $= IK$. p.l.

$CI \simeq$ et $= DK$. §139

Id quod simili discursu de rectis DC, QK DK, CI applicet.

Quare

$AL : LM = AI : IK$. §349.

$AL : AI = LM : IK$. §150 Ar

cumque $AI = IK$ p.l.

$AL = LM$. §152. Ar

simili discursu ostendam

$LM : MN = IK : KG$ §350.

foreque $LM = MN$. §150. 152 Ar.

id quod cum eodem modo de rectis NO, OP evincatur

Ergo $AL = LM = MN = NO = OP$. 41 Ar.

cumque etiam

$DP : PO = GD : GF$. §349.

erit $DP = PO$. §150: 152. Ar

Ergo

$AL = LM = MN = NO = OP = PD$. §41 Ar.

Ergo

$AD = b \times AL$ aut $b \times LM$. §47 Ar.

Ergo.

$\frac{1}{b} AD = AL$. §450 Ar

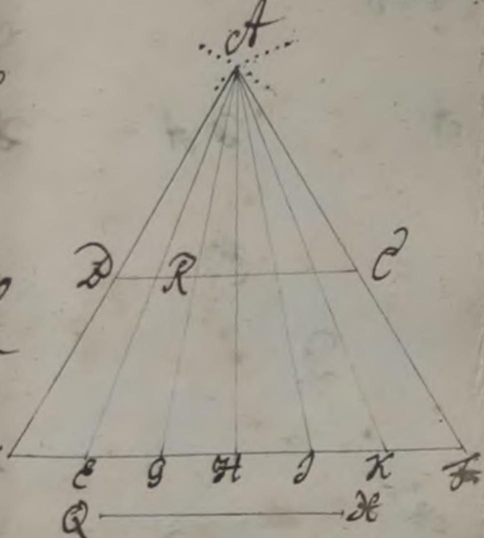
III) Subdivide quamcunque Rectam DF in imperatas æquales partes v. c. in 6.

2) Super ipsa describe Triangulum æquilaterum DFA. §96

3) Accipe rectam QX eamque Centro A applica adutrumque Crus Trianguli æquilateri in D et C.

4) Duc DC. §81.

5) Per A et puncta Divisionum E, G, H, I, K duc Rectas AE, AG etc. secantes Rectam DC in R, Q. D. F.



Nam

$\angle A = \angle A$. §40. Ar.

et $DF : AF = AD : AC$.

$\angle D = \angle ADC$. §356.

ΔADF æq. ang. Δlo ADC. §153. 305.

Ergo

$AD : AD = DF : DC$. §353.

Sed $AD = DF$ p. c.

Ergo $AD = DC$. §152. Ar.

Cumque $AD = QX$. p. c.

$DC = QX$

* *

* *

Porro

$\angle ADC = \angle D$ p. d.

$\angle DAE = \angle DAR$. §40. Ar.

Δum DAE æq. ang. Δo DAR. §153. 305.

Ergo

$AD : DE = AD : DK$. §352.

h. e.

$DF : DE = QX : DK$. §10 Ar.

Sed

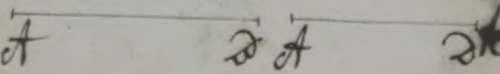
$DF : DE = 6 : 1$. p. c.

$QX : DK = 6 : 1$. §144 Ar.

Q. E. D.

§365. Problema LI

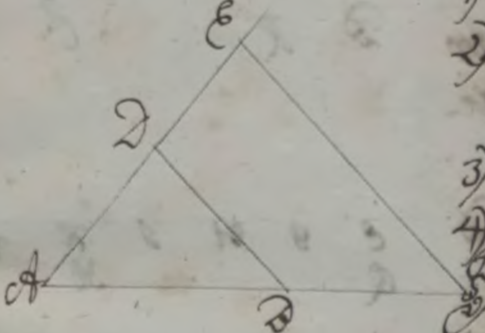
Datis duabus rectis Lineis AB, A tertiam ppalem invenire.



Resolutio.

1) Facto quolibet ∠lo rectilineo A
2) Lateri uni ex A applica AB, alter vero ex eodem A, AD.
3) Fac BC = AD.
4) Junge BD. §81.
5) Cum hac duc EC ∥lam §138.

Dico DE esse quaesitam.



Demonstratio

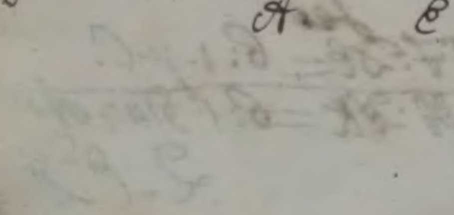
DB ∥ EC p. C.
AB : BC = AD : DE. §349.
Sed BC = AD p. C.
AB : AD = AD : DE. §10 Ar.

Q. E. D.

Aliter.

1) Junge Rectam AB alteri AD ad
∠ R.R. §158.
2) Duc DB. §81
3) In D excita ⊥lem DE &c. occur
rentem producta AB in E.

Dico AE esse quaesit



Demonstratio

$\triangle BDE$ est Æglm p. C. atq. DA ∥lis ad DE p. C. Ergo.
BA : AD = AD : AE - § 359. M. 1.
Q. E. D.

§ 366. Problema L. II
Datis tribus Rectis DE, DG, EF invenire quartam ppalem GH.

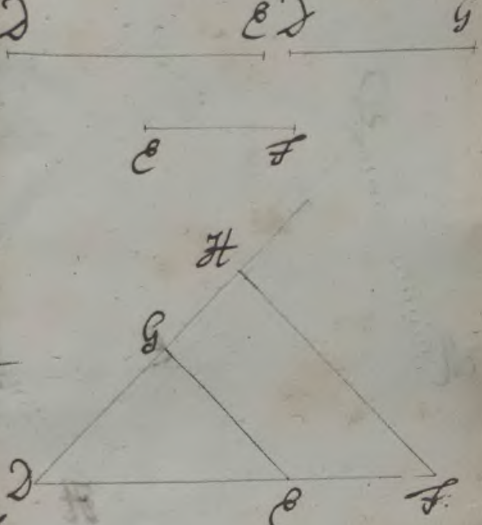
Resolutio.

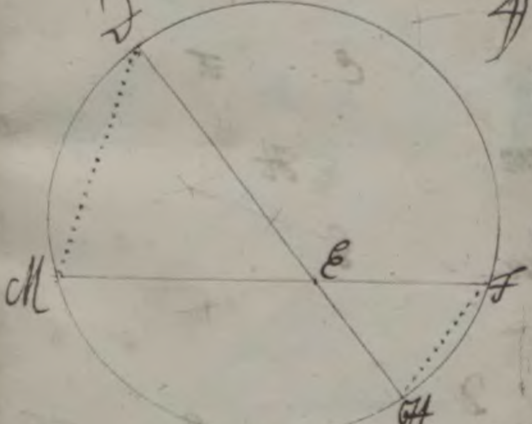
1) Facto quolibet ∠lo rectilineo.
2) Applica ad Verticem D primam datarum DE
3) Applica ad alterum crus ad eundem Verticem secundam datarum DG.
4) Atq. ipsi DE pone indirectum tertiam EF. § 52.
5) Junge GE § 81 huicq. duc HF ∥lam per p(unc)tum F. § 135.

Dico GH esse quartam.

Demonstratio.

Quia GE ∥ HF p. C.
DE : EF = DG : GH. § 349.
Q. E. D.





Aliter

1) Medias datarum EF et ME pone in directum §82.

2) Ad E sub quolibet ∠lo rectilineo stra primam DE.

3) Per pcta M, D, F, describe Circulu §317.

4) Produc DE in H ad concursum cum Pphia. §82.

Dico EH esse quart

Demonstratio

MF et DH sunt Rectae semet secantes in E. §29.

Ergo.

$ME \times EF = DE \times EH$. §292.

Ergo

$DE : ME = EF : EH$. §311 Ar. Q. E. D.

Vel h. m.:

Duc MD et HF. §81.

$\left.\begin{array}{l}\angle D = \angle F\\ \angle M = \angle H\end{array}\right\}$ §282.

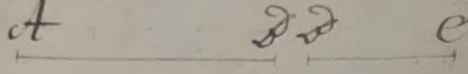
△ DME aeqlg. △lo EFH. §155. 305

Ergo

$DE : EM = EF : EH$. §382.

§367. Problema LIII

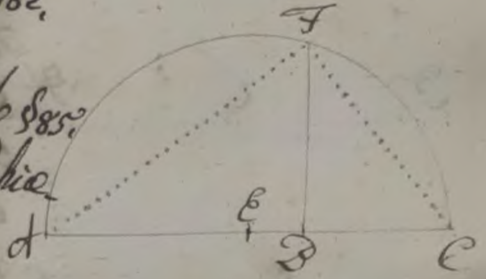
Datis duabus rectis Lineis AB et BC mediam ppalem invenire.



Resolutio.

1) Junge datas AB, BC in directum §82.
2) Diseca totam AC in E. §112.
3) Descriptoque Radio AE semicirculo §85.
4) Ex B educ ⊥lem occurrentem pphiae BF. §120.

Dico BF esse quaesitam.



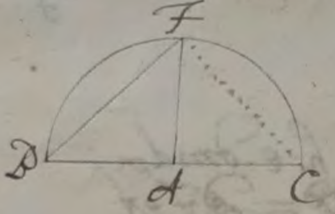
Demonstratio.

Ductis AF, FC. §81
∠AFC = R. §288 atque
BF ⊥lis ad AC p. C.

AB : BF = BF : BC. §359.

Vel

Manentibus iisdem datis rectis AB, BC.



1) Super Majorem BC describe semicirculum. §85.
2) In diametro abscinde minorem BA.
3) Ex A erige ⊥lem occurrentem pphiae in F, AF §120.
4) Duc BF. §81. Hanc dico esse quaesitam

¤

¤ Demonstratio.

Duc FC. §81.
Ergo ∠BFC = R. §288.
cumque AF ⊥ ad BC p. C.

CB : BF = BF : BA §359.

Q. E. D.

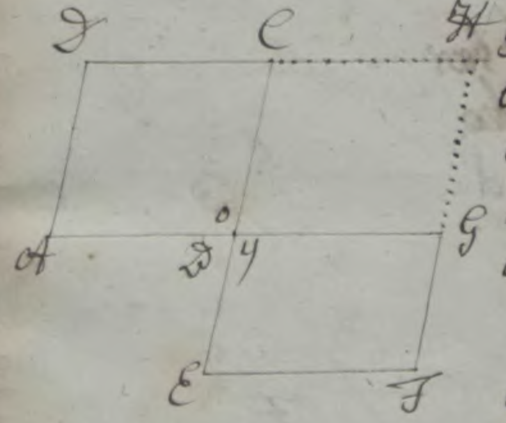
§368. Corollarium.

Hinc omnis Recta quae a quovis Diametri pcto ad Pphiam usq; producta, normaliter educitur est media ppalis inter duo Diametri segmenta.

§369. Theorema 103.

Aequalium et unum ABC uni EBG aequalem habentium ∠um Parallelogrammorum BD et BF, reciproca sunt Latera quae circum aequales ∠los.

Et: Quorum Pllgmorum BD et BF unum ∠um ABC uni ∠lo EBG aequalem habentium, reciproca sunt Latera circum aequales ∠los, illa sunt aequalia.



h. e. dmdm:

1) Si BD = BF et
$\angle o = \angle y$
AB : BG = BE : BC vel
AB : BE = BG : BC.

2) Si in Pllgmis BD et BF
$\angle o = \angle y$ et
AB : BG = BE : BC vel
AB : BE = BG : BC
Pllgm BD = Pllgo BF.

Demonstratio.

Super Rectis AB et EC ad eundem Verticem B non tamen ad easdem partes ordina Parallelogramma BD et BF. §135.

Produc DC et FG usq; ad Concursum in H. §82. Ergo
Pllgma BD et BH } sunt in iisdem
item BF et BH } ≈lis.

Ergo:

AD : DG = DB : DH. §347.
CD : DL = DF : DH. §c.
Sed DB = DF p. H.

AD : DG = CD : DL. §144. Ar
AD : CD = DG : DL. §150. Ar
Q. E. D.

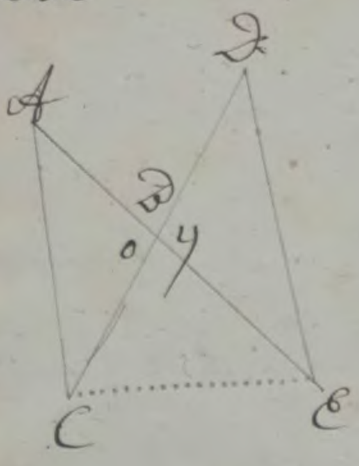
AD : DG = DE : DL. p. H.
Sed AD : DG = DB : DH. A} §347.
CD : DL = DF : DH }

DB : DH = DF : DH. §144. Ar.
DF = DB. §152. Ar
Q. E. D.

§370. Theorema 104.

Aequalium et unum ∠lum ADC uni DBE aequalem habentium Triangulorum ADC, DBE reciproca ſunt Latera, quae circum aequales ∠los conſtituta ſunt.

Et: Quorum Triangulorum ADC et DBE unum ∠lum ADC uni DBE aequalem habentium reciproca ſunt Latera, quae circum aequales ∠los, illa ſunt aequalia.



Demonstratio.

Productis Lateribus AB, DB stata Latera BE et BC in directum. Ergo ABE est Recta.

Ductaq CE §91 erit

$AB : BE = \triangle ABC : \triangle BCE$. §347.

$DB : BC = \triangle DBE : \triangle BCE$. §c.

Sed $\triangle ABC = DBE$: p. Hyp.

$AB : BE = DB : BC$. §144 Ar.

Q. E. 1.

$AB : BE = DB : BC$. p. Hyp.

$\triangle ABC : \triangle BCE = AB : BE$ et $\Big\}$ §347.

$\triangle DBE : \triangle BCE = DB : BC$. $\Big\}$

$\triangle ABC : \triangle BCE = \triangle DBE : \triangle BCE$. §144 Ar.

adeoq

$\triangle ABC = \triangle DBE$. §152 Ar

Q. E. M. D.

§371. Theorema 105.

Si fuerint quatuor Rectæ ppales, quod sub extremis comprehenditur Rectangulum, æquale est ei quod sub mediis comprehenditur Rectangulo. Et:

Si sub extremis comprehensum Rglum AC, aequale fuerit ei, quod sub mediis comprehenditur Rglo, illae quatuor Rectae sunt ppales.

Demonstratio.

AC et EG sunt Rgla p. H.

Ergo ∠D = ∠F §175. ge.

Cumque AD : FG = EF : CD p. H.

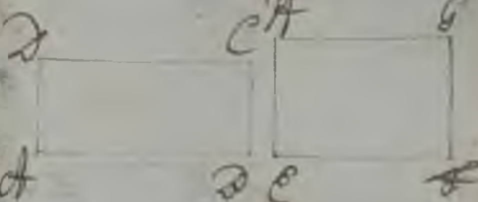
Ergo AC = EG. Q.E.I.

Quoniam uti ante

∠D = ∠F p.d. et H.

et AC ~~= EG = FE : CD §369.~~ p.H

AD : FG = FE : CD §369. Q.E.//D.



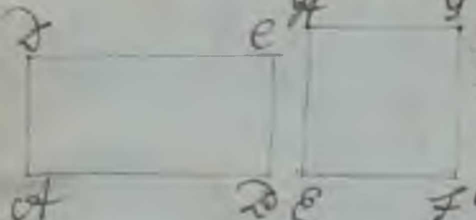
§372. Corollarium.

Hinc ad datam rectam Lineam ~~AD~~ facile est Rectangulum dato EG aequale applicare quaerendo p. quartam ppalem ad datas tres AD, GF, EF §366. ut scil. sit AD : GF = EF : CD.

§373 Theorema 106.

Si tres Lineæ fuerint ppales quod sub extremis comprehenditur, Rectangulum, æquale est ei, quod a media describitur Quadrato, Et:

Si sub extremis comprehensum Rectangulum, æquale sit ei, quod a media describitur, Quadrato, tres illæ Rectæ ppales erunt.



Demonstratio.

Accipe EF = GF et
describe Rectangulum §171.
Ergo ∠D = ∠F. §68. 92. 70.
et AD : EF = GF : DC. p. H.

AC = EG. §369.
$AC = EF^2$ Q.E.I.

∠D = ∠F p. C.
atq. $AC = EF^2$ p. H.

AD : EF = EF : DC. §369.   Q.E.IID.

§374. Problema LIV.

A data recta Linea AB dato Rectilineo CEFD simile similiterq. positum Rectilineum ABHG describere.

Resolutio.

1) Resolve Rectilineum datum CE DF in Triangula per Diagonales.

2.) Fac $\angle$lum $B = \angle D$

$\left.\begin{array}{l}\angle BAH = \angle DCF \\ \angle AHG = \angle CFE \\ \angle HAG = \angle ECF\end{array}\right\}$ §107.

3) Produc Crura $\angle$lorum ad Concursum usq. in G et H. §82.

Dico ABHG esse Rectilineum quaesitum.

Demonstratio.

$\angle o = \angle i$ } p.C.

$\angle y = \angle z$ }

$\angle x = \angle k$ §158.

Porro $\angle m = \angle p$ } p.C.

$\angle n = \angle q$ }

$\angle g = \angle w$ §c.

Ergo $\angle p + k = \angle x + m$ §42.

h.e $\angle F = \angle H$. §47.

Sic et $q + z = \angle n + y$. §42.

h.e $\angle C = \angle A$. §47. } Ar

cum et $\angle D = \angle B$ p.C. Ergo Rectilinea sunt aequiangula §300.

Cum vero et
△la CDF et ADH } sint æquila pd.
atque CEF et AGH }

Ergo:
$AD : DH = CD : DF$. §352.
Q.E. I.

Sed et $DH : HA = DF : FC$. } §c.
et $GH : HA = EF : FC$ }

$DH : GH = DF : EF$. §173. Ar.
Q.E. II.

Porro etiam:
$HG : GA = FE : EC$. §352.
Q.E. III.

cumque $GA : AH = EC : CF$. } §c.
et $AD : AH = CD : CF$ }

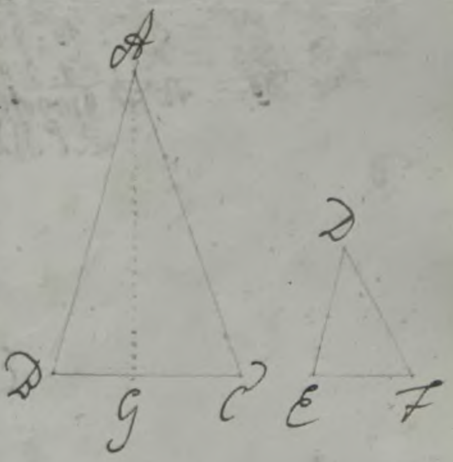
$GA : AD = EC : CD$. §173. Ar.
Q.E. IV.

Ergo.
Singula Latera circum æquales ∠los sunt ppalia. Proinde
ADHG ~ CDFE. §341.
atque ex ipsa Constructione similiter descripta §342.

Q.E.D.

§ 375. Theorema 107.

Similia Triangula ADC et DEF ſunt in duplicata Ratione Laterum homologorum DC et EF.



Demonſtratio.

Fac DC : EF = EF : DG. § 366.
atq; duc AG § 81.
Quia Δ ADC ~ Δ DEF. p. H.
Ergo ∠D = ∠E. § 341.
Ergo AD : DE = DC : EF. § 353.
Sed DC : EF = EF : DG. ~~[illegible]~~ p. C.

AD : DE = EF : DG. § 144 Ar.
Ergo Δ ADG = Δ EDF. § 370.

Verum.

Δ ADC : Δ ADG = DC : DG § 347.
et DC : EF = EF : DG p. C.

DC : DG = $DC^2 : EF^2$. § 189 225. Ar.

Δ ADC : Δ ADG = $DC^2 : EF^2$. § 144. Ar.
Sed Δ ADG = Δ DEF p. d.

Δ ADC : Δ DEF = $DC^2 : EF^2$. § 10 Ar.
Q. E. D.

h. e. dmdimi
Si Δ ADC ~ Δ DEF.
eſſe
Δ ADC : Δ DEF = $DC^2 : EG^2$.

Eſto Δlum ſuper DC = T.
EF = t
DG = θ
Quia
DC : EF = EF : DG p. H.
Ergo DC : DG = $DC^2 : EF^2$. § 189 Ar.
Verum T ~ t. p. H.
Ergo T : t = $DC^2 : EF^2$. § 375

DC : DG = T : t. § 144. Ar.
Q. E. D.

§ 376. Corollarium

Hinc ſi tres Lineae DC, EF, DG ſint ~~pp~~ales, uti eſt prima ad tertiam

Est autem et $t \backsim \theta$ p. H.

288. Ergo $t : \theta = EF^2 : DG^2$. §375

sed $DC : EF = EF : DG$ p. H.

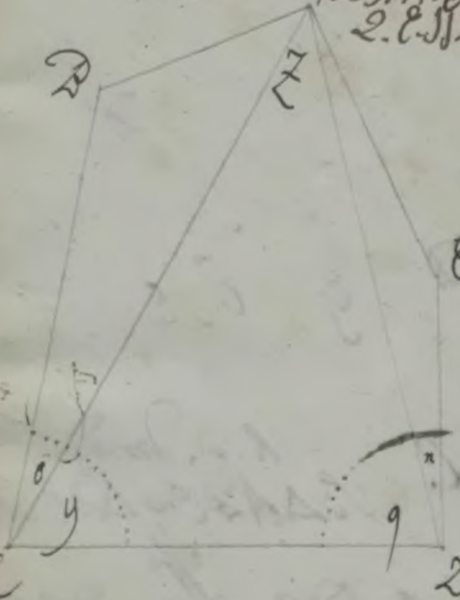
Ergo $DC^2 : EF^2 = EF^2 : DG^2$. §182

Cumq; $DC : DG = DC^2 : EF^2$. §189

Ergo $DC : DG = t : \theta$. §144

Ita Triangulum super primam ad Triangulum super secundam simile similiterq; descriptum. Vel ita est

2. E. S. D. Triangulum super secundam ad Triangulum super tertiam simile similiterq; descriptum.



§ 377. Theorema 108.

Similia Polygona ABCDE atq; FGHIK in similia Triangula ~~ABC, FGH~~, ADC, FIH; atq; AED, FKI dividuntur et Numero aequalia et homologa Totis n. p. Polygonis. Et Polygona ABCDE et FGHIK duplicatam habent eam inter se Rationem quam Latus homologum BC ad Latus homologum GH.

Demonstratio.

Mbr 1. $\angle B = \angle G$. §341 et H.

et $AB : BC = FG : GH$. §c. et H.

$\triangle ABC$ aeqsl. $\triangle FGH$. §386.

Sic etiam:

et $AE : ED = FK : KI$. $\angle E = \angle K$. } p. H. §341.

$\triangle AED$ aeqsl. $\triangle FKI$. §356.

$\angle C = \angle H$ p. H.
hc $\angle o+y = \angle x+u$. §47. Ar.
sed $\angle o = \angle x$ p.d.

$\angle y = \angle u$. §43 Ar

$\angle D = \angle I$. p. H.
hc $\angle q+n = \angle p+i$. §47 Ar
sed $\angle n = \angle i$ p.d.

$\angle q = \angle p$. §43 Ar.

$\triangle$ ACD æqllum $\triangle$lo FHI. §155. 305.

Ergo

Triangulorum horum æquiangulorum latera homologa sunt proportionalia §352 adeoque

$\triangle ABC \sim FGH$
$\triangle ACD \sim FHI$ } §341
$\triangle AED \sim FKI$

Q.E.I.

Nbr 2. Quia ABC $\sim \triangle FGH$ p.d.

Ergo:

ABCDE : Pol: FGHI.K
$\triangle ABC : \triangle FGH$ = Pol. ABCDE : Pol. FGHIK
sicut $\triangle ACD : \triangle FHI$ = Pol. ABCDE : Pol. FGHIK } §148.
ut et $\triangle AED : \triangle FKI$ = Pol. ABCDE : Pol. FGHIK

$\triangle ABC : \triangle FGH = \triangle ACD : \triangle FHI$ = AED : $\triangle FKI$. = Pol. ABCDE : Pol. FGHIK §144. Ar.

Q.E.II.

Membr 3.

$$\left.\begin{array}{l}\Delta ABC : \Delta FGH = BC^2 : GH^2 \\ \Delta ACD : \Delta FHI = CD^2 : HI^2 \\ \Delta ADE : \Delta FIK = ED^2 : KI^2\end{array}\right\}\text{§325.}$$

$$\Delta ABC + ACD + ADE : \Delta FGH + FHI + FIK = BC^2 + CD^2 + ED^2 : GH^2 + HI^2 + KI^2 \text{ §170 Ar.}$$

cumque $BC : GH = CD : HI$. p. Hyp et §353.

$BC : GH = CD : HI$.

$BC^2 : GH^2 = CD^2 : HI^2$ §187 et 225 Ar.

Similiter quia

$CD : HI = ED : IK$. p. Hyp. et §353.

$CD : HI = ED : IK$.

$CD^2 : HI^2 = ED^2 : IK^2$ §187 et 225 Ar

$BC^2 : GH^2 = CD^2 : HI^2 = ED^2 : IK^2$ §144 Ar.

Proinde:

$BC^2 + CD^2 + ED^2 : GH^2 + HI^2 + IK^2 = BC^2 : GH^2$ §165 Ar.

$\Delta ABC + ACD + ADE : \Delta FGH + FHI + FIK = BC^2 : GH^2$. §144 Ar

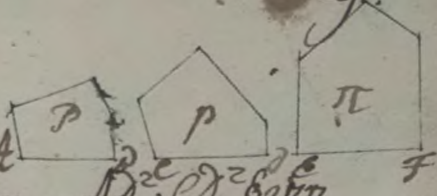
h.e.

Polyg: $ABCDE$: Polyg $FGHIK = BC^2 : GH^2$ §47 A

Q. E. D.

§378. Corollarium. 1.

Hinc si fuerint tres Lineæ rectæ ppales, uti est prima ad tertiam ita Polygonum super primam ad Polygonum super secundam simile similiterque descriptum; vel ita erit Polygonum super secundam ad Polygonum super tertiam simile similiterque descriptum.



P : p = AB² : CD². §377.

Sed e AB : CD = CD : EF. p. H.

AB² : CD² = AB : EF. §189. Ar.

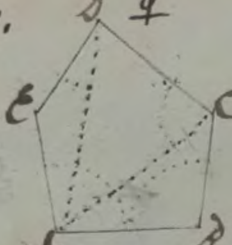
P : p = AB : EF. §144. Ar.

Q.E.I.

♀

§379. Scholion.

Inde quidem elicitur Methodus figuram quamlibet augendi vel minuendi in ratione data. Ut si velis Pentagoni ABCDE cujus Latus sit AB aliud facere quintuplum.



Quare.

1) Inter AB et 5×AB quære mediam ppalem. §367.

2) Super hac construe Pentagonum simile dato. §374.

Iste erit quintuplum dati.

Nam

AB : GH = GH : 5×AB p. C.

AB : 5×AB = Pol. ABCDE : Pol. P. §378.

1 : 5 = Pol ABCDE : Pol. P. §160 Ar.

Ergo 5×Pol. ABCDE = Polyg. P. §209. Ar. Q.E.D.

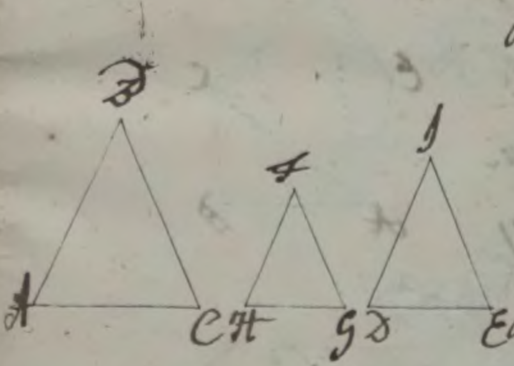
♀ p.π = CD² : EF².

§380. Corollarium 2.

Hinc etiam ſi Figurarum ſimilium homologa Latera nota fuerint, etiam Proportio figurarum innoteſcet nſ. inveniendo tertiam proportionalem.

§381. Theorema 109.

Rectilinea ABC et DIE ſimilia eidem Rectilineo HFG et inter ſe ſunt ſimilia.



Demonſtratio.

Quia Δlum ABC ∼ ΔFHG. p. H.

Ergo $\left.\begin{array}{l}\angle A = \angle H \\ \angle B = \angle F \\ \angle C = \angle G\end{array}\right\}$ §341.

Sed et DIE ∼ FHG p. H.

$\left.\begin{array}{l}\angle H = \angle D \\ \angle F = \angle I \\ \angle G = \angle E\end{array}\right\}$ §c.

ΔABC aeqlgl. ΔDIE. §41 Ar. 305.

adeoque

$\left.\begin{array}{l}AB : AC = ID : DE \\ AC : CB = DE : EI\end{array}\right\}$ §352.

$AB : CB = ID : EI$. §172. Ar.

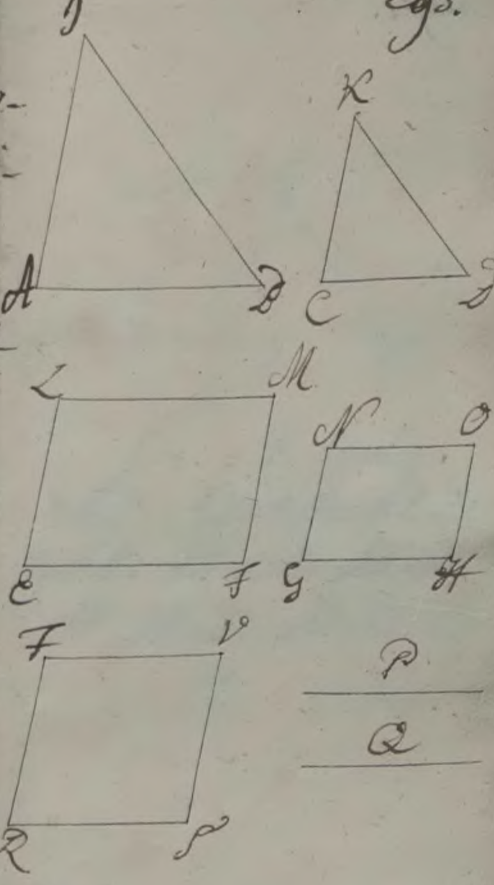
Ergo

ABC ∼ DIE. §341. Q. E. D.

§382. Theorema 110.

Si quatuor Rectæ fuerint ppales et ab eis Rectilinea similia similiterq; descripta ppalia erunt.

Et contra: Si a rectis Lineis similia similiterq; descripta Rectilinea ppalia fuerint, ipsæ etiam Lineæ rectæ ppales erunt.



Demonstratio.

Mbr. 1. Inveni ad AB et CD tertiam ppalem P. §

et ad rectas EF, GH tertiam ppl. Q §365

Ergo

AB : CD = CD : P } p. C.
EF : GH = GH : Q }
sed AB : CD = EF : GH. p. H.

CD : P = GH : Q. §144 Ar.

AB : P = EF : Q. §172. Ar.
sed AB : P = ABI : CDK. §378.
et EF : Q = EM : GO. §c.

ABI : CDK = EM : GO. §144 Ar.

Q.E.I.

Si Rectilinea EM, GO fuerint Δla argumentatio procedit per §376

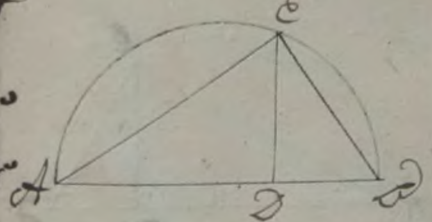
h.e.

1) Si AB : CD = EF : GH
erit ABI : CDK = EM : GO.
2) Si ABI : CDK = EM : GO,
erit AB : CD = EF : GH.

Membrum II.
Invenj ad AB, CD, EF quartam ppa-
lem RS. §366.
Super istam fac Rectilineum simile
similiterque positum ipsi EM. §374.
Quare cum $EM \sim G\theta$. p. H.
et $EM \sim RV$. p. C.
$G\theta \sim RV$. §381.
Ergo cum
$AB : CD = EF : RS$ p. C. Ergo
$ABI : CDK = EM : RV$. p. Cas. I.
$ABI : CDK = EM : G\theta$. p. H.
$EM : RV = EM : G\theta$ §144. Ar.
$RV = G\theta$. §152. A.
sed $RV \sim G\theta$ p. d.
RV congruit $G\theta$ §88.
$RS = GH$. §87.
Ergo.
$AB : CD = EF : GH$. §100. Ar.
Q. E. II. D.

§383. Theorema III.
Si Recta AB secta sit utcunque in D
Rectangulum sub partibus AD et DB

contentum est medium ppale inter earum Quadrata. Item: Rectangulum contentum sub tota AB et una parte AD vel DB est medium ppale inter Quadratum totius AB et Quadratum dicte partis AD vel DB.



h. e. p. H. dmdm.

1) $AD^2 : AD\times DB = AD\times DB : DB^2$

2) $AB^2 : AB\times AD = AB\times AD : AD^2$

3) $AB^2 : AB\times DB = AB\times DB : DB^2$

Demonstratio.

Super tota AB describe semicirculum §85. atq ex D erige ⊥lem DE occurrentem Pphiæ in E. §120.

Ergo

Mbr 1. $AD : DE = DE : DB$ §359.

$$\frac{AD : DE = DE : DB}{AD^2 : DE^2 = DE^2 : DB^2}$$ §187 Ar.

sed $DE^2 = AD\times DB$ §373.

$AD^2 : AD\times DB = AD\times DB : DB^2$ §10 Ar.

Q.E.I.

Mbr. 2.

$AB : AE = AE : AD$ §359.

$$\frac{AB : AE = AE : AD}{AB^2 : AE^2 = AE^2 : AD^2}$$ §187 Ar.

$AE^2 = AB\times AD$ §373.

$AB^2 : AB\times AD = AB\times AD : AD^2$ §10 Ar.

Q.E.II.

Mbr III.

$AB : BE = BE : BD$ 359.

$AB^2 : BE^2 = BE^2 : BD^2$ §187 Ar.

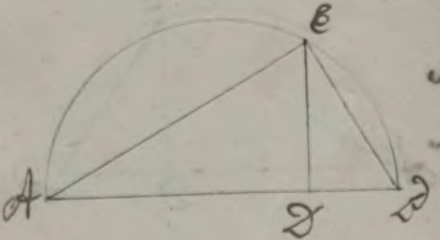
$BE^2 = AB\times BD$ §373.

$AB^2 : AB\times BD = AB\times BD : BD^2$ §10 Ar.

Q.E.III. D.

§384. Scholion.

Poterant autem Proposita expeditius demonstrari f. m.



$AD^2 : AD \times DB = AD : DB$. §347.
$AD \times DB : DB^2 = AD : DB$. §c.

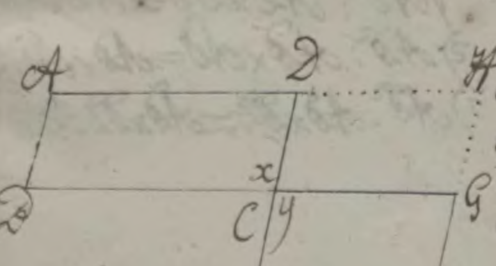
$AD^2 : AD \times DB = AD \times DB : DB^2$. §144 Ar.

Q. E. I.

$AB^2 : AB \times AD = AB : AD$. §347.
$AB \times AD : AD = AB : AD$. §c.

$AB^2 : AB \times AD = AB \times AD : AD^2$. §144.

Q. E. II.



$AB^2 : AB \times DB = AB : DB$. §347.
$AB \times DB : DB^2 = AB : DB$. §c.

$AB^2 : AB \times DB = AB \times DB : DB^2$. §144.

Q. E. III D.

§385. Theorema 112.

Aequiangula Parallelogramma AC et CF inter se Rationem habent eam, quae ex Lateribus componitur.

Demonstratio.

PH. dmdm.

Pllgm AC : Pllm CF = DC × BC : CG × CE.

Conjunge Pllgma AC, CF alio CH ad ∠los aequales x et y productis rp. Rectis AD et FG ad concursum in H; ita tamen ut DC, CG.

itemq; EC, CD in directum jaceant. p

Assume Rectam quamvis I atq; ad DC, CG et I quaere quartam proportionalem K. §366.
Sive et ad DC, CE et K quaere quartam propalem L. §c.

Quia

$DC : CG = AC : CH$ §347.

$DC : CG = I : K$ p.C.

$AC : CH = I : K$ §144 Ar.

Porro

$DC : CE = CH : CF$ §347.

$DC : CE = K : L$ p.C.

$CH : CF = K : L$ §144 Ar.

$AC : CF = I : L$ §172 Ar.

cumq; $DC : CG = I : K$ } p.C.
p $DC : CE = K : L$ }

$CD \times CD : CG \times CE = I \times K : K L$ §187 Ar.

$I \times K : K \times L = I : L$ §160 Ar.

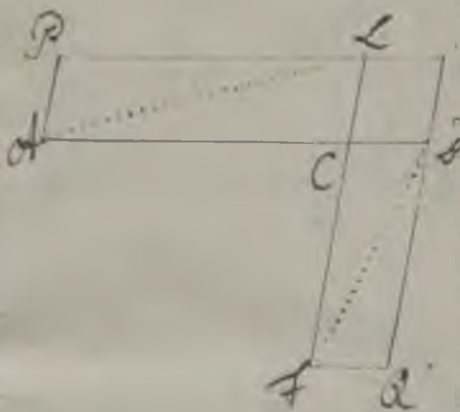
$DC \times DC : CG \times CE = I : L$ §144 Ar.

Quare

$AC : CF = DC \times DC : CG \times CE$ §c. et 142 Ar.

Q. E. D.

298



§386. Corollarium. 1.
Inde patet per §169. Triangula qu
∠lum unum C, aequalem habent, Ra
tionem habere compositam ex Rati
nibus Rectarum AC : CD et LC : CF,
aequalem ∠lum comprehendentium

Nam.

$CP : CQ = AC \times CL : CF \times CD$. §385.
$CP : CQ = \frac{PC}{2} : \frac{CQ}{2}$ §160. Ar.

$\frac{PC}{2} : \frac{CQ}{2} = AC \times CL : CF \times CD$. 144 Ar.

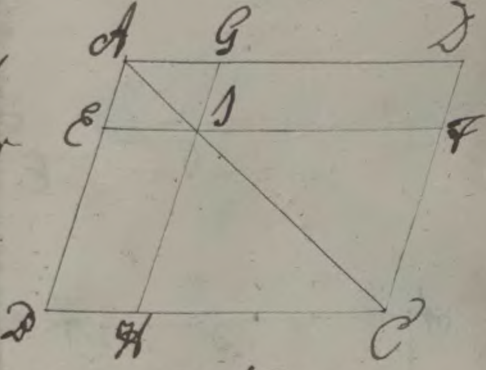
Sed $\frac{CP}{2} = \triangle ACL$ } §169.
et $\frac{CQ}{2} = \triangle CDF$ }

Ergo
$\triangle ACL : \triangle CDF = AC \times CL : CF \times CD$.
§10 Ar.

§387. Corollarium 2.
Elucet etiam Rectangula adeoq
et Pllgma per §174. quaecunq Ratio
nem habere inter se compositam
ex Rationibus Basis ad Basin
atq Altitudinis ad Altitudinem
Id quod et de Triangulis ut pote
illorum dimidiis valet §169.

§388. Theorema 113.

In omni Parallelogrammo ABCD quae circa Diametrum AC sunt Pllgma EG et HF et Toti BD inter se sunt similia



P. H. dmdm.
$ABCD \sim EG \sim HF$.

Demonstratio.

In Parallelogrammis
EG et BD

$\angle EAG = \angle BAD$. §40. Ar:
$\angle AGI = \angle ADC$ } §133.
$\angle AEI = \angle ABC$ }
cumq $\angle EIG = \angle BHI$ } &c.
et $\angle BHI = \angle BCD$ }

$\angle EIG = \angle BCD$. §41 Ar.
Ergo EG aeq∠glm BD. §305.
Simili discursu demonstrabis
BD aeq∠glm HF. Ita ut
EG aeq∠lm BD aeq∠lm HF. §40 Ar.
Q. E. I.

$\angle AEI = \angle ABC$ p. d.
$\angle EAI = \angle BAC$ §40 Ar.

$\triangle EAI$ aeq∠lm $\triangle$lo BAC. §155. 305
Id quod cum simili ratione de $\triangle$lis AGI et et ADC evincatur
Ergo: $AE : EI = AB : BC$ §352 Q. E. I.
$EI : IA = BC : CA$ } &c.
$IA : IG = CA : CD$ }

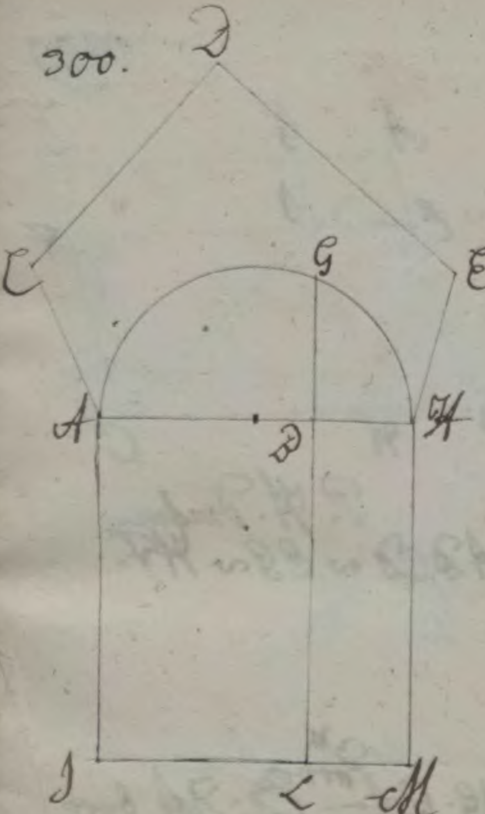
✗ $EI : IG = BC : CD$. §172. Ar. Q. E. II

Porro ✗
$IG : GA = CD : DA$. §352.
Q. E. III.
$AG : AI = AD : AC$ } &c.
$AI : AE = AC : AB$ }

$AG : AE = AD : AB$. §172 Ar.
Q. E. IV.

Ergo
$EG \sim BD$ §341.
Similiter cum sit discursu
$HF \sim BD$.

$BD \sim EG \sim HF$. §381.
Q. E. D.



§389. Problema LV.

Dato Rectilineo ABCDE ſimile ſimiliterq poſitum P, idemq alteri dato Rectilineo F æquale efficere.

Reſolutio.

1) Fac Rectangulum $AL =$ Rectilineo ABCDE §186. 187.

2) Itemq ſuper BL Rectangulum $BM =$ Rectilineo F §§.cc. aut ſi Triangulum fuerit §185.

3) Inveni inter AB et BH mediam ppalem BG. §364.

4) ſuper $BG = NO$ fac Polygonum ſ. Rectilineum $P \sim ABCDE$ ſimile ſimiliterq poſitum §374.

Hoc dico æquale ipſi F.

Demonſtratio

$AB : BG = BG : BH$ p.c.

$ABCDE : P = AB : BH$. §378.

$AL : BM = AB : BH$. §347.

$ABCDE : P = AL : BM$. §144. Ar.

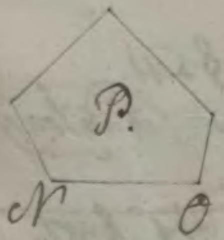
Verum $ABCDE = AL$. p.c.

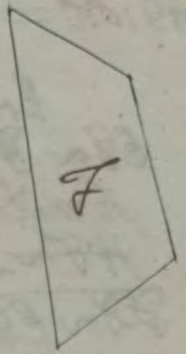
Ergo $P = BM$. §152 Ar.

Sed $BM = F$. p.c.

$P. = F$. §410 Ar.

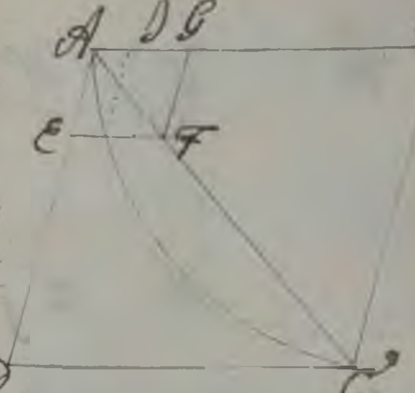
Q. E. D.





§390. Theorema 114.

Si a Parallelogrammo ABCD Parallelogrammum AGFE ablatum sit, et simile Toti et similiter positum communem cum eo habens Angulum EAG, hoc circa eandem cum Toto Diametrum AC consistet.



Demonstratio.

Si negas AC communem esse Diametrum, duc aliam AH quae secat EF in H, praetereaq; duc HI ≈lam AE §135.

Ergo Pllgm EI ~ DB. §.388.

Ergo $AE : EH = AD : DC$. §341.

Sed $AD : DC = AE : EF$ p. H.

$AE : EH = AE : EF$. §144 Ar.

Ergo $EH = EF$. §152. } Ar.

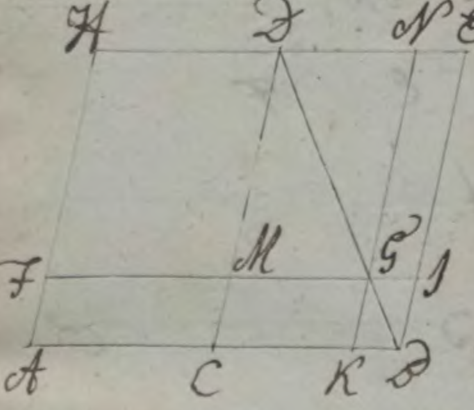
J. Q. E. A. p. §47)

§391. Theorema 115.

Omnium Parallelogrammorum secundum eandem rectam Lineam AB applicatorum deficientiumq; figuris Parallelogrammis similibus similiterq; positis ei quod a dimidia describitur maxi=

mum eſt, quod ad dimidiam eſt applicatum, ſimile exiſtens defectui.

h. e.



Detur Recta AB diviſa bifariam in C ſuperque eius dimidiam BC conſtituatur quodcunque Pllgm CEB, cujus diameter ſit DB. Si igitur compleatur totum Pllgm ABEH erit Pllgm AD super dimidiam AC ipſius AB conſiſtens, applicatum ſecundum AB deficiens Pllgmo CE et exiſtens ſimile defectui CE. Dico Pllgm AD ad dimidiam AC applicatum deficiensque Pllgmo CE maximum eſſe omnium quo ſecundum rectam AB applicantur, deficiuntque Pllgis ſimilibus ſimiliterque poſitis ipſi CE. Ita Clavius in Com. ad Eucl. L. VI. Prop. XXVII.

Demonſtratio

Sumto utcunque in diametro DB puncto G, ductiſque

per G ≈la FI cum AB } §135

et ≈la MK cum DC }

Pllgm AG erit deficiens Pllgo KI. §346.
Pllgm KI autem $\sim$ Pllgo LE. §388.
ſimiliterq; poſitum ipſi LE §342.
Dico ergo $AG \angle AD$.

$GE = GL$. §184
$KI = KI$. §40 Ar

$KE = LI$. §42. Ar
ſed $AL = LB$ p. H.
$FI \approx AD$ p. C.

$AM = CI$ §176.

$KE = AM$. §41 Ar.
$LG = LG$. §40. Ar.

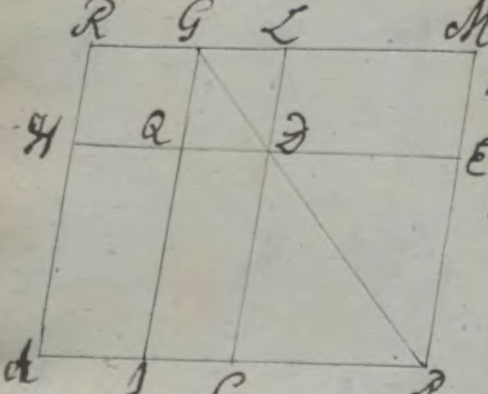
$KE + LG = AG$. §42. Ar
$KE + LG \angle LE$. §47. Ar

$AG \angle LE$. §46. Ar.
cumq; $AL = LB$. p. H.
B HE $\approx AD$. p. C.

$AD = LE$. §176.

$AG \angle AD$. §46 Ar

Q. E. D.



Ponamus porro G non cadere in Diametrum ipsam DB, sed in continuatam ultra HE. ducta ergo per G ≈la RM cum AB vel HE, et ≈la GI cum AH vel EB. §35. productisque AH et BE in R et M. §82.

Erit Pllgm AG applicatum ad Rectam AB deficiens Pllgo IM. §346.

Pllgm vero IM ∼ CE. §388.
similiterque positum ipsi CE. §342.

Dico ergo AG ∠ AD.

Produc ED in L. §82.

Quia RC et LM sunt Pllga p. c.
Ergo RL = AC }
LM = CB } §167.
Sed AC = CB p. H.
RL = LM. §41 Ar.
Sed RM ≈ HE p. c.
Pllgm RD = Pllgo DM. §176.
Sed DI = DM. §184.
RD = DI. §41. Ar.
cumque RQ ∠ RD. §47. Ar

* *

* *
RQ ∠ DI. §46
Sed AQ = AQ. §40.
AQ + QR ∠ AQ + DI. §42 Ar
h. e.
AG ∠ AD. §47.
Q. E. D.

§392. Problema LVI

Ad datam rectam Lineam AB dato Rectilineo C æquale Parallelogrammum AF applicare deficiens figura Parallelograma ZR, quæ simili sit alteri Parallelogramo dato D. Oportet autem datum Rectilineum C, cui æquale AF applicandum est, non majus esse eo AF, quod ad dimidiam applicatur similibus existentibus Defectibus et ejus AF, quod ad dimidiam applicatur, et ejus D, cui simile deesse debet.

cf Fig. pag. 307.

Resolutio.

1) Biseca AB in E. §112

2) Super EB describe Pllgm EG simile ipsi D similiterque positum §374.

3) Comple totum Parallelogrammum AHGB.

Quod si ergo AF = C cum sit adplicatam ad AB deficiens Pllgmo EG, simile ipsi D, F.E. Q.P.

Quod si vero majus sit Pllgm AP
ipso C, minus enim esse nequit
per §391. Oportet enim datum
Rectilineum G, cui aequale AP
applicandum est. Ergo GE major
erit ipso Rectilineo G, quia AP =
CG. — Quare —

4) Investiga Excessum ipsius GE su-
pra Rectilineum C. §188. qui
sit = Rectilineo I.

5) Fac Pllgm KM ∼ dato D ~~el~~
~~GE~~ = I. §389.

6) Duc Diametrum FD. §81.

7) Fac FO = KN et } §26.
FQ = KJ }

8) Per O et Q duc
≈lam SR cum AD } §135.
et ≈lam QZ cum DG }

Dico Pllgm AP esse
quaesitam h.e. AP = C.

Demonstratio.

Parallelogramma DEG, OQ, NT,
et ZR sunt ∾lia p. C. §388.

Porro:

EG = Rectilin. C + D p. C.
Rectilin. D = NT. p. C.
EG = Rectilin. C + NT. §10 Ar.
OQ = NT. p. C.
GZ + EP = C §43 Ar.

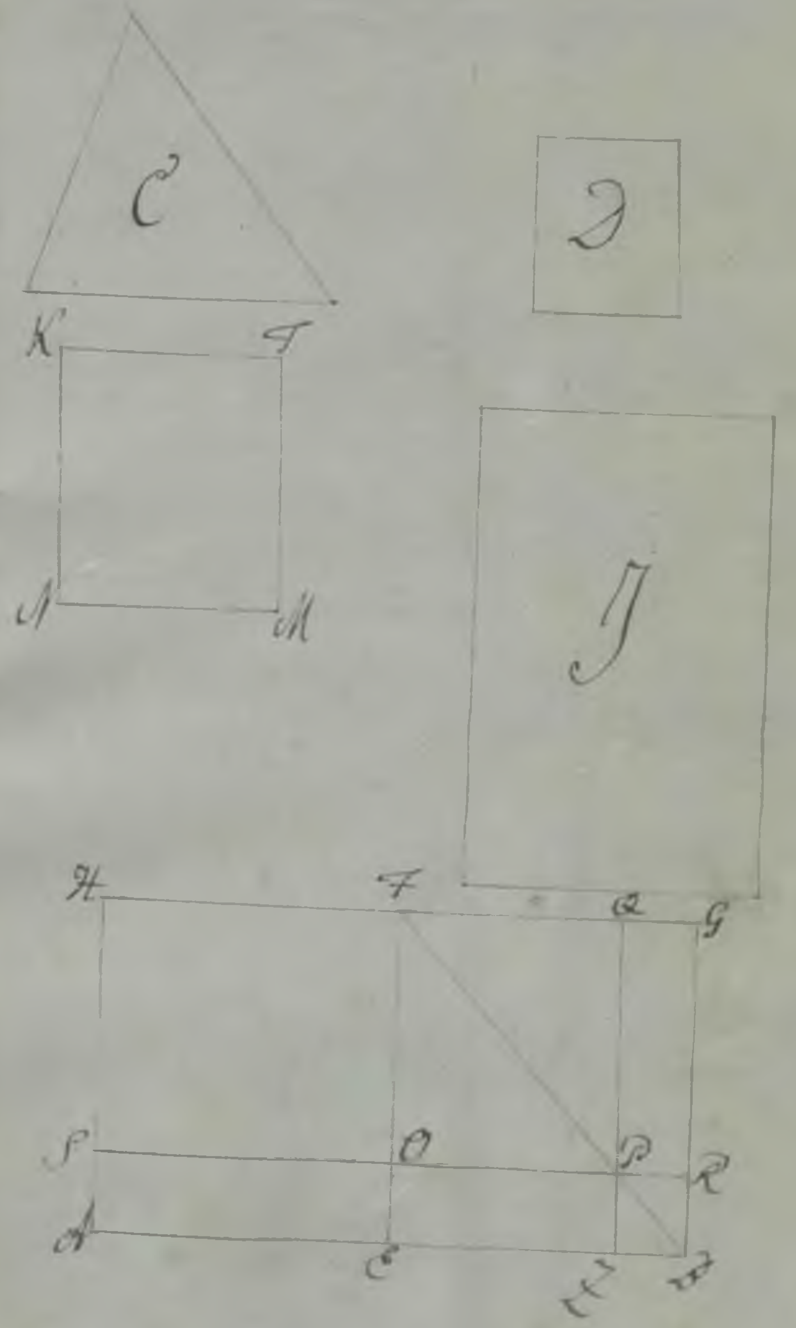
Verum

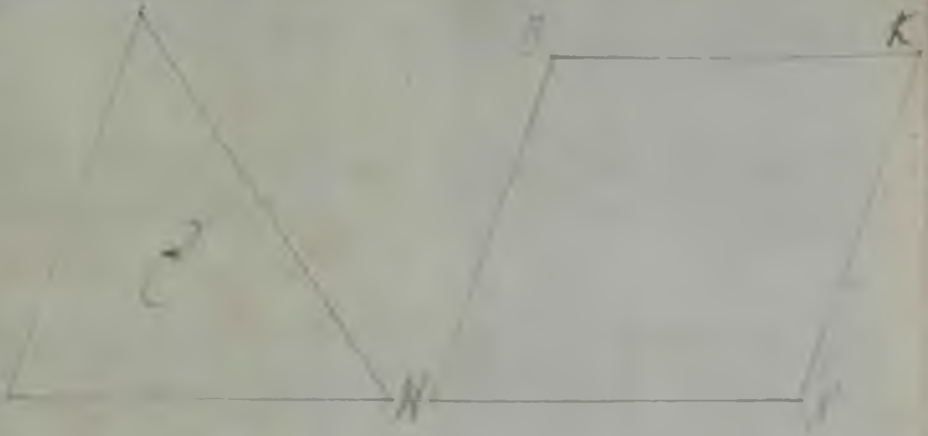
EP = GP. §184.
ZR = ZR. §40. } Ar.
ER = GZ §48
sed AE = ED. p. H.
et AD ≈ AR. p. C.
ER = AO. §176.
GZ = AO. §41 Ar.

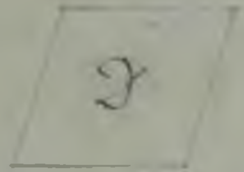
Ergo.

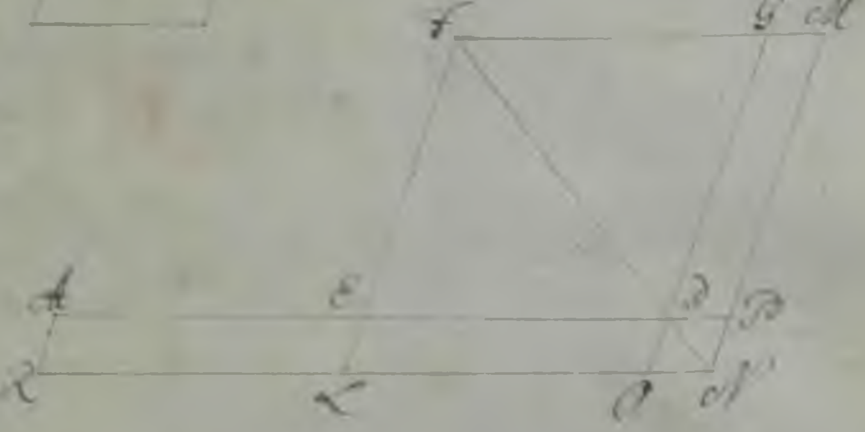
AO + EP = C. §10. } Ar
h. e.
AP = C. §47

Q. E. D.









§393. Problema LVII

Ad datam Rectam AB dato Recti-
lineo C aequale Plgm. AN applican
excedens Figura parallelogramm
OP quae similis sit Plgmo alteri
dato D. Resolutio.

1) Diseca propositam AB §112.
2) Super bisectam EB fac Parallelogra-
mum EG simile dato D. §379
3) Fac Plgm. HK = C+EG §186 simile
dato D vel EG similiterque positum
§389.
4) Fac Rectam FEL = IH et Rectam
FGM = IK.
5) Per L et M duc ≈ MN et LN
FM et FL. §135.
6) Similiter duc AR ≈ FL. §o.
7) Produc rectam AB in P
GB in O. §85
Dico Plgm AN esse quaesitum
h.e. AN = C. Q.F.

Demonſtratio.

Pllgma D, HK, LM, EG ſunt ∼lia p. C.

Ergo Pllgm OP ∼ LM ∼ D. §381.

Cumqꝫ Pllgm LM = HK p. C.

et HK = EG + C. p. C.

Pllgm LM = EG + C. §41 Ar.

EG = EG. §40 } Ar.

LM − EG = C. §42 } Ar.

LM − EG = LP + PG.

LP + DM = C. §41 Ar.

ſed LD = DM. §184.

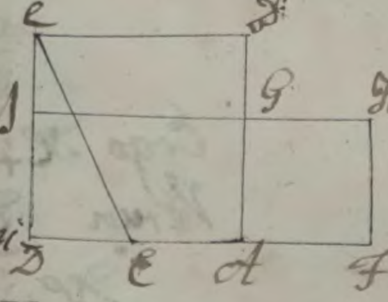
LD = AL. §176.

DM = AL. §41. h. e.

LP + AL = C. §10 } Ar. h. e.

AN = C. §47 }

Q. E. D.



§394. Problema LVIII

Propoſitam rectam lineam terminatam AB extrema et media ratione ſecare

Reſolutio

ſeca AB in G ut AB × GB = AG^2 §230. Q. F.

* *

Demonſtratio

AB × BG = AG^2 p. C.

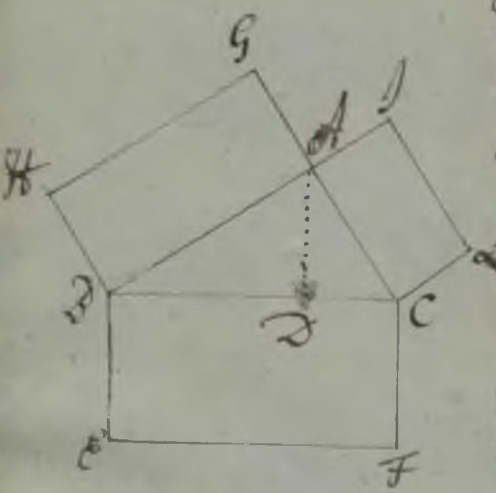
Ergo AB : AG = AG : BG. §373.

* *

Ergo AB media et extrema Ratione ſecta eſt §344. Q. E. D.

§395. Theorema 116.

In rectangulis Triangulis BAC Figu[ra] quovis BF a Latere BC rectum an[-] gulum subtendente, descripta, aequa[-] lis est Figuris BG, et AL quae prior[i] illi BF similes similiterque positae a Lateribus BA, AC rectum ∠um continentibus describuntur.



P.H. dmdm.
BF = BG + AL.

Demonstratio.

Demitte ab ∠lo R ⊥lem AD ad Hypotenusam BC. §119.

Ergo.

DC : CA = CA : CB. §359.
DC : CB = AL : BF §378.
Sed et DB : BA = BA : BC §359.
DB : BC = BG : BF. §378.

DC : DB = AL : BG. §173. Ar.
Ergo DC + DB : DB = AL + BG : BG. §168. Ar.
Verum DB : BC = BG : BF. p.d.

DC + DB : BC = AL + BG : BF. §172. Ar.

h.e.
BC : BC = AL + BG : BF. §117 Ar
Ergo AL + BG = BF. §126. Ar.
Q. E. D.

Aliter:

$AL \sim DF \}$ p. H.
$DG \sim DF \}$

Ergo:

$AC^2 : CD^2 = AL : DF$. §377.
$AD^2 : CD^2 = DG : DF$. §c.

$AC^2 : AD^2 = AL : DG$. §172 Ar.
$AC^2 + AD^2 : AD^2 = AL + DG : DG$ §168 Ar.
Sed $AD^2 : DC^2 = DG : DF$. p. d

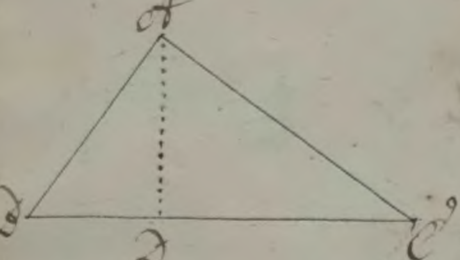
$AC^2 + AD^2 : DC^2 = AL + DG : DF$. §172 Ar.
Sed $AC^2 + AD^2 = DC^2$ §189.
Quibus ſubſtitutis erit
$DC^2 : DC^2 = AL + DG : DF$ §10 Ar.
$DC^2 : AL + DG = DC^2 : DF$. §150 Ar.
Ergo $AL + DG = DF$. §152. Ar.
Q. E. D.

§396. Scholion I.

Facile adparet uſum Theorematis hujus eſſe longe ampliſsimum, cujus præſidio Additio et Subtractio omnium figurarum ſimilium rectilinearum abſolvitur ea Methodo quam §193. 194. expoſuimus.

§ 397. Scholion 2.

Liquet etiam demonstratum Theorema Ambitu suo complecti ipsumque Theorema Pythagoricum, quod ex jactis hucusque principiis brevissima Demonstratione evincitur, ita Dndm autem § collato 189.



$$BC^2 = BA^2 + AC^2.$$

Demisso ex ∠lo R. ⊥lo ad Hypotenusam BC § 119. erit:

$BD : AB = AB : BC.$ § 359.

$BD \times BC = AB^2.$ § 373.

$BC : CA = CA : CD.$ § 359.

$BC \times CD = CA^2.$ § 373.

$BD \times BC + BC \times CD = AB^2 + AC^2.$ § 42 Ar.

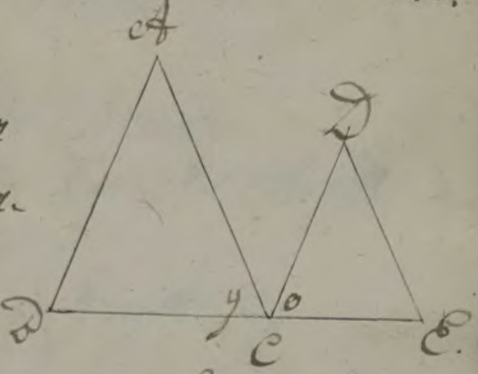
$BD \times BC + BC \times CD = BC^2.$ § 203.

$BC^2 = AB^2 + AC^2.$ § 41. Ar.

Q. E. D.

§ 398. Theorema 117.

Si duo Triangula ADC, DCE, quorum duo Latera duobus Lateribus parallela habeant, secundum unum Angulum ACD composita fuerint, ita ut homologa eorum Latera ſint parallela, tum reliqua illorum Triangulorum Latera BC et CE in rectam Lineam collocata reperientur.



h. e.

Si AB : AC = DC : DE et
AB ≈ DC et DE ≈ AC.

---

erit BC in directum ipſi CE.

Demonſtratio.

$$\text{Quia } \left.\begin{aligned} &AB \approx CD \\ \text{et } &AC \approx ED \end{aligned}\right\} \text{p. H.}$$

$$\left.\begin{aligned} \angle A &= \angle ACD \\ \angle D &= \angle ACD \end{aligned}\right\} \S 132.$$

$$\angle A = \angle D.\ \S 41\ \text{Ar.}$$

Porro, cum.

$$AB : AC = DC : DE.\ \text{p. H.}$$

$$\angle B = \angle o.\ \S 356.$$

Ergo

$$\angle A + B = \angle D + o.\ \S 42\ \text{Ar.}$$

$$\text{sed } \angle D = \angle ACD\ \text{p. d.}$$

$$\angle A + B = \angle ACD + o.\ \S 10.\ \text{Ar.}$$

$$= \angle ACE.\ \S 47.\ \text{Ar.}$$

sed

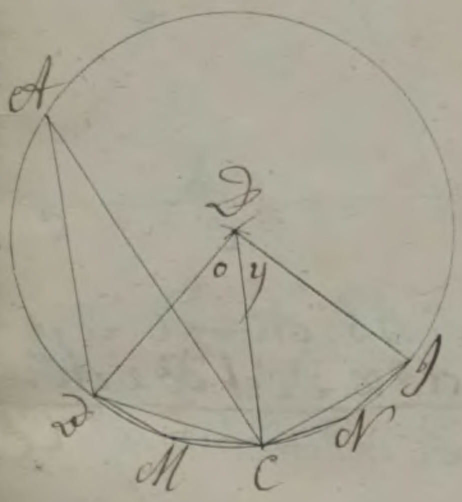
$$A + B + y = \angle y + ACE.\ \S 42\ \text{A.}$$

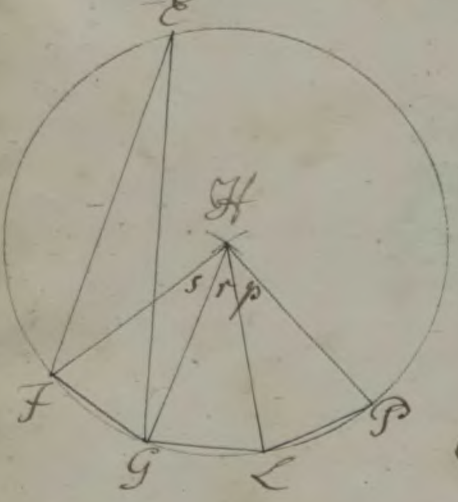
$$\angle A + B + y = 2R.\ \S.\ 128.$$

$$\angle y + \angle ACE = 2R.\ \S 41.\ \text{Ar.}$$

Ergo BC in directum ipſi CE. §93.

Q. E. D.





§ 399. Theorema 118.

In eodem vel aequalibus Circulis DCA et HFG anguli DDC et HF G eandem habent rationem cum Pphiis DC, FG, quibus insistunt sive ad Centra ut DDC et HFG, sive ad Pphias A et E constituti insistant. Insuper vero et sectores DDC et HFG, quippe qui ad Centra consistunt.

Demonstratio.

Duc Rectas DC, FG §81.

Accommoda CI = DC } §307.

itemq GL = LP = GF }

Quia DC = CI. p. C.

Ergo Arc DC = Arc CI. §288.

Ergo $\angle o = \angle y$. §282.

Cumq Arc. DC + CI = Arc. DI. §44 Ar.

Ergo $\angle$DDI = $\angle o + y$. §c.

Ergo

Arc. DC : Arc DCI = $\angle o$: $\angle$DDI. §132. Ar.

Similiter.

Quia FG = GL = LP. p. C.

Ergo Arc FG = Arc GL = Arc LP. §288.

Ergo $\angle s = \angle r = \angle p$. §282.

cumq; Arcus FG + GL + LP = Arc FP. §42 Ar.

Ergo et $\angle s + r + p = \angle o$ FHP. &c.

Ergo Arc FG : Arc FP = $\angle s$: $\angle$ FHP. §132 Ar.

Eſt autem Circulus DCAd = Circ. FLEH. p. H.

Ergo Arcus DCI vel = Arc. FGLP.
vel Arcus DCI > Arc. FGLP. } §39 Ar
vel Arcus DCI < Arc. FGLP.

Ergo et

$\angle$lus DDI aut $\left\{ \begin{matrix} = \\ > \\ < \end{matrix} \right\}$ $\angle$lo FHP. §282.

Quare in omni Caſu:

Arc. DCI : Arc FGLP = $\angle$DDI : $\angle$FHP. §132 Ar.
Sed Arc DC : Arc DCI = $\angle o$: $\angle$DDI. p.d.
Arc FGLP : Arc DC = $\angle$FHP : $\angle o$. §175 Ar.
Arc FG : Arc FGLP = $\angle s$: $\angle$FHP. p.d.
Arc. DC : Arc FG = $\angle o$: $\angle s$. §175. Ar.

Q. E. I.

Cumq; $\angle o : \angle s = \frac{\angle o}{2} : \frac{\angle s}{2}$ §160 Ar.

et $\frac{\angle o}{2} = A$ } §273. Geom. et 45. Ar.
et $\angle \frac{s}{2} = E$

Ergo $\angle o : \angle s = \angle A : \angle E$. §10 Ar

Ergo Arc DC : Arc FG = $\angle A : \angle E$. §244 Ar.

Q. E. II.

Membr. 2. Duc DM. et MI item
CN et MB utcunq ad Pphiam ex Pun-
ctis DGJ. Quare cum
ſubt. DC = ſub. CJ. p. C.
Ergo ∠ DMC = ∠ CMJ. §282.
Ergo Sgmtm DCMD ~ Sgto CJMC. §245.
Ergo Sgmtm DCMD = Sgto CJMC. §278.
Verum △ DDC = △ DJC. §106.

Sector DDCMD = Sectri CDJMC. §42 Ar.
Simili Diſcurſu oſtenditur.
Sectorem FHG = Sectori GHL = Sectori LHP.

Quare ſi
Arc. DC $\left\{\begin{matrix}=\\>\\<\end{matrix}\right\}$ Arc FG. §39 Ar.

ergo et
ſector DDC $\left\{\begin{matrix}=\\>\\<\end{matrix}\right\}$ Sect FHG. §0.

adeoq
DC : FG = Sect DDC : Sect FHG.
§132. Ar.

Q. E. D.

§400. Corollarium 1.

Quia

DC : FG = Sect. DAC : sect FAG. p.d. ad §399

DC : FG = $\angle o$: $\angle s$. p.d. §c.

Sect. DAC : Sect. FAG = $\angle o$: $\angle s$. §144 Ar.

§401. Corollarium 2.

Angulus o ad Centrum est ad quatuor Rectos uti Arcus DC ad Peripheriam. Cum enim

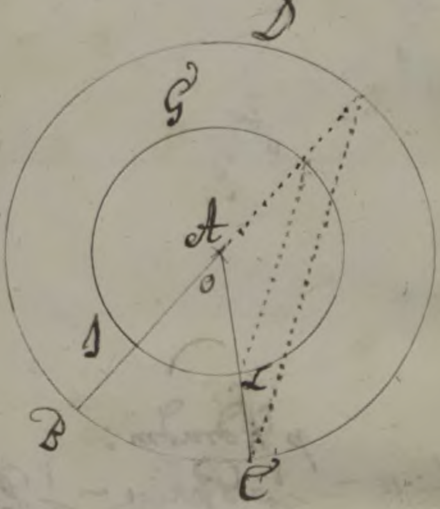
$\angle$i R mensura = $\frac{1}{4}$ Pphiae §91.

Ergo

$\angle o : \angle R$ = Arc. DC : $\frac{1}{4}$ Pphiae §399.

Ergo

$\angle o$: 4R = Arc. DC : Pphiam §162 Ar.



§402 Corollarium 3.

Inaequalium Circulorum Arcus δL, DC, qui aequales subtendunt $\angle$los sive ad Centra sive ad Pphias constitutos, sunt similes.

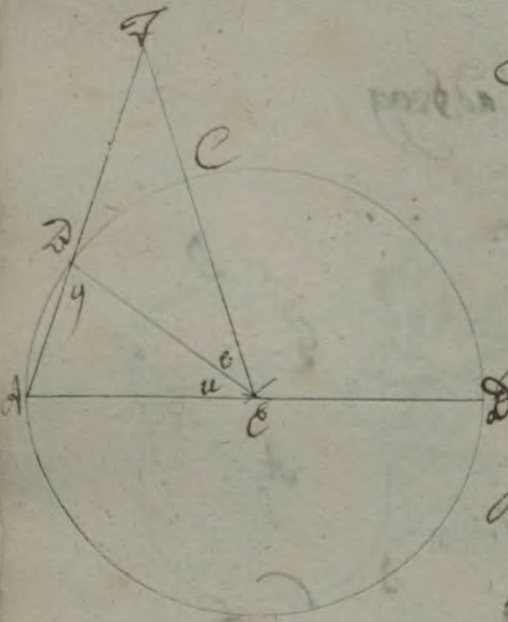
Nam:

$\angle o$: 4R. = Arc δL : Pphiam δLG } §401.

$\angle o$: 4R = Arc. DC : Pphiam DCD }

Arc. δL ÷ ~~Arc. DC. §146 atq.~~ Pph. δLG = Arc. DC : Pph. DCD §142 Ar.

Arcus δL ∾ Arc. DC. §146 atq. 142 Ar.



§403. Corallarium 4.

Atq; inde simul apparet duas semi diametros AD, AC a Peripheriis concentricis auferre Arcus similes IL et DC. §166. Ar.

§404. Theorema 119

In Circulo ADCD majus segmentum Radii proportionaliter secti subtendit Decimam totius, s. q. i. quintam Semicirculi partem.

Demonstratio.

Produc AD in F §81 et fac DF = AE = DE.

p. Admdm:

$\angle u = \frac{\frac{1}{5}\text{Pphiae}}{2} = \frac{1}{10}\text{Pph}$

Quia ∠A = ∠A. §40. Ar

et FA : AE = AE : AD. §231.

Ergo ∠F = ∠u. §356.

cumq; DF = DE. p. C.

∠F = ∠o §100.

∠u = ∠o §41. Ar.

Sed ∠AEC = ∠u + o. §47. Ar.

Ergo ∠AEC = 2×∠u.

Porro:

$\angle y = \angle A + \angle o$ §142.
$= \angle u + \angle u$. p.d.
$= 2 \times \angle u$.
Cumq; $DE = AE$ §26.
Ergo $\angle y = \angle A$. §100.
Sed $\underline{\angle y = 2 \times \angle u}$. p.d.
$\angle A = 2 \times \angle u$ §41 Ar.
Tandem quia.
$\angle CED = \angle A + T$ §142.
$= 2 \times \angle u + \angle u$ p.d.
$= 3 \times \angle u$. et quia.
$\underline{\angle AEC = 2 \times \angle u}$. p.d.
$\angle CED + \angle AEC = 5 \times \angle u$ §42. Ar.
$\underline{\angle CED + \angle AEC = \frac{1}{2}}$ Pphiæ §93. 91. 84.
$5 \times \angle u = \frac{1}{2}$ Pphiæ §41 Ar.
$\angle u = \frac{\frac{1}{2} \text{ Pphiæ}}{5}$ §46 Ar.

h.e.
$\angle u = \frac{1}{10}$ Pphiæ §220. Ar.

Q.E.D.

§405. Corollarium

Atq unde expeditiſsima Pentago
Circulo inſcribendi Praxis elucid
1) Radii proportionaliter ſecti §230.
2) Accommoda ſegmentum majus
bis in Circulo Radio dato deſcript
in AD et DC. §307.
3) Duc AC. §81.
Dico AC eſſe Latus Pentagoni ordin

Nam.

Subtenſa AD = Sub. DC. p. C.
Ergo Arcus AD = Arc. DC. §28.
cumq AD = $\frac{1}{10}$ Pphiae. §404.
Ergo AD + DC = $\frac{2}{10}$ Pph. §42 Ar.
h. e. Arc. ADC = $\frac{1}{5}$ Pphiae. §47. Ar
atq §204. Ar.
Ergo et AC ſubtendit $\frac{1}{5}$ totius Pphi
Ductis itaq ſubtenſis CQ, QM, M
N A aequalibus ipſi AC §81. 26. ſimi
ut ante diſcurſu demonſtrabitur

demonstrabitur, singulas quintam totius Pphæ auferre partem §285.

Ergo Pentagonum ACQMN est æquilaterum §56.

Cumque singuli Anguli A, C, Q, M et N insistant tribus quintis totius Pphiæ partibus, Ergo

$\angle A = \angle C = \angle Q = \angle M = \angle N$ §282. 9. et 41 Ar.

Ergo Pentagonum descriptum est æquiangulum §77.

Ergo Pentagonum est ordinatum §299.

Q. E. D.

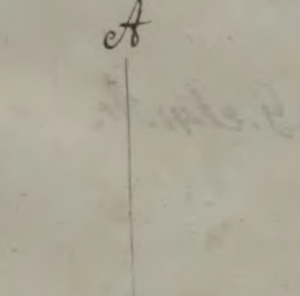
Caput $\mathrm{VI}^{\mathrm{tum}}$

De Situ atq; sectione Planorum itemq; de Angulorum Solidorum atq; Parallelepipedorum affectionibus.

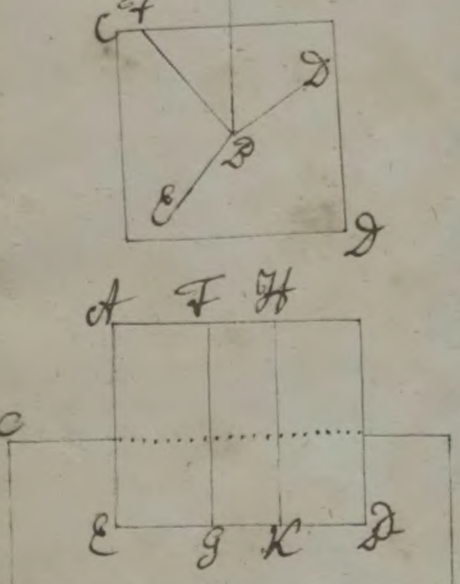
§406. Definitio LXXXII.
Solidum est quod Longitudinem Latitudinem et Crassitudinem habet.

§407. Definitio LXXXIII.
Solidi extremum est superficies.

§408. Definitio LXXXIV.
Linea Recta AB est ad Planum CD Recta cum ad rectas omnes Lineas BD, BE, BF a quibus illa tangitur, quaeq; in proposito sunt Plano rectos efficit ∟los ABD, ABE, ABF.

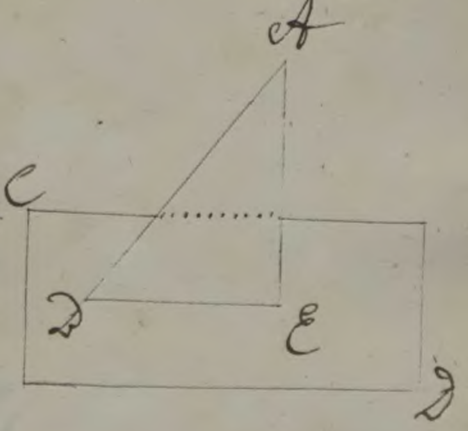


§409. Definitio LXXXV.
Planum AB ad Planum CD Rectum est, cum rectae lineae FG, HK, quae comuni Planorum Sectioni CD ad ∟los Rectos in uno Plano AB ducuntur alteri Plano CD ad Rectos sunt ∟los.



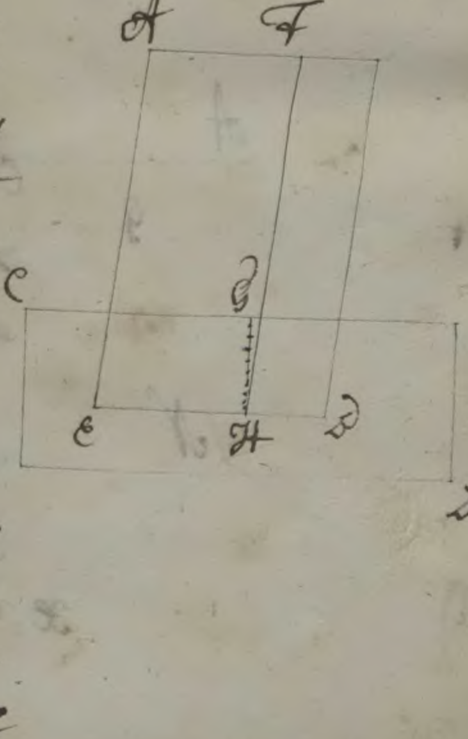
§410. Definitio LXXXVI.

Rectæ Lineæ AD ad Planum CD inclinatio est, cum a sublimi Termino A rectæ illius Lineæ AD ad Planum CD deducta fuerit ⊥lis AE, atq; a puncto E, quod ⊥lis AE in ipso Plano CD fecerit, ad propositæ illius Lineæ extremum D, quod in eodem est Plano altera recta Linea fuerit adjuncta: est, inquam Angulus acutus ADE insistente Linea AD et adjuncta ED contentus.



§411 Definitio LXXXVII.

Plani AB ad Planum CD Inclinatio est ∠lus acutus GHF rectis Lineis FH, GH contentus, quæ in utroq; Planorum AB, CD ad idem communis sectionis BE punctum H ductæ, Rectos cum sectione BE efficiunt ∠los FHB, GHB.



§412. Definitio LXXXVIII.

Planum ad Planum similiter inclinatum esse dicitur atque alterum ad alterum eum dicti Inclinationum ∠li fuerint æquales.

§ 413. Definitio LXXXIX.

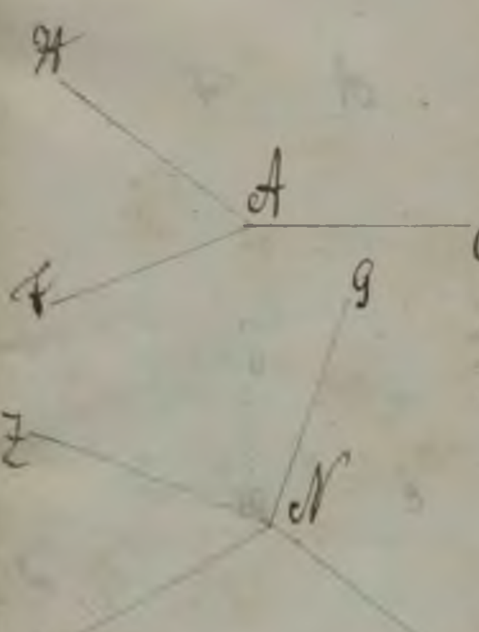
Parallela Plana ſunt, quæ inter ſe non conveniunt.

§ 414 Definitio XC

Similes ſolidæ Figuræ ſunt quæ ſimilibus Planis continentur multitudine æqualibus.

§ 415 Definitio XCI

Æquales et ſimiles ſolidæ Figuræ ſunt quæ æqualibus et ſimilibus Planis multitudine æqualibus continentur.

§ 416. Definitio XCII

Solidus ∠lus et ACFH, NGZUX eſt plurium quam duarum Linearum ſe mutuo contingentium nec in eadem ſuperficie exiſtentium ad omnes Lineas inclinatio. Vel: Solidus ∠lus est qui pluribus, quam duobus ∠lis planis in eodem Plano non conſiſtentibus, ſed ad unum punctum conſtitutis continetur.

§ 417. Hypotheſis.

Angulum ſolidum ita ſignificabimus, ut litera prima ſemper

Verticem, reliquo autem crura deſignent, qualis eſt ∠lus ACFH, aut ∠lus NBZYX.

§418. Definitio XCIII

Pyramis eſt Figura ſolida Planis comprehenſa quae ab uno plano ad unum punctum conſtituuntur.

§419 Definitio XCIV.

Priſma eſt Figura ſolida, quae Planis continetur, quorum adverſa duo ſunt et aequalia et ſimilia et parallela, alia vero parallelogramma.

§420. Definitio XCV.

Sphæra eſt, quando ſemicirculi manente Diametro circumductus ſemicirculus in ſe ipſum rurſus revolvitur, unde moveri coeperat circum aſſumta figura.

§421. Definitio XCVI

Axis autem Sphæræ eſt quieſcens Linea illa recta circum quam ſemicirculus convertitur. Centrum Sphæræ eſt idem quod et Semicirculi Diameter tandem Sphæræ eſt

recta quædam Linea per Centrum ducta atq; utrinq; superficie Sphæræ terminata

§422. Definitio XCVII

Conus ABCDE est, quando rectanguli Trianguli BDC latere uno manente eorum, quæ circa rectum Angulum circumductum Triangulum in seipsum rursus revolvitur, unde moveri coeperat circum assumta figura. Atq; si quiescens Linea CD æqualis sit reliquæ BD quæ circum rectum Lum continetur, Orthogonius erit Conus, si vero minor Amblygonius, si vero major Oxygonius.

§423. Definitio XCVIII.

Axis Coni est quiescens illa Linea CD circa, quam Triangulum CDB movetur. Basis Coni est Circulus qui a circumducta recta Linea BD describitur. Latus autem coni est Hypothenusa BC.

§424. Definitio XCIX.

Cylindrus est, quando rectanguli

Parallelogrammi ABCD manente uno Latere DC eorum, quæ circa rectum Angulum, circumductum Pllgm in ſe ipſum rurſus revolvitur unde moveri cœperat circumaſſumta figura. Axis Cylindri DC eſt quieſcens illa Linea recta, circum quam Pllgm convertitur. Baſes autem Cylindri ſunt Circuli a duobus adverſis Lateribus quæ circumaguntur, deſcripti.

§425. Definitio C.

Similes Coni et Cylindri ſunt quorum et Axes et Baſium Diametri proportionales ſunt.

§426. Defininitio CI.

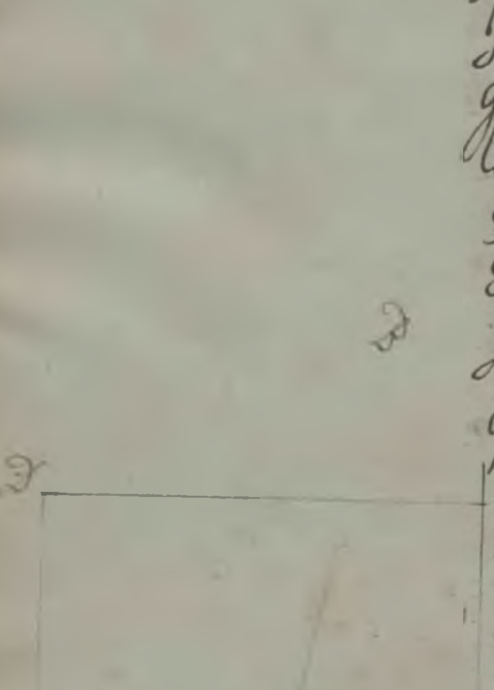
Cubus eſt Figura ſolida ſub ſex quadratis æqualibus contenta.

§427. Definitio CII.

Tetraëdrum eſt Figura ſolida ſub quatuor Triangulis æqualibus et æquilateris contenta.

§428. Definitio CIII.

Octaëdrum eſt Figura ſolida ſub



octo Triangulis æqualibus et æquilateris contenta.

§429. Definitio CIV.

Dodecaëdrum est Figura solida sub duodecim Pentagonis æqualibus et æquilateris et æquiangulis contenta.

§430. Definitio CV.

Icosaëdrum est Figura solida sub viginti Triangulis æqualibus et æquilateris contenta.

§431. Definitio CVI.

Parallelepipedum est Figura solida sex Figuris quadrilateris quarum quæ ex adverso parallelæ sunt contenta.

§432. Theorema 120.

Rectæ Lineæ pars quædam AC non est in subjecto Plano DE altera CB in sublimi. Demonstratio.

Ponamus, si fieri possit partem Lineæ rectæ AC esse in subjecto Plano DE partem CB in sublimi. Produco ergo AC in F §82. Erit itaq AC pars Rectæ

AF, ſed eadem AC eſt etiam pars Rectæ AB p. H. Hinc punctum A. deſcribens Rectam in C mutat directionem ſuam, cum et verſus D et verſus F tendat.

I. Q. E. A. p. §12. 13. 80.

§433. Theorema 121.

Si duæ Lineæ rectæ AB, CD ſeſe mutuo ſecent, in uno ſunt Plano, atq; omne Triangulum DEB in uno eſt Plano.

Demonſtratio.

Concipiamus fieri poſſe △ DBE partem EF eſſe in uno Plano, alteram vero DFGB in altero Plano. Ergo Rectæ CD pars erit in ſubjecto Plano altera vero in ſublimi;

I. Q. E. A. p. §432.

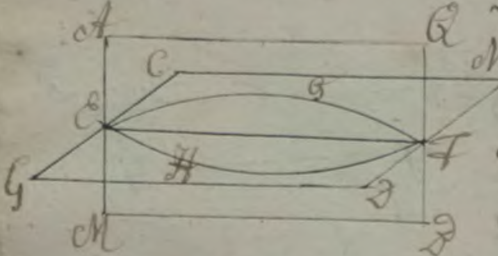
Triangulum ergo EBD eſt in uno eodemq; Plano, proinde et Rectæ ED et EB ſunt in uno eodemq; Plano, adeoq; et totæ AB et CD in eodem Plano exiſtunt. §432.

Q. E. D.

§434. Theorema 122.

Si duo Plana AB, CD ſe mutuo ſecent communis eorum ſectio EF eſt Linea recta

Demonſtratio



Patet ex ipſa Plani Definitione contenta §15. Facile enim adparet Rectas quae Plana terminant CG, AM itemque ND et DQ ſemet ſecare in unico puncto E et F. §80. Quare cum inter duo puncta non niſi unica recta EF cadat §90 erit utique EF recta illa Linea conſtituens utriusque Plani ſectionem. Quodſi vero Adverſarius inſtet non eſſe duci itaque alias Rectas ex punctis E et F v.g. EGF et EHF. Claudent ergo ſpatium, quod cum ſit abſurdum apparet ſectionem fieri per unicam illam rectam EF. Q. E. D.

§435. Theorema 123.

Si Linea recta EF, rectis duabus Lineis AB, CD ſeſe mutuo ſecantibus in communi ſectione Ead rectos ∠los insistat, illa ducto etiam per ipsas Plano ABCD, ad rectos erit Angulos.

Demonstratio.

Fac $AE = EB$, $EC = DE$ §26.

Junge Rectas AC, CB, BD, DA §81.

Per E duc rectam quamlibet GH &c.

Ergo quia $AE = EB$, $DE = EC$ p. C.

$\angle AED = \angle CEB$ §94.

$AD = CB$ §99.

$\angle EAG = \angle EBH$ &c.

Ergo $AD \approx CB$ §133.

Ergo $\angle AGE = \angle EHB$ §132.

Ergo et $\angle AEG = \angle BEH$ §156.

cumque $AE = EB$ p. C.

$\therefore GE = EH$, $AG = HB$ §114.

Porro: $AE = EB$ p. C.

$EC = DE$ p. C.

$\angle AEC = \angle DEB$ §94.

$AC = DB$ §99.

Ex puncto F duc rectas lineas AF, FC, FB, FD §81. Quoniam $AE = EB$ p. C.

$\angle AEF = \angle BEF$ §92 et H.

et $EF = EF$ §40.

$AF = FB$ §99. Sic et evincatur $DF = FC$.

cumque $AD = CB$ p. d.

$\angle DAF = \angle CBF$ §106.

Sed $AG = HB$, $AF = FB$ p. d.

$GF = HF$ §99.

Sed $GE = EH$ p. d.

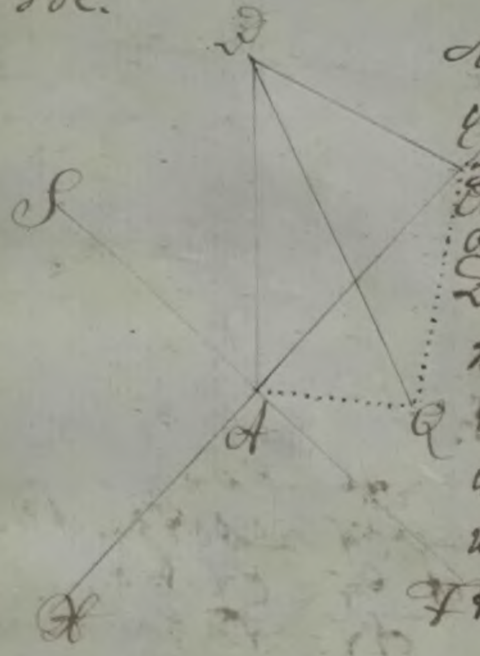
et $EF = EF$ §40. Ar

$\angle FEG = \angle FEH$ §106.

$= R$. §38.

Simili discursu ostendatur Rectam FE cum omnibus in Plano ABCD per E ductis Rectis Rectos efficere ∠los, adeoque ad idem Planum Rectam esse §408. Q. E. D.

332.



Aliter:

Si negas DA Rectam esse ad Planum in quo sunt ductae Lineae rectae AF et XC, dabitur alia Linea quae Recta sit ad Planum AF n.p. DQ.

Duc ergo AQ §81 et excita ex Q normalem Qθ in Plano AF §110 vel 118. quae producta necessario secabit aliquam Rectarum CA, AX et AF aut utramque ubicunque punctum Q constituas. §141.

Ponamus itaque illam in C rectam secare Lineam XA in θ junge Dθ. §81.

Quia $\angle DAC = R.$ p. H. θ n.p. ad rectam lineam XC normalis est DA p. H. non autem ad Planum AFC, ad quod e contendente Adversario Rectam cum ipso statuimus DQ

Ergo

$D\theta^2 = DA^2 + A\theta^2$ §189.

Sed et DQ Recta h.e. normalis est ad Planum CAF p. Hyp.

$DA^2 = DQ^2 + AQ^2$. §189 Ergo

$D\theta^2 = DQ^2 + AQ^2 + A\theta^2$. §100 Ar.

Sed et $\angle AQ\theta = R.$ p. C.

$A\theta^2 = AQ^2 + Q\theta^2$. §189.

Ergo $D\theta^2 = DQ^2 + AQ^2 + AQ^2 + Q\theta^2$. §100 Ar.

h.e.

$DO^2 = DQ^2 + QO^2 + 2 \times AQ^2$.

Ergo

$DO^2 > DQ^2 + QO^2$. §47. Ar.

Ergo.

∠lus DQO non est Rectus §198.

Proinde.

DQ non est Recta ad Planum AH. §408.

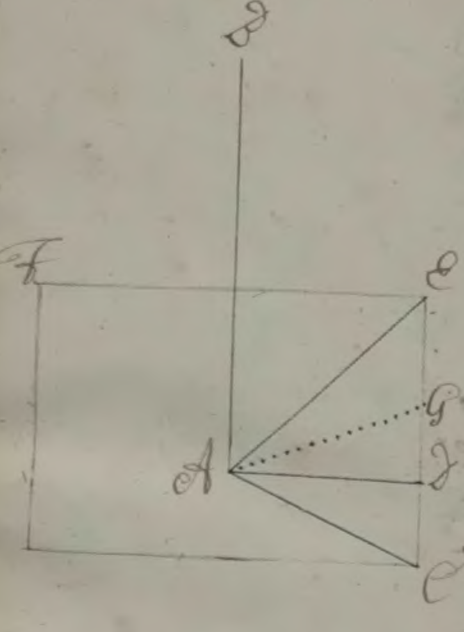
Q. E. D.

§436. Scholion.

Ex eo, quod ponebatur DQ Rectam esse debere ad Planum CAF, demonstratum est, DQ non esse Rectam ad dictum Planum CAF, ac proinde, quod negaretur assertio Theorematis eadem assertio directe probata est. Est autem Demonstratio allata quoad substantiam a Joh. Cietmanno, ita fere et Andr. Tacquet in Geom. Eucl. p. m. 227.

§437. Theorema 124.

Si recta Linea AB tribus Lineis rectis AC, AD, AE sese mutuo tangentibus in communi sectione ad rectos ∠los insistat, illae tres rectae Lineae in uno sunt Plano.



Demonſtratio.

AC tangit ſ. q. i. e. ſecat AD p. H. §

Ergo

AC et AD ſunt in eodem Plano §4

Similiter

AD tangit ſ. q. i. e. ſecat AE. p. H. et

Ergo

AD et AE ſunt in eodem Plano §

Ponamus ergo Plana iſta eſſe di
verſa ut ſcilicet aliud ſit AEG aliud
ADG atq. eorum comunem ſectionem
fieri in recta Linea AG. §434.

Quia DA ⊥lis ad AC et AD p. H.

Ergo DA ⊥lis ad AG. §435.

Ergo ∠BAD = ∠BAG. §92.

I. Q. E. A. §477. Ar.

§438. Theorema 125.

Si duæ rectæ Lineæ AB, DC eidem Plano EF ad rectos ſint ∠los, parallelæ erunt rectæ illæ Lineæ AB, DC.

Demonſtratio.

Duc AD. §81.

In Plano EF ad pctm D excita ⊥lem

GD = AB §120. 26.

Junge BD, BG, AG. §81. Quare
cum $\angle BAD = R$. p. H.
$\angle ADG = R$. p. C.

$\angle BAD = ADG$. §92.
$AB = DG$. p. C.
$AD = AD$ §40. Ar.

$BD = AG$. §99.
$BG = BG$. §40 Ar.
$DG = AB$ p. C.

$\angle BAG = \angle BDG$ §106.
Sed $\angle BAG = R$. p. H. §408.

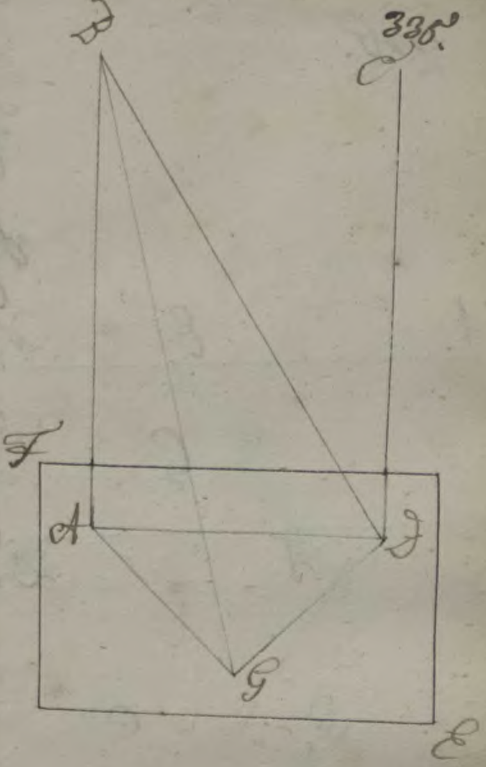
$\angle BDG = R$. §92.
Sed et $\angle GDC = R$. p. H. §408.

Ergo GD ⊥lis ad AD, DC, DB.
Ergo AD, DC, DB sunt in eodem Plano
in quo existit AB §437.
h. e in Plano BADC.

Proinde, cum
AB, ADC in eodem sint Plano p. d.
et ∠lus $BAD = \angle ADC = R$. p. H.
Ergo ∠lus $BAD + ADC = 2R$. §42. Ar

Ergo
AB ≈ DC. §133.

Q. E. D.



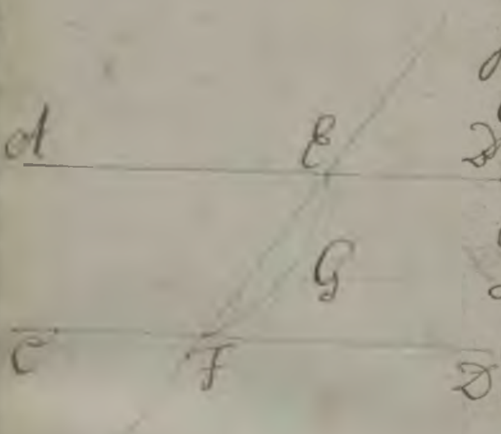
§439. Theorema 126.

Si duæ sint rectæ Lineæ parallelæ AB et CD in quarum utraq; sumta sint qualibet puncta E, F, illa Linea EF, quæ ad hæc puncta adjungitur in eodem est cum =lis AB, CD Plano ABCD.

Demonstratio.

Secetur Planum in quo sunt Lineæ AB, CD alio Plano per pcta E, F. Quodsi EF non sit in Plano ABCD non erit EF sectio communis Plani utriusq; Quare ponamus Rectam illam esse posse EGF. Ergo duæ Rectæ spatium concludunt.

J. 2. l. A.



§440. Theorema 127.

Si duæ sint =læ rectæ Lineæ AB et CD, quarum altera AB ad Rectos cuipiam Plano EF sit ∠los et reliqua CD eidem Plano EF ad ∠los Rectos erit.

Demonstratio.

Præparatis omnibus uti §438. ex ejusdem §. Demonstratione liquet ∠DBA = ∠GDB. §92.

GD Recta est Plano per AD, DD ducto h. e. Plano ADCD §408. 439.

Ergo

GD ⊥lis ad CD in Plano ADCD. §435.

cumque DA ≈ CD p. H.

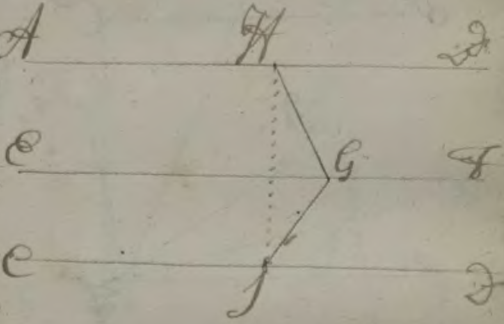
∠DAD + ∠CDA = 2R. §132.

∠DAD = R. p. H. Coor.

∠CDA = R. §43 Ar.

Proinde

CD est Recta ad Planum CF §435. et §408. Q. E. D.



§441. Theorema 128.

Quae AD et CD eidem rectae Lineae EF sunt parallelae, sed non in eodem cum illa plano, hae quoque inter se sunt parallelae.

Demonstratio.

In Plano ≈larum AD et EF. ADFE duc HG ⊥ ad EF. §119 atque in Plano ≈larum EF et CD demitte IG ⊥ ad EF. &c.

Ergo

GE est Recta ad Planum Rectarum HG et GI h. e. HGIH. §408. h. e.

∠EGH = R. cumque

EG ≈ AH. p. H.

AH est Recta ad Planum HGIH Similiter. §440.

Quia EG ≈ CI. p. H.

et ∠CIG = R. p. C.

---

h. e. CI est Recta ad Pl. HGIH &c.

Proinde cum

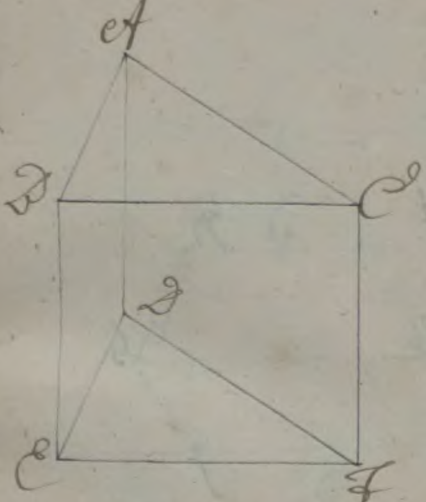
CI sit Recta ad Pl. HGIH p. d.

AH sit Recta ad Pl. HGIH. p. d.

AH ≈ CI. §438. Ergo et

AD ≈ CD. §82.

Q. E. D.



§442. Theorema 129.

Si duae Lineae rectae AB et AC sese mutuo tangentes ad duas Rectas ED et DF sese mutuo tangentes sint ≈lae non autem in eodem Plano illae ∠los aequales BAC et EDF comprehendunt.

Demonstratio.

Fac AB = ED } §86.
AC = DF }
et junge AD, BE, CF, BC, EF §81.
Quia AB = et ≈la ED p. H. et C.
Ergo.
BE = et ≈ AD. §139.
similiter quia
AC = et ≈ DF p. H. et C.
Ergo.
CF = et ≈ AD. §139.

CF ≈ BE. §441. sed
et CF = BE. §41. Ar.

BC = EF §139.
sed BA = ED } p. C.
AC = DF }

∠BAC = ∠EDF. §106.
Q. E. D.

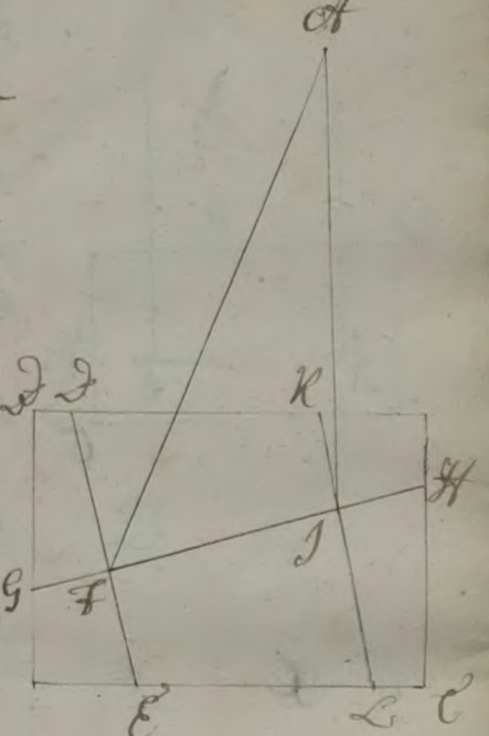
§443. Problema $\boxed{\text{LIX}}$

A dato in sublimi Puncto A ad subjectum Planum DC perpendicularem rectam Lineam AI ducere.

Resolutio.

1) In Plano subjecto DC duc rectam quamcunque Lineam DE. §81.
2) Ex puncto A duc normalem ad illam AF. §119.
3) Ad eandem DE in Plano DC per pctm F duc illam FH. §120.
4) Ad istam FH demitte ex A illam AI. §119.

Dico AI esse Rectam ad Planum DC.



Demonstratio.

Per I duc KL $\approx$ cum DE. §125.

$$\left.\begin{array}{l}\text{quia DE} \perp \text{ad AF} \\ \text{et DE} \perp \text{ad FH}\end{array}\right\}\text{p. C.}$$

DF $\perp$ ad Plan FAI. §435.

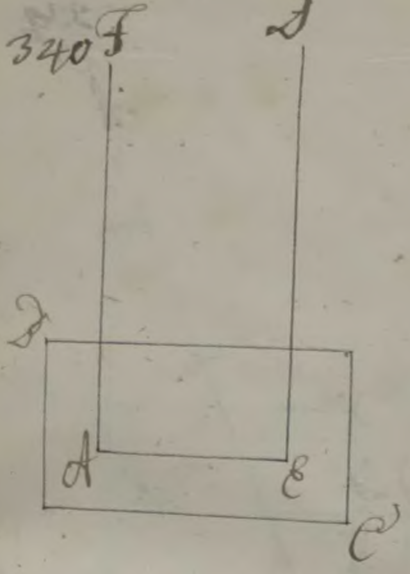
Sed et KI $\approx$ DF p. C. Ergo

$\angle$ KIA = R. §440.

cumque $\angle$ AIF = R. p. C.

AI $\perp$ ad Planum DC. §436.

Q. E. D.



§ 444. Problema LX

Dato Plano BC a puncto A quod in illo datum est ad Rectos Angulos Lineam rectam AF excitare.

Resolutio et Demonstratio.

1) A quovis puncto in sublimi D demitte ⊥lem DE ad Pl. BC. §444.

2) Junge AE. §81.

3) Cum DE duc ∥ AF. §135

Eritq AF ⊥ ad Pl. BC. §440.

Q. E. R. et D.

§ 445 Theorema 130.

Dato Plano AB a puncto D, quod in illo datum est duae rectae lineae CD, DE ad Rectos ∠los non excitabuntur ab eadem parte.

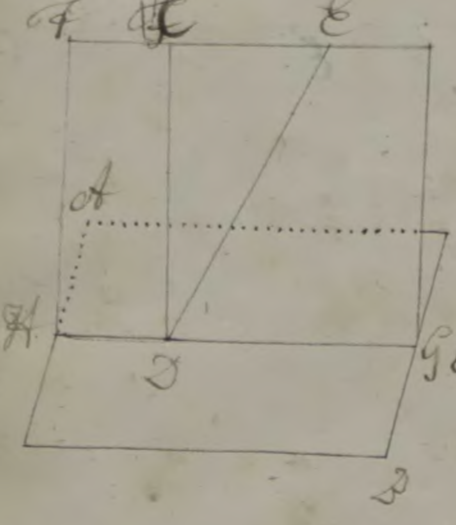
Demonstratio.

Ponamus fieri posse ut CD et DE. sint ad ∠los Rectos excitatae et quidem ad easdem partes

Ergo CD ∥ DE. §438.

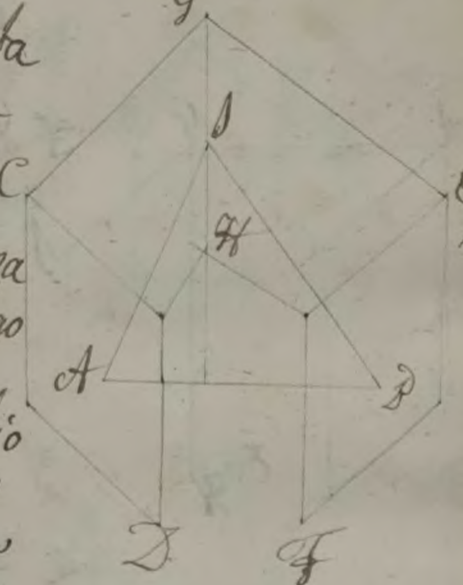
I. Q. E. A. cum in puncto D coëant contra parallelarum definitionem. §48.

Q. E. D.



§446 Theorema 131.

Ad quæ Plana CD, FE eadem recta Linea AB Recta eſt, illa ſunt parallela.



Demonſtratio.

Ponamus ſub data Conditione Plana CD et FE parallela non eſſe. Ergo coibunt. §413. Eſto itaq; Concurſus iſtius ſectio communis Recta HG: aſſumto in illa quovis puncto I, duc Rectas AI et IB §81. Quare cum

$$\left.\begin{array}{l}\angle IAB = R.\\ \underline{\angle IBA = R.}\end{array}\right\}\text{p. H.}$$

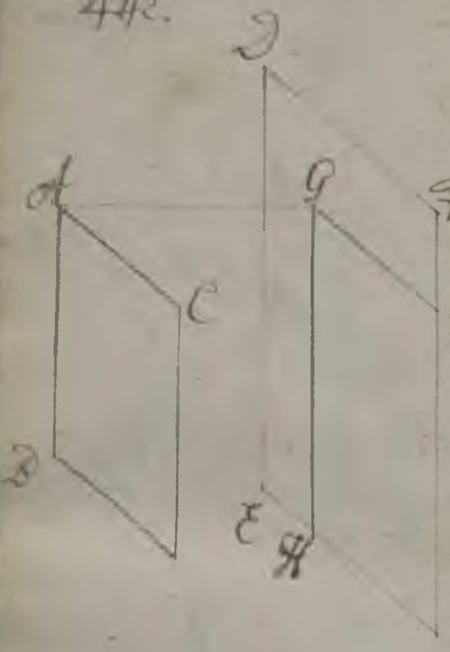
$$\angle IAB + IBA = 2R.\ \text{§42 Ar.}$$

I. Q. E. A. per §144.

§447. Theorema 132.

Si duæ rectæ Lineæ AB, AC ſeſe mutuo tangentes ad duas Rectas DE, DF ſeſe mutuo tangentes ſint parallelæ non in eodem Plano conſiſtentes, parallela ſunt, quæ per illas ducuntur Plana BAC, EDF.

442.



Demonstratio.

Ex pcto A duc AG $\perp$lem ad Planum EF §440.

Perq G duc $GI \approx DF$ } §135.
et $GH \approx DE$ }

Sed et $AC \approx DF$ } p. H.
et $AB \approx DE$ }

$AC \approx GI$ } §441.
$AB \approx GH$ }

Quare.

$\angle IGA + CAG = 2R$. §132.
Sed $\angle IGA = R$. p. C.
$\angle CAG = R$. §43. Ar.

Porro, quia.

$\angle HGA + BAG = 2R$. §132.
et $HGA = R$. p. C.
$\angle BAG = R$. §43. Ar.

Ergo.

GA $\perp$lis ad Plan BC. §425.
Sed GA $\perp$lis ad Plan. EF. p. C.
$BC \approx EF$. §446. Q. E. D.

§448. Theorema 133.

Si duo Plana parallela AB, CD Plano quopiam HE et GF secentur, communes illorum sectiones EH et GF sunt parallela.

Demonstratio.

Lineæ EH et GF quæ sunt in eodem Plano secante EF aut erunt ≈læ, aut non erunt.

Ponamus non esse, ergo EH et GF coibunt alicubi v. c. in I.

sunt autem illæ in Planis AD atq; CD p. H.

adeoq; et hæc, si producantur coibunt.

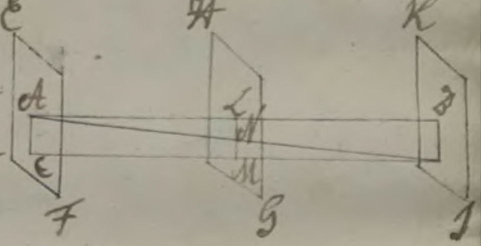
I. Q. C. H. quippe quæ Plana AD et CD parallela præstruit.

Q. E. D.

§449. Theorema 134.

Si duæ rectæ Lineæ AB, CD parallelis Planis EF, GH, IK secentur in easdem rationes secabuntur, h. e. erit

AL : LB = CM : MD.



Demonstratio.

In Planis EF, IK duc AC, BD §81. atq; rectam AD occurrentem Plano GH in N. &c.

junge LN et MN &c.

Quia GH ≈ IK p. H.

LN ≈ BD §448.

et Quia EF ≈ GH p. H.

AC ≈ NM. § c.

Ergo AL : LB = AN : NB. §349.

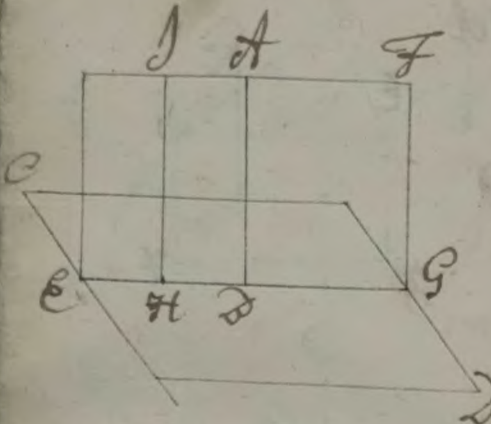
et DM : MC = DN : NA

MC : DM = AN : ND.

§146 Ar.

AL : LB = MC : DM §144 Ar.

Q. E. D.



§450 Theorema 135.

Si recta Linea AB Plano cuipiam CD ad ∠los Rectos fuerit et omnia, quæ per ipsam AB plana EF producuntur eidem Plano CD ad ∠los Rectos erunt.

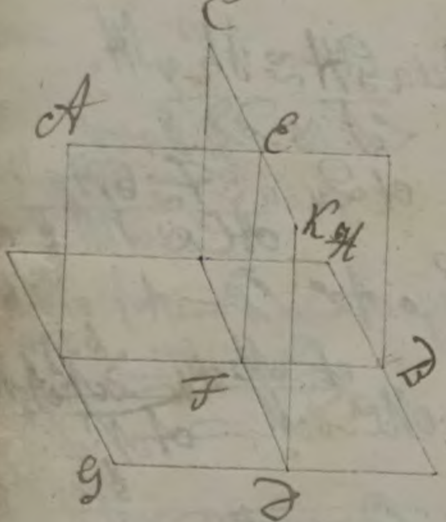
Demonstratio

Concipe per rectam AB Planum aliquod EF ductum esse, cujus cum altero Plano CD sectio sit Linea EG cum Recta ergo AB in quocunque ipsius EG pcto v.c. H. duc ⊥lam HI. §135.

Quare cum AB ⊥ ad CD p. H.
IH ⊥ ad CD. §440.

Similiter demonstrabis alias quasvis ⊥les ad EG esse ⊥les ad CD.

Ergo
Planum EG ⊥le ad Pl. CD. §409.
2. CD.



§451. Theorema 136.

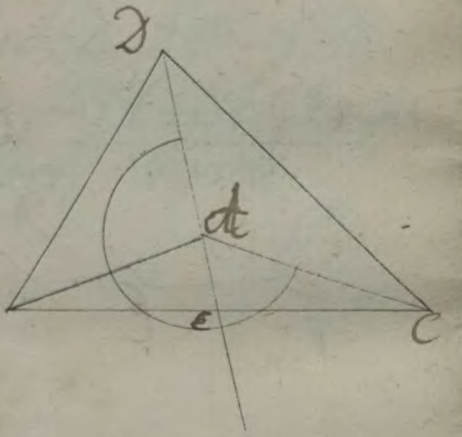
Si duo Plana AB, CD sese mutuo secantia Plano cuidam GH ad Rectos sint ∠los communis etiam illorum sectio EF ad Rectos eidem Plano GH ∠los erit.

Demonstratio.

Plana AB et CD Recta sunt Plano GH p. H.
Expeto intersectioni EF in Plano AB fac ∥lem EF §447
in Plano CD ex F duc ∥lem EF. §
Quo cum solummodo unica esse possit. §446.
Ergo etiam EF ∥lis Plano GH. §435. Q. E. D.

§452. Theorema 137

Si solidus ∠lus ABCD tribus ∠lis planis DAB, DAC, CAB contineatur, ex his duo quilibet utcunque sumti, tertio sunt majores.



Demonstratio

Si tres illi ∠li plani fuerint inter se aequales, per se veritas Theorematis elucet.

Quod si vero inaequales, maximus esto DAC. Ex hoc aufer

$\angle DAE = \angle DAB$. §1017.

et fac $AB = AE$. §26. et
duc rectas DE, DC, DB §81.

Quare cum $DA = DA$. §44 Ar.

$AB = AE$. p. C.

$\angle DAB = \angle DAE$. p. C.

$DB = DE$. §99

Verum $DB + BC > DE + EC$. §116.

$BC > EC$ §43 Ar.

cum que $AB = EA$. p. C.

et $AC = AC$. §40 Ar.

$\angle BAC > \angle$lo EAC. §166.

sed $\angle DAB = \angle$lo DAE. p. C.

$\angle DAB + \angle BAC > \angle$lis DAE + EAC §42 Ar.

h. e. $\angle DAB + \angle BAC > \angle$lo DAC §47 Ar.

Q. E. D.

§ 453. Theorema 138.

Omnis solidus Angulus sub minoribus quam quatuor Rectis terminatur ∠lis. Demonstratio.

Sit ∠us solidus A planis ∠lis BAC, o, y, z, u, ipsum solidum efficientibus, subtende rectas BC, CD, DE, EF, FB in uno Plano existentes. §81. Atq; inde liquet effici Pyramidem cujus Basis est Polygonum BCDEF, vertex autem A. §418. totq; cinctam Triangulis quot Anguli plani constituunt ∠um solidum.

Quare

$$\angle ABF + BFA + o = 2R.$$
$$\angle AFE + FEA + y = 2R.$$
$$\angle AED + EDA + y = 2R.$$
$$\angle ADC + DCA + u = 2R.$$
$$\angle ABC + BCA + CAB = 2R.$$

$$\angle ABF + BFA + o + AFE + FEA + y + AED + EDA + z + ADC + DCA + u + ABC + BCA + CAB = 10R.$$ [illegible] Ar.

Verum $\angle FBC + BCD + CDE + DEF + EFB = 10 - 4R.$ §309.

Ergo

$$\angle FBC + BCD + CDE + DEF + EFB + 4R = 10R.$$ [illegible] Ar.

$\angle\angle ABF + BFA + o + AFE + FEA + y + AED + EDA + z + ADC + DCA + u$
$+ ABC + BCA + CAB = \angle\angle FBC + BCD + CDE + DEF + EFB$
$+ 4R.$ § 410 Ar.

Enimvero $\angle ABF + ABC > FBC$
$\angle ACB + ACD > BCD$
$\angle ADC + ADE > CDE$
$\angle AED + AEF > DEF.$
$\angle AFE + AFB > EFB$
§ 42.

$\angle\angle ABF + ABC + ACB + ACD + ADC + ADE + AED + AEF + AFE +$
$AFB > FBC + BCD + CDE + DEF + EFB.$ § 420 Ar.

$\angle o + y + z + u + CAB <$ res 4 Rectis § 430 Ar.

Q. E. D.

§ 454. Theorema 139.

i fuerint tres ∠li Plani quorum
uo quilibet utlibet assumti reliquo sunt majores A, B, HCI. com-
rehendunt autem eos recta Lineae
equales AD, AE, BF, BG, HC, CI fieri
otest, ut ex rectis Lineis DE, FG,
HI. aequales Rectas illas connectenti-
us Triangulum constituatur.

Demonstratio.

num istud ostendendum, duas quas-
is trium illarum Rectarum DE, FG,
HI esse majores tertia, tum enim inde-
ribi poterit Triangulum § 93. et 116

348.

Idquod hoc modo evincitur:

Fac ∠lum HCK = ∠D. §107.

duc rectas CK = CH } §81.
ut et KH atq KI }

Unde liquet dari tres Casus

1) aut enim KI cadet ad partes ∠lorum KCH + HCI h.e. D + HCI, idquod fiet si ∠li dicti ∠res 2R.

2) KI in directum et continuum sit ipsi CI, quod fiet si dicti ∠li = 2R.

3) KI ad partes oppositas ∠lorum KCH + HCI, id quod eveniet si dicti ∠li fuerint 7res 2R.

Quare in Casu 1. quia

$CK = CH$. p. C.
$CH = DF$. p. H.
$DF = CK$. §41 Ar.
$\angle D = KCH$. p. C.
$DG = HC$ p. H.
$FG = KH$. §99.

omnq $DF = CK$ p. d.
$DF = AD$. p. H.
$CK = AD$. §41 Ar.
$CI = AE$. p. H.
$KI > DE$. §16
$KI < KH + HI$ §11b.
$KH + HI > DE$
sed $KH = FG$. p. d.
$FG + HI > DE$. §10 Ar.
Q. E. I.

Porro $\angle KCI = \angle KCH + HCI$. §
$\angle KCI = \angle D + HCI$. §10 Ar.
sed $\angle A < \angle lis\ D + HCI$. p. H.
$\angle KCI > \angle lo\ A$. §46 Ar.

Casu 2do: Quia uti ante

$CK = CH$. p. C.

$CH = FD$ p. H.

$CK = FD$. §41. Ar.

utrumque $\angle HCK = \angle D$ p. C.

$HC = DG$. p. H.

$KH = FG$. §99.

Sed $KH + HI > KI$. §116.

$KI = CK + CI$. §42. Ar.

$KH + HI > CK + CI$. §46. Ar.

Sed $CK = FD$ p. d.

$FD = AD$ p. H.

$AD = KC$. §41. Ar.

$AE = CI$. p. H.

$AD + AE = KC + CI$. §42. Ar.

$KH + HI > AD + AE$. §46. Ar.

$AD + AE > ED$ §116.

$KH + HI$ multo majus ED h. e.

$FG + HI$ multo $>$ ED. §10. Ar.

Q. E. ll.

Casus 3io Patet ex Constructione generali atque hactenus demonstratis.

✗ ✗

✗ ✗

$KH = FG$.

et $CK = DF = AD$.

Quarecum

$CK + CI < KH + HI$. §103.

$CI = AE$. p. H.

$AD + AE < KH + HI$. §10. Ar.

sed

$AD + AE > DE$. §116.

$DE < KH + HI$. h. e.

$DE < FG + HI$ §10. Ar.

Q. E. M. D.

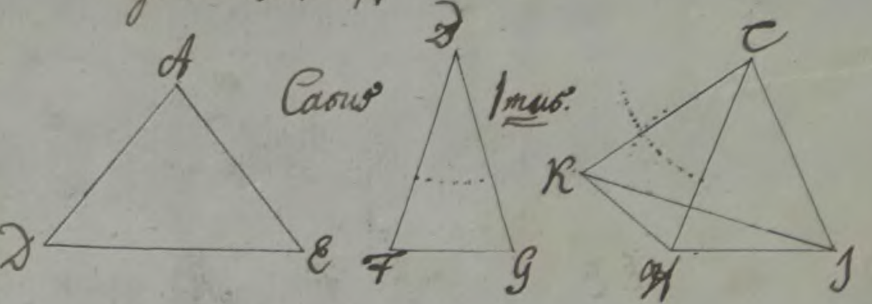
Simili Discursu ostenditur:

1) $FG + DE > HI$.

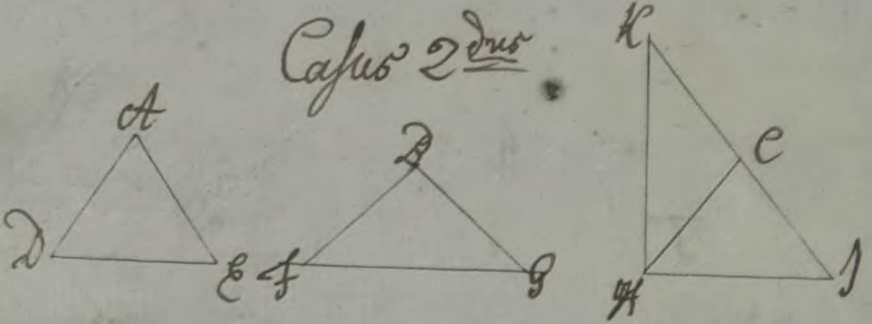
2) $HI + DE > FG$.

Q. E. D.

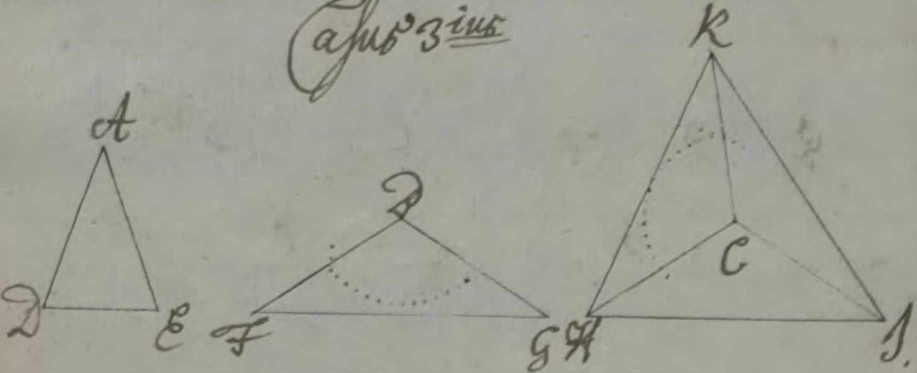
Figura §254.



Casus 1mus.

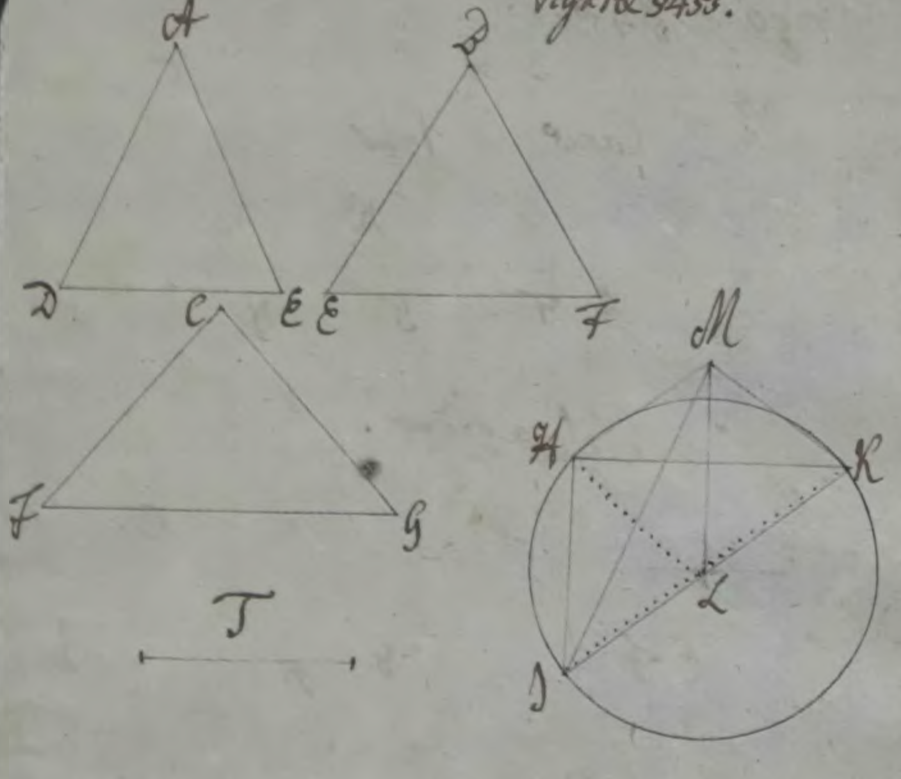


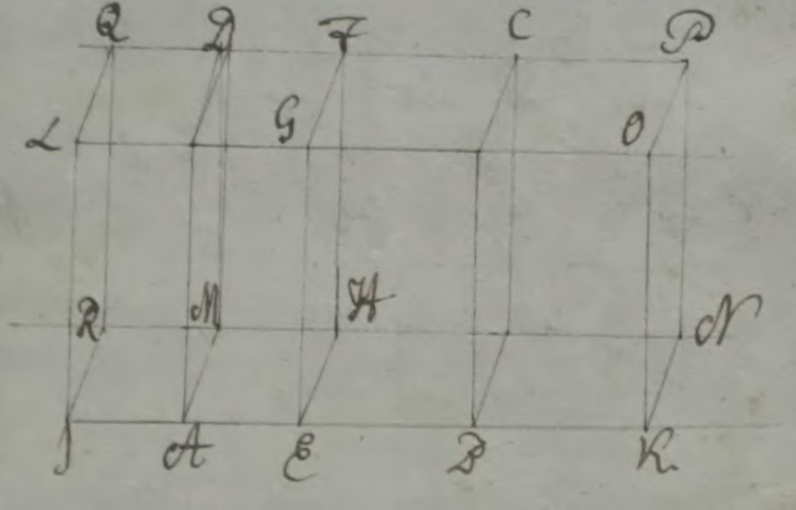
Casus 2dus



Casus 3ius

Figura §455.





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§455. Problema LXI.

Ex tribus ∠lis planis A, B, C quorum duo quomodocunq. assum reliquo sunt majores solidum ∠um MHIK constituere. Oportet autem illos tres ∠los quatuor Rectis es minores cf. §453.

Resolutio.

1) Fac AD = AE = EB = BF = CF = CG &c.
2) Subtende DE, EF, FG. §81 et
3) Construe ex illis Triangulum np. HIK ita ut
HI = DE
HK = EF
KI = FG
4) Circumscribe Circulum §315.
5) Ductisq. radiis HL, LK, LI §8
6) Quaere Excessum Quadrati Lateris AD supra HL. §194.
7) Latusq. inventum T ex cen Circuli circumscripti L sta ad ∠∠ R. ut T = LM. §404.
8) Junge Rectas MH, MI, MK §8

D. F.

Demonstratio.

Primo quidem loco evincendum AD excedere HL, id quod fieri in Constructione assumsimus. Est autem vel 1) $AD = HL$ 2) $AD < HL$ 3) $AD > HL$ } §29 Ar.

Deinde
$\angle KMR = D$,
$\angle KMI = E$,
$\angle KMH = A$.

Quare in Casu 1. Sit $AD = HL$ p. Hyp. quia
$AD = AE$ p. C.
$HL = LI$. §26.
$AE = LI$. §21 Ar.
et $DE = HI$. p. Constr.
$\angle A = HLI$. §106 similiter
$\angle D = HLK$
et $\angle E = KLI$.
$\angle\angle A + D + E = HLI + HLK + KLI$. §42 Ar.
$\angle\angle HLI + HLK + KLI = 4R$. §95.
$\angle\angle A + D + E = 4R$. §41 Ar.
I. Q. E A. quo $\angle\angle$os $A + D + E$ $\angle$res $4R$ supponit.

Casu 2 Esto $AD < HL$ p. Hyp.
quia $AE = AD$ p. C.
et $HL = LI$. §26.
$AE < LI$. §46. Ar
sed $DE = HI$. p. C.

$\angle A > \angle$lo HLI. §163.

Sic et $\angle D > \angle$lo HLK.

$\angle C > \angle$lo KLI.

$\angle A + D + C >$ res $\angle$lis $HLI + HLK + KLI$ §42. Ar.

Verum $\angle HLI + HLK + KLI = 4R$. §95.

$\angle A + D + C >$ res $4R$. §46. Ar.

I. Q. C. H. quæ ponit $\angle$los $A + D + C <$ res 4 Rectis.

Proindecum

neq. $AD = HL$.

neq. $AD < HL$ Ergo utiq.

$AD > HL$

Q. E. I.

$AD^2 = HL^2 + ML^2$ p. C.

cumq. $ML \perp$ ad Circ. p. C.

$ML \perp$ ad HL. §408.

$MH^2 = HL^2 + ML^2$. §189.

$AD^2 = MH^2$ §41. Ar.

Ergo $AD = MH$. §197.

Porro, quia

$AD = AE$ p. c.

$AD^2 = AE^2$ §44 et 225 Ar.

Sic et $HL = LI$. §26.

$HL^2 = LI^2$. §44 et 225. Ar.

Quare cum

$AD^2 = HL^2 + LM^2$ p. c. A

$AE^2 = LI^2 + LM^2$. §10. Ar.

cumque $LM \perp$ ad LI. §408.

$MI^2 = LI^2 + LM^2$. §189.

$AE^2 = MI^2$. §41 Ar. adeoque

$AE = MI$. §197. sed et

$AD = MH$ p. c.

$DE = HI$. p. c.

$\angle A = \angle HMI$. §106.

Simili omnino Demonstratione evincitur Aequalitas ∠orum reliquorum, ut tandem fit

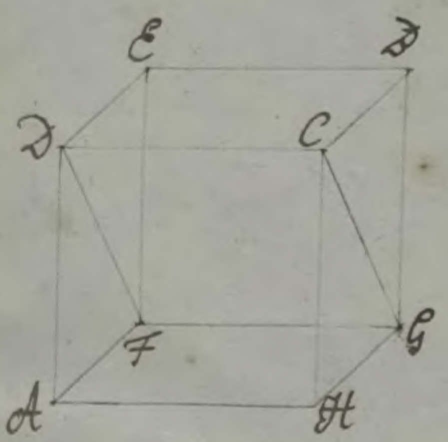
$\angle HMK = \angle B$

$\angle KMI = \angle C$.

Q. E. D.

§456. Theorema 140.

Si solidum AB parallelis Planis contineatur, adversa illius Plana AE, HB; AG, DB; et GE, AC Parallelogramma sunt similia atque aequalia.



Demonstratio

$DB \approx AG$ p. H.
et AC secat DB atque AG.

$DC \approx AH$. §448.

$HB \approx AE$ p. H.
et AC secat HB et AE.

$AD \approx CH$. §c.

AC est Parallelogrammum §42.
Simili discursu ostendatur et reliqua Plana esse Parallelogramma
Q. E. D.

Admdm.
1) Plana omnia quibus solidum AB continetur esse Parallelogramma.
2) Adversa quaevis bina Pllgma esse 1) aequalia 2) similia.

Duc DF et CG. §81

Quia $\left.\begin{array}{l} AF \approx HG \\ AD \approx CH \end{array}\right\}$ p. d.

$\angle FAD = \angle GHC$. §442.

sed $\left.\begin{array}{l} AF = HG \\ AD = CH \end{array}\right\}$ §167.

$\triangle FAD = \triangle GHC$. § 99. Ergo et
$AF : AD = GH : HC$. § 145 Ar.

$\triangle FAD \sim \triangle GHC$. § 356. 341.
sed $\triangle FAD = \frac{1}{2} AE$
$\triangle GHC = \frac{1}{2} HB$ } § 169.

$\frac{1}{2} AE =$ et $\sim \frac{1}{2} HB$. § 341 Ar. et 381.
$AE =$ et $\sim$ HB. § 44 aut 451 Ar.
Similiter et Aequalitas et Similitudo reliquorum adversorum Pllgmorum evincitur.

Q. E. D.

§ 457. Theorema 144.

Si solidum Parallepipedum ABCD Plano EF secetur adversis Planis AD, CB parallelo, erit quemadmodum Basis ad Basin, ita solidum AHD ad solidum BHC.

cf. Fig. pag. 350.

p. H. dendm:
$AH : HB = AHD : BHC$.

Demonstratio.

Concipe Parallelepipedum ABCD produci utrinque, fac AB = AE atq. BK = EB atque pone Plana BG et KP ∥la Planis AD et CB.

Ergo
$IM = AH$. §176 b.
cumq. IR. & AM. §72.
$\angle RIA = \angle MAE$. §132.
$\angle RIA = \angle RMA$ }
$\angle MAE = \angle MHE$ } §160.

$\angle MHE = RMA$. §41 Ar.
$\angle RIA + IAM = 2R$ }
$\angle MAE + AEH = 2R$ } §169.

$\angle IAM = \angle AEH$. §41 et 43 Ar.
Ergo et
$\angle IRM = \angle AMH$. §169. G.
et 41 Ar.
Cumq. $IR = AM = EH$
et $IA = RM = AE = MH$
§167. G. 41 Ar.
Ergo
$RI : IA = MA : AE$ }
$IA : AM = AE : EH$ } §145 Ar.
Ergo
$IM \sim AH$. §341. cumq.
$LD \sim$ et $= IM$ }
$DG \sim$ et $= AH$ } §45 b.

Similiter est $LD \sim$ et $= DG$. §381. G. et 41 Ar
$LA \sim$ et $= AG$
$RM \sim$ et $= MF$ tandem
$LR \sim$ et $= AD$ }
$AD \sim$ et $= EF$ } §45 b

Ergo.
Pppdm SD ~ AH = Pppdo AF. §415.
simili Discursu ostenditur
Pppdm CK ~ et = Pppdo GD.
Proinde liquet:

1) Pallelpppdm SF continere Pppdm AF eodem modo, quo Pppdm ES continet Pppdm DF. §115 etc.

2) Basin SH continere Basin AH eodem modo, quo Basis EC continet Basin HD &c. Ergo.
AF ~ GD §133. Totorum SF et ES Pppdorum
AH ~ HD &c. Totarum SH et HK Basium.

Quapropter.

1) Si SH = HK erit
Pppdm SF = Pppdo ES §415.
Sunt enim bina singula Pllga adversa prioris Pppdi = singulis binis Pllgis adversis posterioris, per demita id est Secilla quibus continetur SF = Secaillis quibus ES terminatur Pllgmis.

2) Si SH > HK erit etiam
Pppdm SF > Pppdo ES.
i.q.h.m. ostenditur.

$DH > HK$ p. H. Ergo
$\angle F > FO$. §456.
cumq ob $KN \approx DK$ p. H et C.
$\angle DEH = \angle EKN$. §132.

$DH : HK = DE \times EH : EK \times KN$.
atq ob $EH = KN$. §167 §385.
$DH : HK = DE : EK$. §160 144. Ar.
Hinc cum
$DH > HK$ p. H.

$DE > EK$. §132 } Ar.
$EG = EG$. §40 }

$\angle E > EO$ §175. et
$GH > HO$. §456.
$DQ = GH$ } §c.
$GH = ON$ }

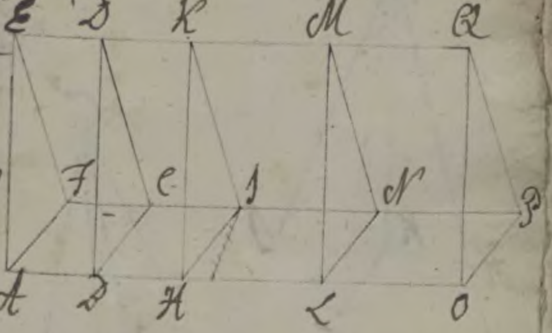
Summa sex Planorum quibus terminatur Ppdm $DF >$ Suma sex Planorum qbus terminatur Ppdm EP.

3. Si $DH < HK$ erit etiam
$DF < EP$. idq quod similimodo quo Casus 2dus demonstrat. Quare in omni Casu:
$DF : EP = DH : HK$. §91 Ar.
$DF : EP = AF : EC$ } §144.
$DH : HK = AH : HB$ } Ar.
$AF : EC = AH : GHB$. §44 } Q.E.D.

§458. Scholion.

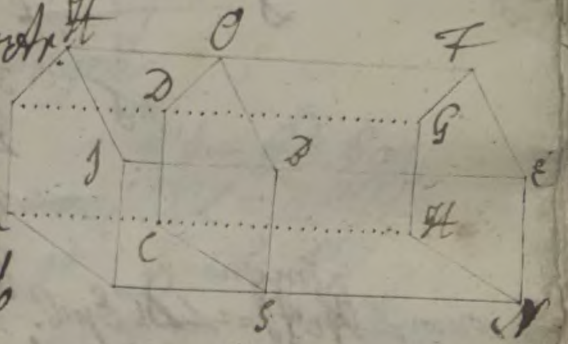
Quae §357 de Pppdo demonstrata sunt, mutatis mutandis de omni quoque Prismate demonstrabuntur. Facto enim Prismate.

ABCDEF = Pr. BCHDIK.
et NLMOPQ = Pr. HIKLMN.



Liquet omnino per similem §357. demonstrationem esse:

Pr. AK : Pr. HQ = AI : HP. §91 Ar.
AK : HQ = Pr. DK : Pr. LQ. §148 Ar.
Bas. AI : HP = DI : HN. §144. Ar.



§459. Corollarium.

Liquet etiamsi Prisma quodvis secetur Plano adversis Planis ∥, sectionem esse Figuram aequalem et similem oppositis Planis:

Nam

HF ∥ IE. §419.
OD ∥ FE. p. H.

OD = FE. §72. 167.

OD = FG
DC = GH etc.

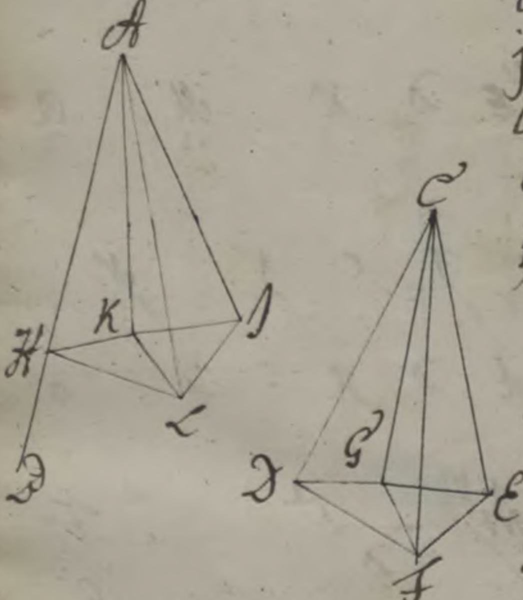
Porro quia:
✗

✗
FE ∥ OB } p. H.
FG ∥ DO }

∠EFG = ∠BOD. §142.
Sic et ∠FGH = ∠ODC
∠GHN = ∠DCS etc.

Similiter liquet. Figura OS congruit figurae FN. §4. OS ∼ et = FN.
§. 86.

Q. E. D.



§ 460. Problema LXII

Ad datam Lineam rectam AD, ejusque punctum A constituere ∠lum solidum AHIL aequalem solido ∠lo CDEF.

Resolutio.

1) Ex Recta CF quolibet puncto v.c. F, demitte GF ⊥lem ad Planum DCE. §443.

2) Junge Rectas DF, FG, EG, GD et GC. §81.

3) Fac AH = CD et

4) ∠lum HAI = ∠DCE. §107.

5) Itemque AI = CE.

6) Sic et in Plano HAI fac ∠HAK = ∠DCG et AK = CG. §107. 26.

7) Ex K erige KL ⊥ ad Pl. HAI §444. et fac KL = FG.

8) Duc AL §81. D. F. h. e. ∠ AHIL = ∠ CDEF.

Demonstratio

Dmqdm cum ∠HAI = ∠DCE p.C.
1) ∠HAL = ∠DCF et
2) ∠LAI = ∠FCE fore.
Quonp. summa ∠lorum planorum ∠lum solidum CDFE componentium, sit aequalis summae ∠lorum planorum ex quibus ∠lus AHLI componitur.

$\angle HAK = \angle DCG$. p.C.

$HA = DC$ } p.C.
$AK = CG$ }

$HK = DG$. §99.

$KL \perp$ ad lf. HAI. p.C.

$\angle HKL = R$. §408.

$FG \perp$ ad lf. DCE. p.C.

$\angle DGF = R$. &c.

$\angle HKL = \angle DGF$. §92.

$HL = FG$. p.C.

$HL = DF$. §99.

Porro $\angle AKL = \angle CGF$. §408. 92.

$AK = CG$ } p.C.
$KL = FG$ }

$AL = CF$. §99.

cumque $HL = DF$ p.d.

et $AH = DC$. p.C.

$\angle HAL = \angle DCF$. §106.

Q.E.I.

$\angle HAI = \angle DCE$. p.C.

$\angle HAK = \angle DCG$. p.C.

$\angle KAI = \angle GCE$. §43 Ar.

$AK = CG$ } p.C.
$AI = CE$ }

✠ ✠

✠ $KI = GE$. §99.

cumque $\angle IKL = \angle EGF$. §408. 92.

et $KL = FG$. p.C.

$LI = FE$. §99.

sed $AL = CF$. p.d. ad M. 1.

et $IA = EC$. p.C.

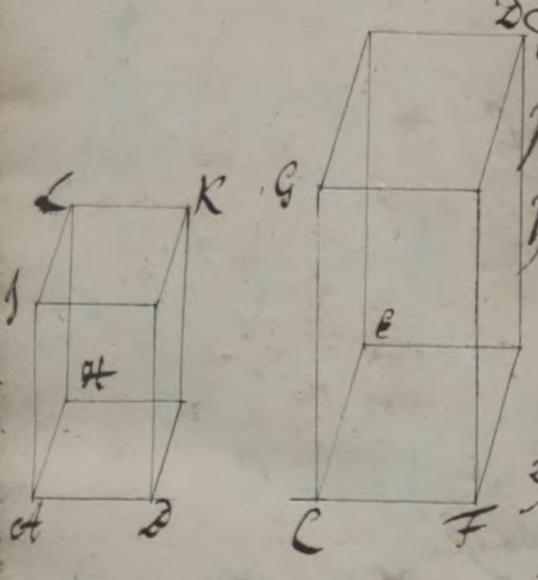
$\angle LAI = \angle FCE$. §106.

Q.E. IId.

362.

§461. Problema LXIII

Adata recta Linea AB dato solido Pprdo CD simile similiterq. positum describere.



Resolutio.

1) Ex ∠lis BAH = FCE } §107.
HAI = ECG
BAI = FCG

fac ∠lum ABHI = ∠ CFEG. §460.

2) Fac, FC : CE = BA : AH } §366.
CE : CG = AH : AI

3) Completis Pllgmis AB, BL,
IH, IK, KB, KA. §170 – 173.
Q. F.

Demonstratio.

∠FCE = ∠BAH. p. C.
FC : CE = BA : AH. p. C.

EF ∾ similiterq. pos. HB. §341.
Sic et ∠ECG = ∠HAI } p. C.
et CE : CG = AH : AI }

GE ∾ similiterq. pos. IH. &c.

Porro quia.
FC : CE = BA : AH } p. C.
CE : CG = AH : AI }

FC : CG = BA : AI. §172. Ar

cumq. ∠FCG = ∠BAI. p. C.

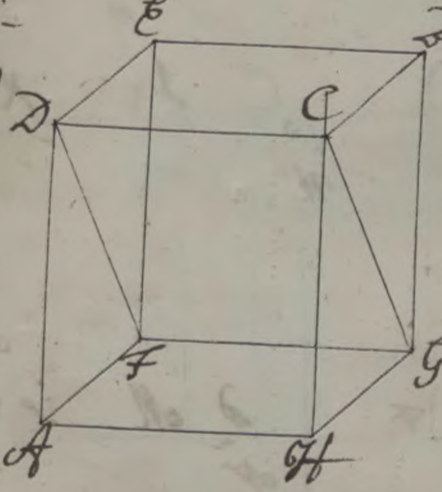
FG ∾ similiq. pos. AL. §341.

Et quia Planis similibus similiterq. positis modo demonstratis, etiam adversa similia sunt et similiter posita, nempe GD, LB, ED item IK, BK, HK. §406.

Ergo AK ∾ CD. §414.
Q. E. D.

§462. Theorema 142.

Si ſolidum Pppdm. AB Plano CDFG ſecetur per diagonios DF et CG adverſorum Planorum AE, HB bifariam ſecabitur ſolidum AB a Plano FGCD.



Demonſtratio.

DC = EB } §164.
EB = FG }

DC = FG. §241 Ar.

DC ≈ EB } §72.
EB ≈ FG }

DC ≈ FG. §441.

DF ≈ et = CG. §139.

DG eſt Parallelogrammum §72.

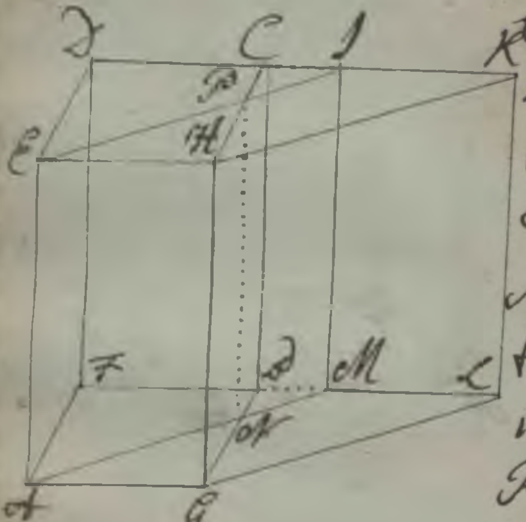
AE ∾ et = HB. §456.
△DAF ∾ et = △DEF. §169.
△CHG ∾ et = △CBG. §c.
AC ∾ et = FB } §456.
DB ∾ et = FH }
ſed et DG ∾ et = DG. §40. Ar.

Prism: DFGA ∾ et = Pr. DFGC. §415.
Prism: DFGA + Pr. DFGC = AB. §47. Ar.
2 × Pr. DFGA = AB. §10. Ar.

h. e.
Pr. DFGA = $\frac{1}{2}$ AB §48. Ar.
vel
Pr. DFGE = $\frac{1}{2}$ AB. §c. et Ar.
Q. E. D.

§463. Theorema 143.

Solida Parallelepipeda AGHED CBF et AGHE IKLM, ſuper eandem Baſin AGHE conſtituta et in eadem Altitudine. h.e. inter Plana zla, quorum inſiſtentes Lineæ AF et AM, ſunt in iisdem Rectis AG et FL ſunt inter ſe æqualia. h.e. ſuper Baſin eandem AEHG conſtituta ſint duo Ppp.da AL; AK. et in eadem Altitudine h.e. inter Plana zla ut ſcil. AH z FK, quorum inſiſtentes Lineæ rectæ ex quatuor Baſis ∠lis A, E, H, G, videlicet AF, AM; GB, GM; ED, EI; CH, HK; in iisdem terminentur rectis Lineis FL atque DK, vel iisdem rectis Lineis AF, BD, DC productis in K et L, hoc dico ipſa AL et AK eſſe inter ſe æqualia, ita fere Claud. Richardus in Comment. Eucl. LXI. P. 29. add. Clavius ad Lgt. c.



Demonstratio.

$AB = FB$ }
$AG = LM$ } §431. 72.

$FB = LM$. §41 Ar.
$BM = BM$. §40 Ar.

$FM = BL$. §42. Ar.
$AF = DG$ }
$AM = GL$ } §431. 72.

$\triangle AFM = \triangle GBL$. §106 Similiter.
$\triangle EDI = \triangle HCK$.

$FM = BL$. p. d.
$DF = CB$. §431: 72.

$DM = CL$. §175. G. et 43 Ar.
$EM = HL$ }
$EF = HD$ } §456.

Pris. $EAMDA$ = Prism. $HGLCG$. §415.
Pris. $ACIMBN$ = Prism. $KCIMOD$. §40. Ar.

Solid. $EANPCDFDA$ = Solido $HGNPIMKG$. §43 Ar.
Pris. $EAHN$ = Pris. $EAHN$. §40. Ar.

Pppdm. $AGHEDCDF$ = Ppp. do $AGHEIKLM$. §42 Ar

Q. E. D.

§464. Theorema 144.

Solida Pppda AF et ABCDEMKL ſuper eandem Baſin ABCD conſtituta atq; in eadem Altitudine, quorum inſiſtentes Lineæ AH et AM in iisdem Lineis rectis collocantur inter ſe ſunt æqualia.

h.e.

Super Baſin eandem CABD ſita ſint Pppda AF, AL in eadem altitudine ny. ut Planum s. Baſis AC ≈la Plano HGFKLMIE, et horum Pppdorum rectæ Lineæ quatuor Baſis ∠lis A, B, C, D, inſiſtentes AH, AI; DE, DM, BG, BK, CF, CL, non ſint terminatæ s. collocatæ in iisdem rectis Lineis, h.e. neq; HG, EF productæ tranſeant per puncta K, I, L, M, neq; HE, GF productæ incedant per eandem puncta K, I, L, M, terminantia Rectas AI, DM, BK, CL. Dico:

Pppda AF et AL esse aequalia.
Ita fere Cl. Richardus ad Eucl. L. XI.
Prop. 30 add. Clavius ad Librum et
Propos. citat.

Demonstratio.

Produc Rectas HE, GF, LM, KI ad
occursum in G, M, Q, P, §82.
duc AB, DO, DQ, CN. §81.

Hinc quia.

DC $\cong$ AD $\cong$ HG $\cong$ EF $\cong$ PQ $\cong$ CN §167.

et

AD $\cong$ HE $\cong$ GF $\cong$ DC $\cong$ KL $\cong$ IM $\cong$ QN $\cong$ PO. &c.

Ergo

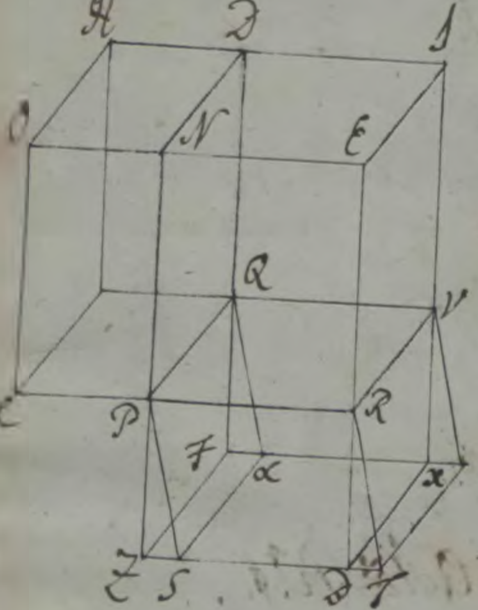
Pppdm ADCBQO = AF §463.
Pppdm ADCBQO = ADCBKLIM. &c.

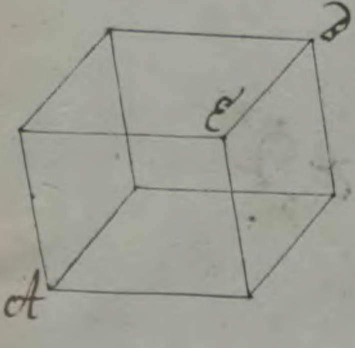
Pppdm AF = Pppda ADCBKLIM &c. erit,

Q. E. D.

§465. Theorema 145.

Solida Pppda AB, CD super aequales
Bases AE et OP constituta et in
eadem Altitudine sunt inter
se aequalia





Demonstratio.

Casus 1. Si Pppda AB et CD latera
ad bases recta habuerint.

Produc CP in R. §82. et
fac Pllgm PT ~ et = AE §374.
comple Pppdum SV. §135.
Quia Pppdi AB altitudo = Pppdi
CD altitudini p. H.
et Pppdi SV altitudo = Pppdi
CD altitudini p. C. ∴

Pppdi AB altitudo = Pppdi SV
altitudini §41 Ar.
sed et basis PT = basi AE. p. C.

Pppdm AB ~ et = SV. §461.
Produc CN, HD, NP, DQ §82
Ductisq per R et V = ER et DV.
cum NP et DQ §135.
et ipsas ER, DV, TS, Hx produc
ad concursum I, E Z, F, d,
atq X §82.
et tandem duc FZ, Xd, EI. §81

Quia.

Planum PS ≈ CV ≈ FT. §431.

et AE = CN. p. H.

AE = PT. p. C.

cumq. PR ≈ ZT p. C.

et PR = PR. §40 Ar.

PT = ZR. §174.

AE = CN = PT = ZR. §41 Ar.

eritq.

CD : PS = CN : PE. §457.

ZDXFQPRV : PS = ZR : PE. §c.

CD : ZDXFQPRV = CN : ZR. §173 Ar.

sed CN = ZR. p. d.

Ergo CD = ZDXFQPRV. §132 et 126 Ar.

Pppdum SV = ZDXFQPRV. §463.

atq. SV = AD. p. d.

Pppdum AD = CD. §41 Ar.

Q. E. I.

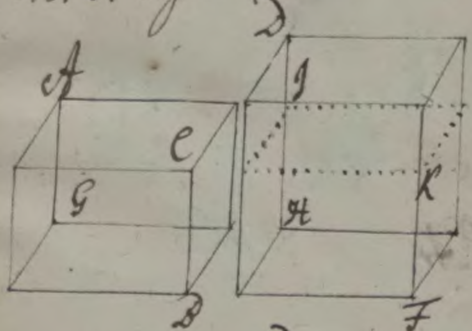
Casus 2dus Si Pppda AD et CD Latera ad Bases obliqua habuerint. Super easdem Bases et in eadem Altitudine pone Pppda alia, quorum Latera Basibus sint recta, quae et inter se et obliquis aequalia sunt. §463. 464. 9. et 41 Ar. Q. E. N. D.

§466. Corollarium.

Solida Pppda aequalia super aequales bases sunt etiam in eadem Altitudine. Et Pppda aequalia in eadem Altitudine, super aequales bases sunt, si non habuerint eandem

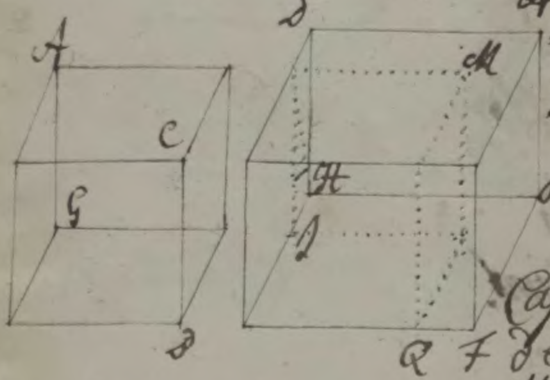
Ex Claudio Richardo
Vamin

h. e.
1) Si AB = DF et basis BG = FH erit Altit. Pppdi AB = Alt Pppdi DF.
2) Si Altit: Pppdi AB = Alt Pp DF et Pppdm AB = Pppdo DF erit et basis BG = FH.



Casu 1. Ponamus Pppda DF altius esse Pppdo AB; concipe ergo ex DF solidum rescissum esse DK, quo tandem solidum IF fiat ejusdem altitudinis cum Pppdo AB. Ergo quia

Alt. IF = Alt. AB. p. C.
et BG = HF p. H.
Sol. IF = Sol. AB §465.
Sed AB = DF. p. H.
IF = DF. §41 Ar
J. Q. E. A. §42 Ar.

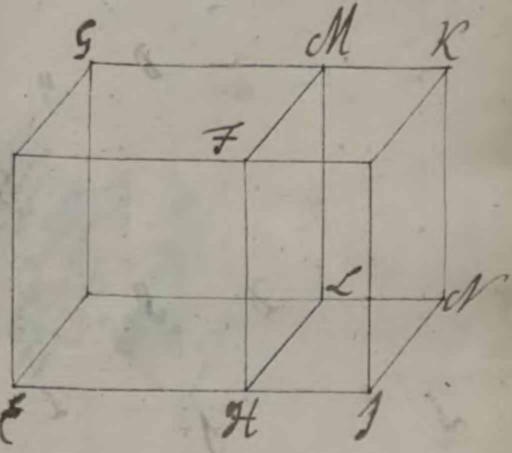


Casu 2do. sit bas. HF > bas. G B et abscinde ex HF basin IQ = bas. GB. p. H.
cum ergo Alt Pppdi AB = Alt Pppdi IQM. p. H.
et basis GB = basi IQ p. C.
Pppdm AB = Pppdo IQM. §465.
Sed AB = HFD p. H.
Pppdm HFD = Pppdo IQM. §410 Ar.
J. Q. E. A. §42 Ar.

§ 467. Theorema 146.

Solida Ppda ABCD et EFGL ſub eadem Altitudine inter ſe ſunt uti Baſes AB et EF. h. e. p. H.

ABCD : EFGL = AB : EF.



Demonſtratio.

Produc EH in I. § 82.

Fac FI = AB. § 116.

et comple Ppdm. HK. § 135.

Quarecum

Ppdon EM ejusd. Altitu.; cum Ppdo HK. p. C.

Ppdm EM ejusd. Altit: cum Ppdo BACD. p. H.

Ppdm HK ejusd. Altit. cum Ppdo BACD. § 410 Ax.

ſed et baſis FI = Baſi AB. p. C.

Ppdum HK = Ppdo ABCD. § 465.

cumq HM ≈ IK ≈ EG p. C. et H. cum ſit Ppdm § 431.

Ergo

HK : EM = FI : EF. § 457.

ſed FI = AB p. Conſtr.

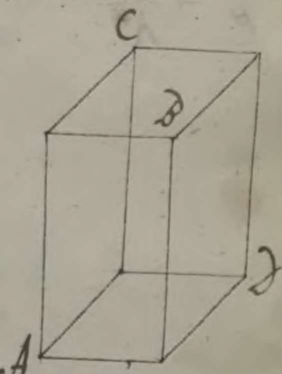
HK : EM = AB : EF. § 100 Ax.

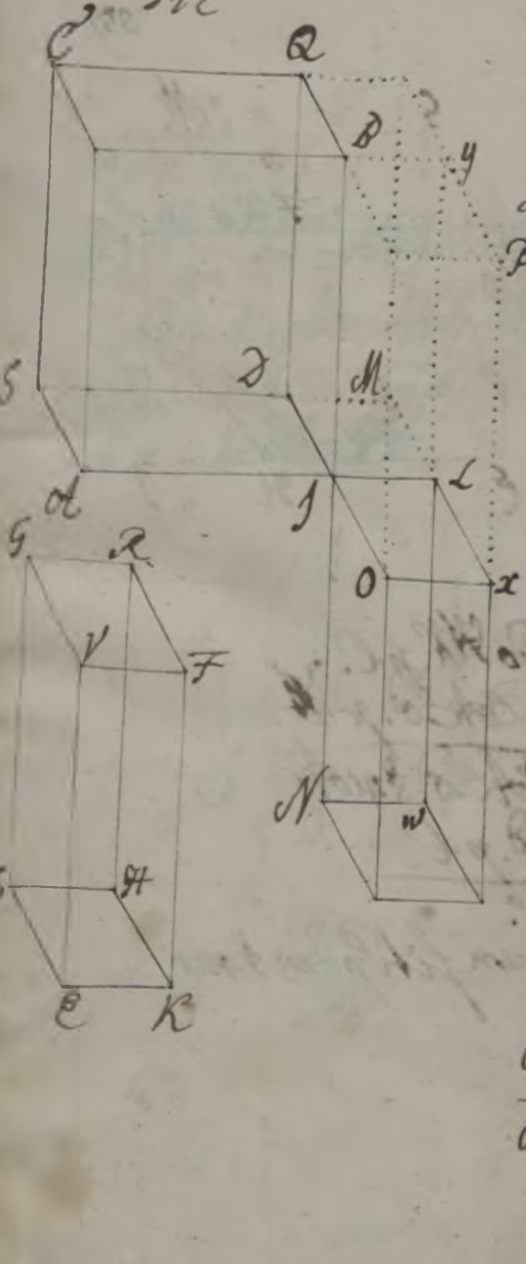
cumq et HK = ABCD p. d.

Ergo tandem

ABCD : EM = AB : EF. &c.

Q. E. D.





§468. Theorema 147.

Similia solida Pppda QA, GK inter se sunt in triplicata ratione Laterum homologorum AI et ER.

Demonstratio.

Produc Rectas
AI in L ut IL = GR.
DI in Θ ut IΘ = GV
SI in N ut IN = GS.
Comple Pppdm NX. §135. Ergo
Pppdm NX ~ et = Pppdo GK. §461. 415.
Perfice Pppdm LQ. §135
Quia Pppdm QA ~ Pppdo ER. p. H.
Ergo.
AI: GR = DI: GV. §341.
AI: IL = DI: IO. §10 Ar.
AI: GR = IS: GS. §341.
AI: IL = IS: IN §10. Ar.

AI: IL = DI: IΘ = IS: IN. §142. Ar.

Porro: LM ≅ AL
DΘ ≅ MX } §431. 72.
SN ≅ yw
Ergo:

AI : IL = AD : DL
DI : IO = DL : LO } §347.
ID : IN = DL : LN

AD : DL = DL : LO = DL : LN. §144 Ar.

Sed et Alt. Pppdi AQ = Alt. Pppdi DY
Alt. Pppdi DY = Alt. Pppdi DX.
Alt. Pppdi DX = Alt. Pppdi NX.

Ergo AD : DL = AQ : QL
DL : LO = QL : DX } §467.
DL : LN = DX : NX

AQ : NX in ratione triplicata AQ : QL. §189.

Sed AQ : QL = AD : DL
AD : DL = AI : IL } p.d.

AQ : QL = AI : IL. §144 Ar.

Est autem IL = GK p.C.
GR = VF
VF = EK } §167.

IL = EK. §41 Ar.

cumque et NX = GK. p.d.

Ergo

AQ : QX in ratione triplicata AI : EK. §10 Ar. h.e.
$AQ : GK = AI^3 : EK^3$. §o. et 189 et 225 Ar.

Q.E.D.

§469. Corollarium.

Hinc ſi fuerint quatuor Lineae continue ppales, ut eſt prima ad quartam ita quoq; eſt Pppdm ſuper primam ad Pppdm ſuper ſecundam ſimile ſimiliterq; deſcriptum.

Nam eſto.

$A:B = B:C = C:D$. p. H.

Pone Pppdm ſuper A factum $= P$, et Pppdm ſuper B ſimile ſimiliterq; poſitum ipſi A $= \Pi$. Ar.

Quia $A:D = A^3:B^3$. §189. 225

et $P:\Pi = A^3:B^3$. §468.

$A:D = P:\Pi$. §144. Ar.

§470. Theorema 148.

A Fig. pag 375.

Aequalium Solidorum Pppdorum AB et EF baſes et Altitudines reciprocantur. Et contra: Quorum Solidorum Pppdorum baſes et Altitudines reciprocantur, illa ſunt aequalia.

h. e dmdm.

1) ſi $AB = EF$ erit $GE:AC = AD:EH$.

2) ſi $GE:AC = AD:EH$ erit $AB = EF$.

Demonſtratio

I. Sunto Latera CA, GE ad baſes recta et Altitudines aequales.

Quia AB = EF p. H.
et alt^{ido} AB = Alt^{ido} EF. p. H hujus Casus
AB = EH §466.

Ergo
AB : EH = EG : AC. §126. Ar
Q. E. I.

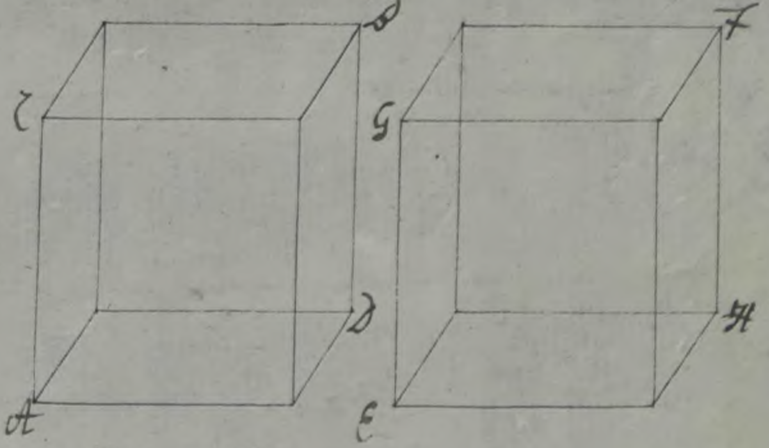
II Sumto Altitudines inaequales:
A majore ergo GE aufer minorem
Altitudinem AC = IE.
Per I duc Planum IKzLe. Bas. EH

Ppdi AB alt = Ppdi EK Alt p. C.
AB : EH = AB : EK. §467.
Ppdi EF Alt = Ppdi EK Alt §40 Ar.
GL : IL = EF : EK. §467.
Pllgmi GL Alt = Pllgmi IL Alt §40 Ar
GE : IE = GL : IL. §347.
GE : IE = EF : EK. §144 Ar.
sed IE = CA p. C.
EF = AB. p. H.
GE : CA = AB : EK. §100 Ar.

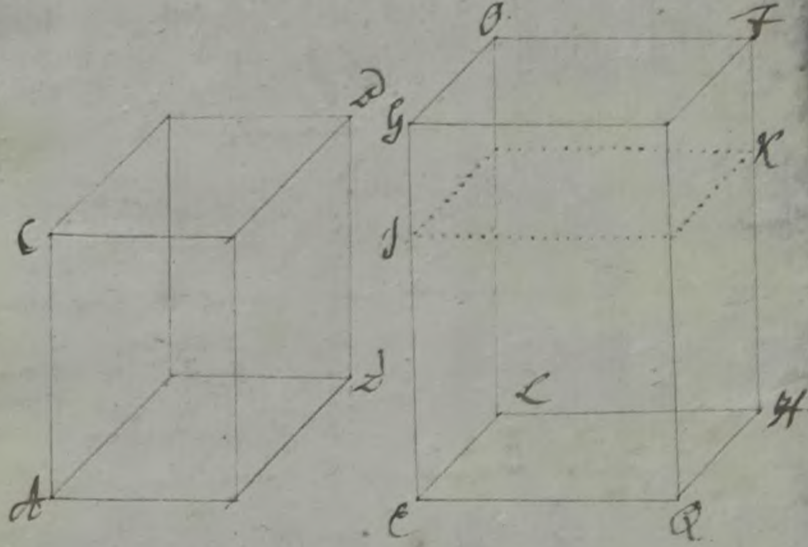
Ergo
GE : CA = AB : EH. §144 Ar.
Q. E. II.

AB : EH = GE : AC p. H.
IE = AC. p. C.
AB : EH = GE : IE §100 Ar.
Ppdi AB alt = Ppdi EK Alt p. C.
AB : EK = AB : EH. §467.
Pllgmi GL Alt = Pllgmi IL Alt. §40 Ar
GE : IE = GL : IL §347.
cumque et GL ≅ FQ §431.
GL : IL = EF : EK. §467.
AB : EK = EF : EK. §144 Ar

Ergo
AB = EF. §152 Ar.
Q. E. III D.

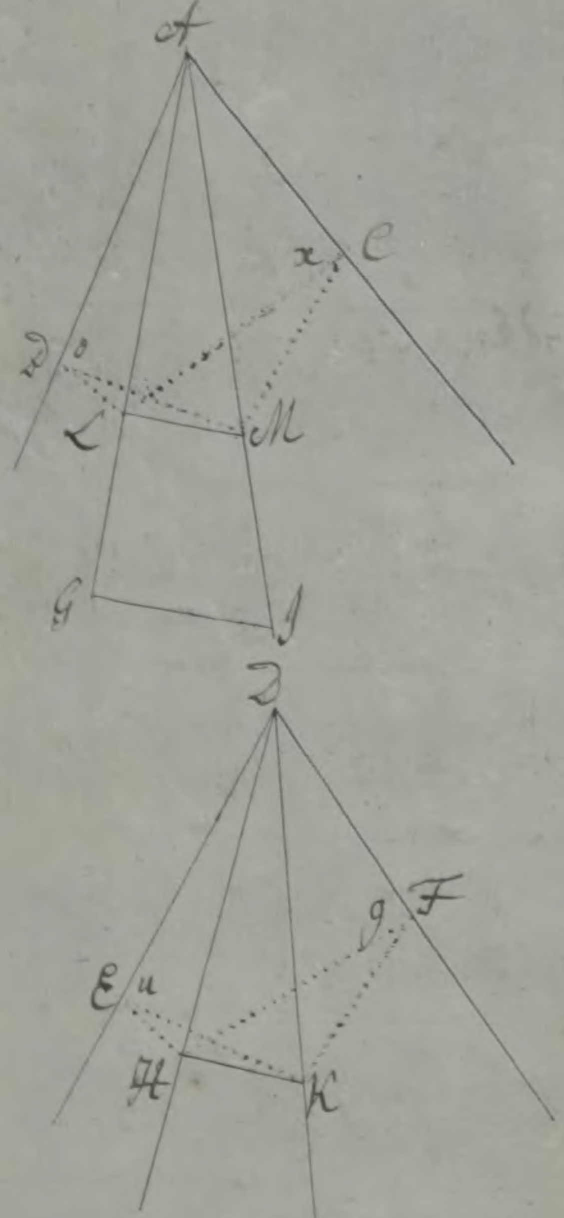


Ad §470. Cas. 1



Cas. 2di.

Figura §phi 472



III Sunto Latera ad Bases obliqua. Erige super Bases easdem in Altitudine eadem Pppda recta, eruntq; recta = Pppdis obliquis §465. Enimvero Pppda recta, reciprocant Bases et Altitudines. §10 Tr. Q. E. DD.

§471 Corollarium.

Quæ §. §. 463.—470 demonstrata sunt Prismatibus etiam Triangularibus conveniunt, utpote Pppdorum dimidiis. §462.

Proinde.

1) Prismata Triangularia æque alta sunt inter se ut Bases.

2) Si Prismata Triangularia eandem vel æqualem Basin habuerint eandemq; vel æqualem Altitudinem, æqualia sunt.

3) Si Prismata Triangularia sint similia eorum Proportio est triplicata homologorum laterum.

4) Si æqualia sint Prismata illa reciprocant Bases et Altitudines et contra.

§ 472. Theorema 149

Si fuerint duo Anguli Plani BAC et EDF æquales, quorum Verticibus A et D sublimes rectæ Lineæ AG et DH insistant, quæ cum Lineis primo positis Angulos contineant æquales utrumque utrique h.e. $\angle GAB = \angle^{lo} HDE$ et $\angle^{lum} GAC = \angle^{lo} HDF$ in sublimibus autem Lineis AG et DH quælibet sumta fuerint puncta G et H, et ab his ad Plana BAC et EDF in quibus consistunt Anguli primum positi BAC et EDF ductæ fuerint normales GL et HK a punctis vero L et K, quæ in Planis a perpendicularibus fiunt ad $\angle^{los}$ primum positos adjunctæ fuerint rectæ Lineæ AL et DK, hæ cum sublimibus AG et DH æquales Angulos GAL et HDK comprehendent. h.e. Q.E. dmdm $\angle GAL = \angle HDK$

cf. Fig. p. 376

Demonstratio.

Si $AB > HD$ aut contra
fac $AL = DH$ aut conversim
atq; in Plano ABD per L duc ∥lam
LM cum GD §135.
Ergo LM ⊥ ~~cum GD §13~~ ad Planum ADE
§440. Porro.
Ex pcto M itemq; ex pcto K demitte
MD ⊥lem ad AB et EK ⊥ ad ED §119.
et ME ⊥lem ad AE, et KF ⊥ ad AB
atq; connecte
DE, DL, LE. itemq; EF, EH, HF. §81.

Quare cum
LM ⊥ ad Plan. ADE. p.c.
$\angle LMA = R$. §408.
$AL^2 = LM^2 + AM^2$. §189.
ME ⊥ ad AE. p.c.
$AM^2 = ME^2 + AE^2$. §c.
$AL^2 = LM^2 + ME^2 + AE^2$. §10 Ar.
sed $\angle LME = R$. §408.
$LE^2 = LM^2 + ME^2$. §189.
$AL^2 = LE^2 + AE^2$. §10 Ar.
Ergo
∠lus $AEL = R$. §198.

Porro: $\angle ADM = R.$ p. C.

$AM^2 = AD^2 + DM^2$. §189. ſed

$AL^2 = LM^2 + AM^2$. p. D.

$AL^2 = LM^2 + AD^2 + DM^2$. §10 Ar.

ſed et $\angle DML = R.$ §408.

$DL^2 = LM^2 + DM^2$. §189.

$AL^2 = DL^2 + AD^2$. §10 Ar.

Ergo $\angle ADL = R.$ §198.

Simili diſcurſu evincam:

1) $\angle DFH = R.$

2) $\angle DEH = R.$

Quare cum

$\angle ADL = \angle DEH$. §92.

$\angle LAD = \angle HDE$. p. H.

$\angle ALD = \angle DHE$. §155.

ſed $AL = DH$ p. C.

$\left.\begin{array}{l} AD = ED \\ DL = EH \end{array}\right\}$ §114.

ſimiliter quia $\angle ACL = \angle DFH$. §92.

$\angle LAC = \angle HDF$ p. H.

$\angle CLA = \angle FHD$. §155.

Ergo cumque $AL = DH$. p. C.

$\left.\begin{array}{l} AC = DF \\ LC = HF \end{array}\right\}$ §114.

Porro: $AD = ED$ p. d.

$AC = DF$. p. d.

$\angle DAC = \angle EDF$. p. H.

$DC = EF$. §99.

$\left.\begin{array}{l}\angle o = \angle u \\ \angle x = \angle g\end{array}\right\}$ §c

Est vero et $\angle ADM = \angle DEK$ §92.

$\angle o = \angle u$ p. d.

$\angle CDM = \angle FEK$. §43 Ar.

Sed et $\angle ACM = \angle DFK$. §92.

et $\angle x = \angle g$. p. d.

$\angle DCM = \angle EFK$. §48. Ar

cumque $DC = EF$. p. d.

$\left.\begin{array}{l}DM = EK \\ CM = FK\end{array}\right\}$ §114.

Sed $AC = DF$. p. d.

et $\angle ACM = \angle DFK$. p. d.

$AM = DK$. §99.

Tandem quia

$DL = EH$. p. d.

Ergo $DL^2 = EH^2$ §175. G et §44 Ar.

Enimvero

$DL^2 = DM^2 + LM^2$. p.d.

$EH^2 = EK^2 + HK^2$. p.d.

$DM^2 + LM^2 = EK^2 + HK^2$. §41 Ar.

cumq; $DM = EK$. p.d. Ergo

$DM^2 = EK^2$. §175. G et 44 Ar.

$LM^2 = HK^2$. §43 Ar. Ergo

$LM = HK$. §197. cumq;

$AL = DH$. pr. C.

$AM = DK$. p.d.

$\angle LAM = \angle HDK$. §10 b.

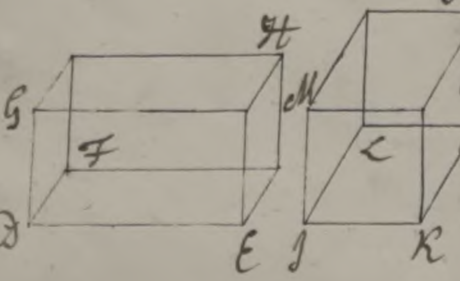
Q.E.D.

§473. Corollarium.

Quare, si sint duo Anguli plani aequales, quorum verticibus sublimes rectae Lineae aequales insistant, quae cum Lineis primo positis ∠los contineant aequales utrumque utrique; erunt a punctis extremis Linearum sublimium ad Plana Angulorum primo positorum demissae les inter se aequales, npe $LM = HK$.

§ 474. Theorema 150.

Si tres rectæ Lineæ DE, DG, DF proportionales fuerint, quod ex his tribus fit solidum Pppdm. DH, æquale est descripto a media Linea DG = IK solido Ppdo IN, quod æquilaterum quidem sit, æquiangulum vero prædicto DH.



Demonstratio:

Super IK = DG. p. C fac.

$\angle KIL = \angle EDF$ }
$\angle KIM = \angle EDG$ } §107.
$\angle LIM = \angle FDG$ }

Solid. $\angle$ IKLM = $\angle$ DEFG. §42. Ar. et 417.
Porro fac IM = IL = IK. §26. absolutoque solido Pppdo §138.
erit Solidum IN æquiangulum solido DH §169. 342.
Porro cum: ED : DG = DG : DF. p. H.
Ergo: ED : IK = IL : DF. §10 Ar.
cumque $\angle$ K = $\angle$ EDF. p. C.

Ergo $\angle$ K = FE. §369.
Sed et $\angle$ LIM = $\angle$ FDG } p. C.
$\angle$ KIM = $\angle$ EDG }
atque Lineæ DG, IM in sublimi posita ad Vertices D et I.
Angulorum KIL atque EDF æquales p. C.
Ergo:
Normalis ex G ad Plan. FE = Ili ex M ad LK §473.
Ergo
Altitudo Pppdi IN = Altitudini Pppdi DH. §126.

Pppdm IN = Pppdo DH. § 468. Q. E. D.

§475. Lemma 7.

Quantitatum proportionalium Potentiae eodem sunt et ipsae proportionales.

Demonstratio.

$$\left.\begin{array}{l} A : B = C : D \\ A : B = C : D \end{array}\right\} \text{p. H.}$$

$$A^2 : B^2 = C^2 : D^2 \text{ §189. et 225 Ar.}$$

$$A : B = C : D. \text{ p. H.}$$

$$A^3 : B^3 = C^3 : D^3 \text{ §§ cc.}$$

gnamus A duci in A vicibus m, et in B, C in C, D in D vicibus m ucendum est p. H. Quare cum

$$\left.\begin{array}{l} A : B = C : D \\ A : B = C : D \end{array}\right\} \text{p. H.}$$

$$A^m : B^m = C^m : D^m \text{ §§ cc.}$$

Q. E. D.

h. e.

Si $A : B = C : D$ erit $A^3 : B^3 = C^3 : D^3$ et in genere $A^m : B^m = C^m : D^m$

Ex. $1 : 2 = 3 : 6$

G. $1 : 8 = 27 : 216$

Hinc exponentes rationes ~~sun~~

$$\frac{1}{8} \quad \frac{27}{216} = \frac{1 \times \cancel{27}}{8 \times \cancel{27}}$$

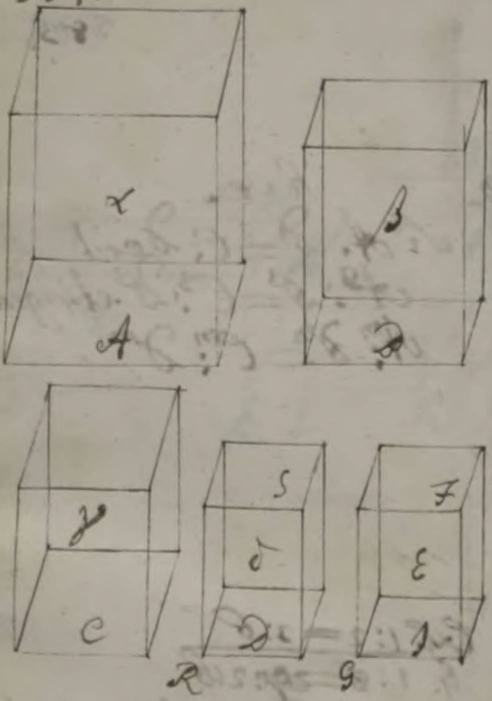
~~Ju [illegible]~~ 5. teln.

$$1 : 32 = 243 : 7776$$

$$\frac{1}{32} = \frac{243}{7776} = \frac{1 \times \cancel{243}}{32 \times \cancel{243}}$$

§476. Theorema 151.

Si quatuor rectae lineae A, B, C, D proportionales fuerint et solida pppda α, β, γ, δ similia similiterque descripta erunt proportionalia. Et contra: si solida pppda quae et similia sunt similiterque posita describuntur sint ppalia et ipsae Rectae ppales erunt.



Demonstratio

Descriptis super rectas ppales Parallelepipedis similibus similiterq; positis α, β, γ, δ. §461. erit

$\alpha : \beta = A^3 : D^3$ §468.

$\gamma : \delta = C^3 : D^3$ §c.

sed $A : D = C : d$ p. Hyp.

$A^3 : D^3 = C^3 : D^3$ §475.

$\alpha : \beta = \gamma : \delta$ §144. Ar.

Q.E.1.

Ad Rectas A, D, C quaere quartam ppalem d. §366.

Super inventam describe solidum Ppedm ε ∼le ∼literq; positum ipsi δ vel γ. §461.

Quia $A : D = C : d$ p. C.

Ergo $\alpha : \beta = \gamma : \varepsilon$ p. M. 1.

Sed $\alpha : \beta = \gamma : \delta$ p. Hyp.

$\gamma : \varepsilon = \gamma : \delta$ §144. Ar.

Ergo $\varepsilon = \delta$ §152. Ar.

Cumq; ε ∼le ∼literq; positum ipsi δ vel γ. p. C.

Planum GF ∼ et = Plano RS §445.

Ergo GF congruit RS. §88.

Ergo Recta d = Recta D. §87.

Ergo

$C : d = C : D$. §145 Ar. sed

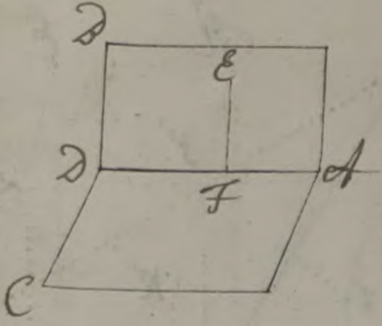
$A : D = C : d$. p. C.

$A : D = C : D$. §144 Ar.

Q.E.D.

§477. Theorema 152

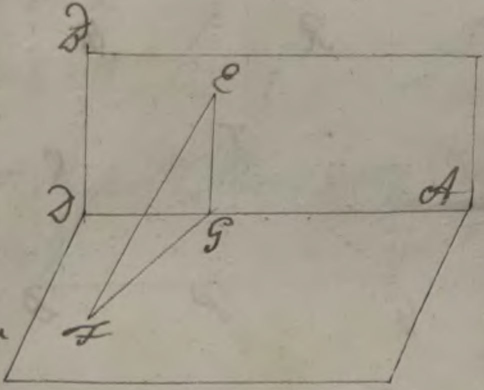
Si Planum AB ad Planum AC rectum fuerit, et ab aliquo puncto E eorum quae sunt in uno Planorum AB ad alterum Planum AC perpendicularis EF ducta fuerit, in Planorum communem sectionem AD cadet ducta perpendicularis Linea EF.



Demonstratio

Aut normalis EF cadet in communem utriq; Plano sectionem AD aut non cadet.

Ponamus non cadere. Punctu ergo F normalis EF extra Intersectionem AD in Planum AC cadet.



Demitte ex F ⊥lem ad AD np.

FG. §119.

et connecte GE. §81.

Quare cum

$\angle FGE = R.$ §408. 409. et

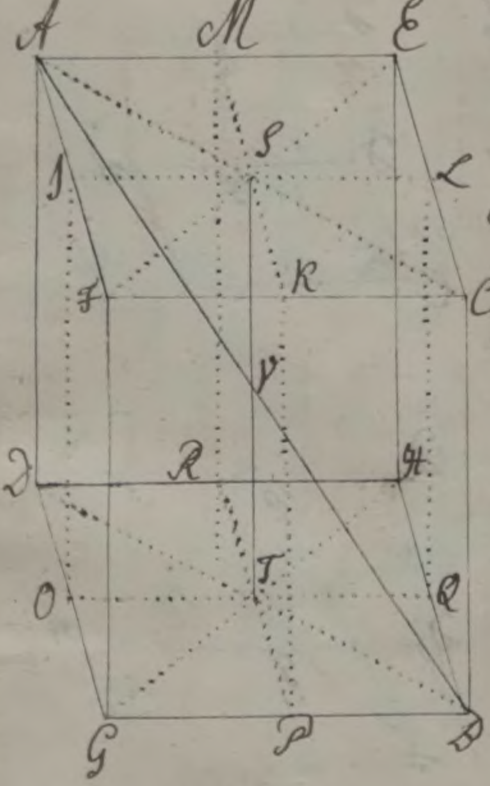
$\angle EFG = R.$ p. H. aſs.

$\angle FGE + \angle EFG = 2R.$ §42 Ar.

I. Q. E. A. §144.

§478. Theorema 153.

Si solidi Pppdi AB eorum, quo ea
adverso Planorum AC, DB; AH, FB,
AG, DE; latera AE, FC, pp bifariam
secta fuerint, per sectiones autem
Plana ILQO, et PKMR sint exte
sa, Planorum communis sectio
ST et solidi Pppdi Diameter
bifariam se mutuo secant.



Demonstratio

AB est Pppdum. p. H.
Ergo DG = et ≈ HB. §167.
OG = et ≈ QB §ca et H.

GB = et ≈ OQ. §139.

Similiter erit
OT = et ≈ GP
PB = et ≈ TQ, cumq;
PB = GP. p. H.

OT = TQ. §410 Ar.

Duc rectas TB, TH, TD, TG } §81.
item SA, SE, SC, SF }

Quia DG ≈ HB p. d.
∠DOT = ∠TQB §182.
sed OT = TQ p. d. H.
OD = QB p. H.

Ergo $DT = Td$ §99.

$\angle DTO = \angle QTd$. §c.

Sed $\angle OTP + PTd + dTQ = 2R$. §93.

Ergo $\angle PTd + OTP + DTO = 2R$. §10. Ax.

DTd est Linea recta §93.

Similiter demonstrabitur ASC esse Lineam rectam.

Porro $Ad =$ et $\approx Fb$ } §167.

$Cd =$ et $\approx Fg$ }

$Ad =$ et $\approx Cd$. §41 Ax. et 441 G.

$AC =$ et $\approx Dd$. §139.

DC est Pllgm §72.

Ergo S T itemque Ad sunt in eodem cum DC Plano §439.

cum itaque $DT = Td = \frac{1}{2} Dd$ } p.d.

$AS = SC = \frac{1}{2} AC$ }

$AS = Td$. §41 Ax.

$\angle AVS = \angle dVT$. §94.

cumque $AC \approx Dd$ p.d.

$\angle SAd = TdA$ §132.

$SV = VT$ } §114.

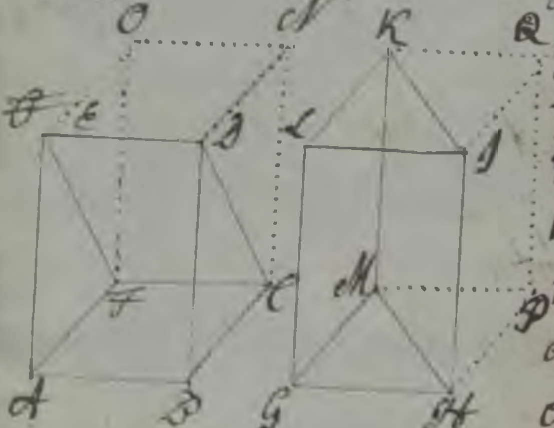
et $AV = Vd$ }

Q.E.D.

§479. Corollarium.

Hinc in omni Pppdo Diametri omnes se mutuo bifariam in unico puncto V intersecant.

388.



§480. Theorema 154.

Si fuerint duo Prismata ADC DEF et GHIKLM aequalis Altitudinis, quorum hoc quidem habeat Basin ADCF Pllgm, illud vero GHM Triangulum, duplum autem sit Pllgm ADCF Trianguli GHM aequalia erunt dicta Prismata.

Demonstratio.

Perfice Pppda AN et GQ §185.
quia Basis AC = GP p.C.
et Altit. Pppdi AN = Alt. Pdi GQ p.H.
Ergo AN = GQ. §465.
Ergo
Pris ADCDEF = Pr GHIKLM.
§466
Q.E.D.

§481. Corollarium.

Ex hactenus demonstratis elucet, quomodo dimensio Prismatum Triangularium et Quadrangularium fiat, ducendo Altitudinem in Basin.

Caput VII

De Pyramidum, Cylindrorum Conorum et Spherarum adfectionibus.

§482. Theorema 155.

Quae sunt in circulis ABD et FGI Polygona similia ABCDE et FGHIK inter se sunt ut Quadrata a Diametris descripta.

Demonstratio.

Duc AL, BL et FM, GM. §81.

$\angle ABC = \angle FGH$. p. H. et §341

et $AB : BC = FG : GH$ p. H. §c.

$\angle ACB = \angle FHG$. §356.

et $\angle ACB = \angle ALB$. §274.

et $\angle FHG = \angle FMG$. §c.

$\angle ALB = \angle FMG$. §41 Ar. sed

$\angle ABL = \angle FGM$. §288. 92.

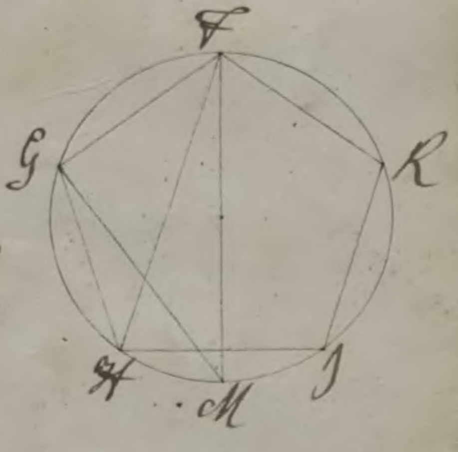
$\triangle ABL$ aequiang. $\triangle$lo FGM. §155. 305.

Ergo

$BA : AL = FG : FM$. §352.

et $BA : FG = AL : FM$. §150. Ar.

$ABCDE : FGHIK = AL^2 : FM^2$. §382.



§ 483. Corollarium

Hinc Polygonorum similium Circulo inscriptorum ambitus sunt, ut Diametri.

Nam quia

AB : FG = AL : FM
BC : GH = AL : FM
CD : HI = AL : FM
DE : IK = AL : FM
EA : KF = AL : FM

p. d. ad § 482.

AB + BC + CD + DE + EA : FG + GH + HI + IK + KF = AL : FM § 144. et § 165. Ar.

h. e.

Polyg. ABCDE : Polyg. FGHIK = AL : FM. § 4

Q. E. D.

§ 484. Theorema. 156.

Omnis Pyramis ABCD triangularem habens basin dividitur in duas Pyramides AEGH et HIKC, aequales et similes inter se triangulares habentes bases atque similes toti ABCD atque in duo Prismata aequalia BFGEHI, et FGCIHK quae duo Prismata majora sunt dimidio totius Pyramidis ABCD.

Demonstratio.

Dissectis omnibus Pyramidis ABCD lateribus in E, F, G, H, I, K §112. junge rectas EF, EI, EG, EH, FI, FG, FK, KI, KH, KG, [illegible], HG. §81.

Quia: $\underline{EH : HA = CI : IB}$ p.C.

$AB \approx HI$. §349.

$\underline{DG : GA = DF : FB}$ p.C.

$\underline{AB \approx FG}$. §340.

$FG \approx HI$. §441.

Similiter demonstratur:

$EH \approx FK$.

$HG \approx IF$.

$EG \approx IK$.

$HG \approx CD$

Quoniam itaque

$IH \approx BA$ } p.d.

$\underline{IK \approx BD}$ }

$\angle HIK = \angle ABD$ §442.

Sic et $\angle IHK = \angle BAD$ } §c.

atque $\angle HKI = \angle ADB$ }

$\triangle HIK$ aequilat. $\triangle$lo ABD. §305.

Porro:

EG ≈ BD p.d

∠AEG = ABD. §132.

∠AGE = ABD. §c.

Δ AEG aeq∠gl. Δ ABD §155. 305.

Sed Δ HIK aeq∠gl. Δ ABD. p.d.

Δ HIK aeq∠gl. Δ AEG. §410 Ar.

Porro:

AB ≈ IH p.d.

∠ABC = ∠HIC } §132.

∠BAC = ∠IHC }

Δ ABC aeq∠gl. IHC. §155. 305.

Sic et

BC ≈ EH p.d.

∠ABC = ∠AEH } §132.

∠BAC = ∠EHA }

Δ ABC aeq∠gl. Δ EHA §155. 305.

Δ IHC aeq∠gl. Δ EHA. §410 Ar.

similiter erit.

Δ KEH aeq∠gl. Δ AHG.

cumq. HG ≈ CD } p.d.

EG ≈ BD }

∠HGE = ∠CDB. §442.

et per similiter demonstrata: ∠HEG = ∠CBD.

Δ EHG aeq∠gl. Δ BDC. §155. 301.

similiter Δ CIK aeq∠gl. Δ BDC.

Δ EIK aeq∠gl. Δ EHG. §410 Ar.

Quare: AH : AE = HC : HI. §352.
Sed AH = HC. p.C.

AE = HI. §152. Ar. Sic et
AE : EH = IH : IC. §352.

EH = IC. §152. Ar.
Sed ∠AEH = ∠HIC. p.d.

△AEH = et ∼ △HIC. §§. 341. Similiter erit
△AGH = et ∼ △HKC
△AGE = et ∼ △HIK. } &c.
△EHG = et ∼ △CIK

Pyr. AEGH = et ∼ Pyr. HICK §415.
Pyr. AEGH ∼ Pyr. ADCB } §414.
Pyr. HICK ∼ Pyr. ADCB
Q.E.I.

Porro, quia HG, GE, EH ∥ CB, BD, DC. p.d.
Ergo etiam Altit. Prism. DFGEIH = Alt. Prism. FGBIHK.
quippe quae inter eadem Plana ∥la sunt.
basis DEGF, Pris. DFGEIH = 2 × bas. GFB, Prismatis FGBIHK p.d.

Prisma DFGEIH = Prism. FGBIHK. §480. Q.E.II.

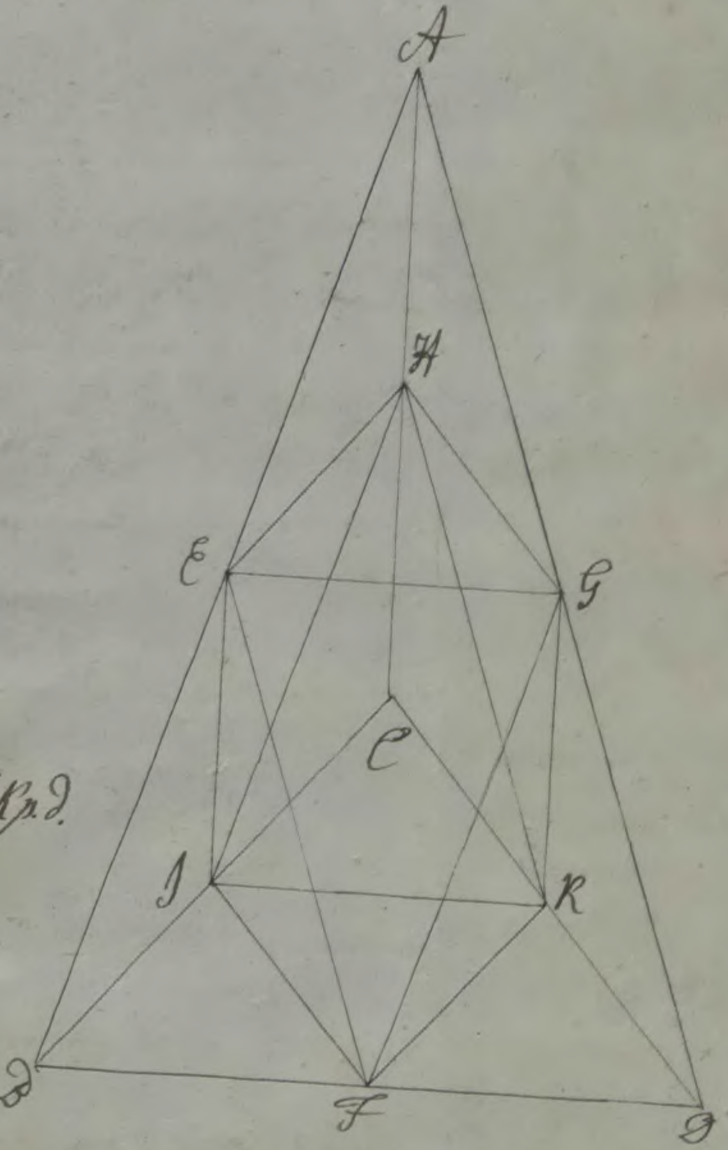
Tandem cum persimilem casus I. Demonstrationem
Pyr. EDFI = Pyr. AHEG
Pyr. FGKB = Pyr. HICK.

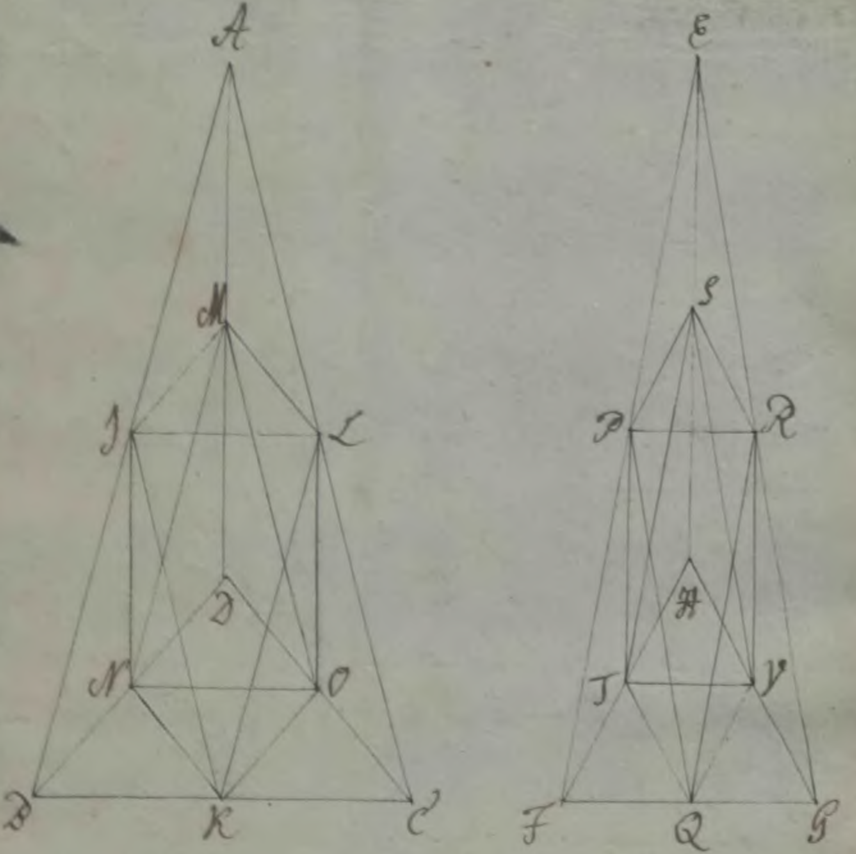
Pyr. EDFI + FGKB = Pyr. AHEG + HICK. §42. Ar.
Sed Pyr. EDFI ∠ Prism. DFGEIH } §42. Ar
Pyr. FGKB ∠ Prism. FGBIHK.

Pyr. EDFI + FGKB ∠ Prism. DFGEIH + FGBIHK. §42. Ar.

Pyram. AHEG + HICK ∠ Prism DFEGIH + FGBIHK. §41. Ar.
Q.E.D.

Fig. §484.





§ 485. Theorema 157.

Si fuerint duae Pyramides ejusdem altitudinis Triangularium Basium ABCD, EFGH, sit autem illarum utraq; divisa et in duo Prismata aequalia et in duas Pyramides aequales et similes Toti; ac eodem modo divisa sit utraq; Pyramidum AIML, MNDO, atq; EPSR et STHV, quae ex superiore Divisione natae sunt, idq; semper fiat, erit: Ut unius Pyramidis Basis ad alterius Pyramidis Basin, ita et omnia quae in una Pyramide Prismata ad omnia quae in altera Pyramide Prismata multitudine aequalia.

Demonstratio

Praeparatis omnibus ut § 484.

erit $BC : FG = \frac{BC}{2} : \frac{FG}{2}$ § 160. } Ar.

h.e. $BC : CK = FG : QG$ § 150 }

$\Delta ABC : \Delta LKC = \Delta EFG : \Delta RQG$ § 382.

¤ $\Delta ABC : \Delta EFG = \Delta LKC : \Delta RQG$ § 150. Ar.

¤ $\Delta LKC : \Delta RQG$ = Prism. LKCMNO : Pr. RQGTSV. § 471 N. 1.

sed Prism: LKCMNO = Pr. IBKLMN } p. H et 484.

Prism: RQGTSV = Pr. PFQRST }

Prism LCKMNO : Pr. RQGTSV = Pr. IBKLMN : PFQRST. § 145 Ar.

¤ Pr. LCKMNO : Pr RQGTSV = Pr LCKMNO + IBKLMN : Pr. RQGTSV + PFQRST. § 165. Ar.

$\Delta ABC : \Delta EFG$ = Pr LCKMNO + IBKLMN : Pr RQGTSV + PFQRST. § 144. Ar. Q. E. D.

Simili Discursu demonstrabo esse Prismata duo Pyramidis ADML ad Prismata duo Pyramidis EPR. sicut est Basis ADL ad Basin EPR, secta scil. utraque Pyramide ex Hypothesi Theorematis aut per §484.

etenim cum DC ≈ DL. p.d. ad §484.

$\triangle ADL \sim \triangle ADC$. §854. similique Discursu erit

& $\triangle EPR \sim \triangle EFG$.

Ergo $\triangle ADL : \triangle EPR = \triangle ADC : \triangle EFG$. §148.

$ADC : \triangle EFG =$ duo Prism. Pyr. ADML : duo Prism. Pyramidis EPRS p.d. ad M.1. et §144. Ar.

Ergo et

$\triangle ADC : \triangle EFG =$ Prism. LKCMNO + DIL KMN + duo Prism. Pyram. ADML : Prism. RQGTSV + PFQ RST + duo Prism. Pyram. EPRS. §144. atque 165. Ar. Q.E.N.D.

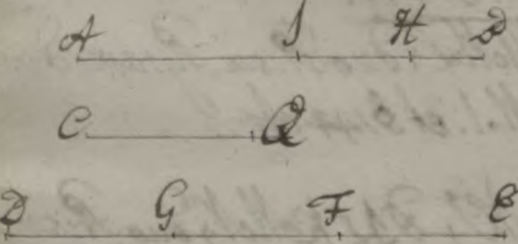
Idem cum simili modo pateat de Pyramidibus MNDC et STVA similiter per §484. sectis, atque ita semper deinceps, erit omnino Bas. Pyr. ADCD npe ADC ad Basin Pyramidis EFGH npe EFG = Omnia Prismata Pyramidis prioris ADCD : omnia Prismata Pyramidis posterioris EFGH. multitudine equalia §144 et 165. Ar.

Q.E.M. et D.

§486. Lemma 2.

Sint duae Magnitudines inaequales propositae, si a majore auferatur majus quam dimidium, et ab eo quod relictum est rursus detrahatur majus quam dimidium, hocque semper fiat, relinquetur tandem magnitudo quaedam, quae minor erit magnitudine proposita minore.

Demonstratio.



Sit Quantitatum propositarum major AB, minor CQ. multiplica CQ toties donec magnitudo quaedam DE fiat quae ipsa magnitudine AB proxime sit major et concipe magnitudinem DE in partes aliquotas DG, GF, FE, aequales ipsi CQ subdivisam esse. Jam ergo detrahe ex AB majus quam dimidium AI, et perge detrahendo ex reliquo IB majus quam dimidium np. IH, et ita porro, donec partes ipsius AB sint multitudine æquales partibus ipsius DE atq; inde liquet, dari duos Casus: aut enim DE ita se habet ut

1) Duplum ipsius CQ np. DG+GE; proxime excedat magnitudinem AB

2) Multiplum ipsius CQ np. DG+GF +FE proxime excedat magnitudinem AB.

Quare in Casu

1mo Quia $DE > AB$ p. C.

$\frac{DE}{2} > \frac{AB}{2}$ §45 Ar.

h. e. $GE > \frac{AB}{2}$.

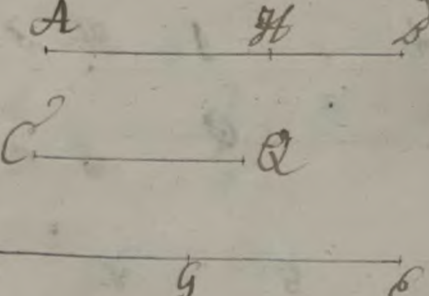
Sed $HB < \frac{AB}{2}$ p. H.

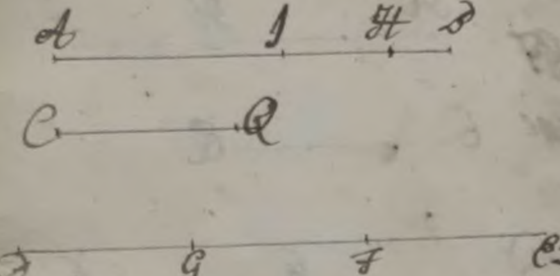
GE multu $> HB$.

Sed $GE = CQ$ p. C. et H. ad hunc Casum.

CQ multo $> HB$. §46 Ar.

Q. E. I.





2do $DE > AB$ p. C.

$\frac{DE}{2} > \frac{AB}{2}$ §45 Ar.

sed $AI > \frac{AB}{2}$ p. H.

et $AI + IB = AB$ §47 Ar.

$IB < \frac{AB}{2}$ §49 Ar.

$\frac{DE}{2}$ multo $> IB$.

$\frac{DE}{2} < GE$ p. C. et H adh. Cas.

GE multo majus IB cumque et

$\frac{GE}{2} > \frac{IB}{2}$ §45 Ar. hic.

$FE > \frac{IB}{2}$. sed

$HB < \frac{IB}{2}$ p. H.

FE multo $>$ HB. cumque

$FE = CQ$ p. C.

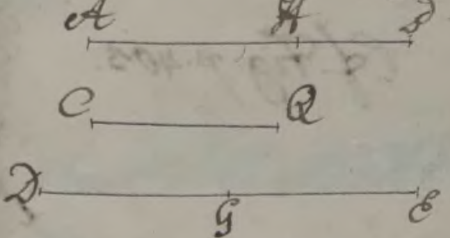
$CQ > HB$. §46. Ar.

Q. E. DD.

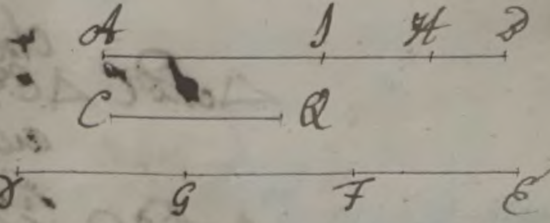
Idem simili prorsus discursu de omnibus multiplis Rectae DE ostendetur multitudine aequalibus ipsius Rectae AB multiplis, quo scil. in omni omnino Casu sit $HB < CQ$.

Sit pro Illustratione in Casu Speciali:

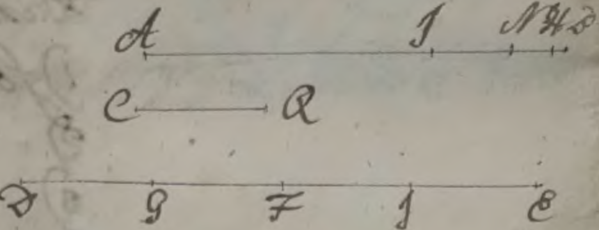
$AB = 8$. $CQ = 5$. Ergo
$DE = DG + GE = 10$
Pone $AH = 6$. Ergo $HB = 2$.
adeoq $CQ \; 7 \; HB$. h.e.
$5 \; 7 \; 2$.



Sit $AB = 12$. $CQ = 5$ erit
$DE = DG + GF + FE = 15$.
Pone $AI = 7$. Ergo $IB = 5$
pone $IH = 3$. Ergo $HB = 2$.
adeoq $CQ \; 7 \; HB$
h.e. $5 \; 7 \; 2$.



Sit $AB = 3$. $CQ = 1$. erit
$DE = DG + GF + FI + IE = 4$.
Pone $AI = 2$. Ergo $IB = 1$.
Pone $IN = \frac{2}{3}$ Ergo $NB = \frac{1}{3}$.
Pone $NH = \frac{3}{12}$ Ergo $HB = \frac{1}{12}$
Ergo $CQ \; 7 \; HB$
h.e. $1 \; 7 \; \frac{1}{12}$.



Similiter in aliis omnibus.

Cf. Fig. p. 403.

§487. Theorema 158.

Sub eadem Altitudine exiſtentes Pyramides ABCD, EFGH triangulares habentes baſes ABC, EFG ſunt inter ſe ut baſes ABC, EFG.

Demonſtratio.

Aut erit

$\triangle ABC : \triangle EFG = \text{Pyr.}\, ABCD : \text{Pyr.}\, EFGH.$

aut non erit:

Sit ergo

$\triangle ABC : \triangle EFG = \text{Pyr.}\, ABCD : \text{Sol.}\, x$

Dico ſolidum $x = \text{Pyr.}\, EFGH.$

Dantur autem tres Caſus aut

$$\left.\begin{array}{l} 1)\ \text{ſolidum}\ x < \text{Pyr.}\, EFGH \\ 2)\ \text{- - - - - -}\ x > \text{Pyr.}\, EFGH \\ 3)\ \text{- - - - - -}\ x = \text{Pyr.}\, EFGH \end{array}\right\}\ \S.\ 29.\ \text{Ar.}$$

Quare in

Caſu 1. Eſto

ſolidum $x < \text{Pyr.}\, EFGH$ ſolido quopiam y. Ergo

$x + y = \text{Pyram.}\, EFGH.$ §47 Ar.

Divide ergo Pyramidem EFGH in Pyramides et Priſmata et reliquas Pyramides ſimiliter in Pyramides et Priſmata et ita deinceps per §484.

401

constructionem ut tandem Pyramides quaedam duo EPδR. et δSTVH sint minores solido y. § 406. idquod fieri posse patet cum duo Prismata QFPTRS + GQRSTV 7ra Pyramidibus EPδR + δSTVH § 404. adeoq. plusquam dimidium subductam est per continuam similem subductionem ex Pyramide EFGH § 406.

Quare cum
Pyramis EFGH = solid. x+y. § 402 dr. et
Pyr. EPδR + δSTVH + Prism. QFPTRS + GQRSTV = Pyr. EFGH
Pyr. EPδR + δSTVH + Prism QFPTRS + GQRSTV = sol. x+y § 410 dr. &c.
Ergo sol. y. ind.
Sed Pyr. EPδR + δSTVH
Prismata QFPTRS + GQRSTV 7ra solido x. § 403 dr.

Concipe et alteram Pyramidem ABCD similiter esse subdivisam p. § 404 constructionem.

Quare cum utriusque eadem sit Altitudo p. H.

Ergo:

$\text{Pr. } DKLIMN + \text{Pr. } MLNKEO : \text{Pr. } FQRPTS + \text{Pr. } SRTQGV = \triangle ABC :$
$\triangle EFG$. § 486.

$\triangle ABC : \triangle EFG = \text{Pyr. } ABCD : \text{Sol. } X$ p. C.

$\text{Pr. } DKLIMN + \text{Pr. } MLNKEO : \text{Pr. } FQRPTS + \text{Pr. } SRTQGV =$
$\text{Pyr. } ABCD : \text{Sol. } X$. § 144. Ar.

Sed Prismata $DKLIMN + MLNKEO < \text{Pyr. } ABCD$. § 472 Ar.
Ergo Prismata $FQRPTS + SRTQGV < \text{Sol. } X$. § 132 Ar.
Sed Prismata $FQRPTS + SRTQGV > \text{Sol. } X$. p. d.
Quae sibi mutuo repugnant.

Patet ergo posita Hypothesi esse non posse: ut Basis ad Basin ita Pyramis prior ad solidum Pyramide posteriore minus. Q. E. I.

Casus 2dus

Sit solidum $X > \text{Pyr. } EFGH$.

Quia $\triangle ABC : \triangle EFG = \text{Pyr. } ABCD : \text{Sol. } X$. p. C.
Ergo $\triangle EFG : \triangle ABC = \text{Sol. } X : \text{Pyr. } ABCD$. § 126. Ar.
Fac $\text{Sol. } X : \text{Pyr. } ABCD = \text{Pyr. } EFGH : \text{Sol. } R$
Quare cum $\text{Sol. } X > \text{Pyram. } EFGH$ p. H. h. casus
erit $\text{Pyr. } ABCD > \text{Sol. } R$. § 132. Ar. et.

$\triangle EFG : \triangle ABC = \text{Pyr } EFGH : \text{Sol. } R$. I. Q. E. A. per Casum 1. quod demonstratum est fieri non posse ut sit basis ad basin, ita Pyramis prior ad solidum Pyramide posteriore minus.

Quoniam ergo
neq; ſolidum X < Pyr. EFGH p. d.
neq; ſolidum X > Pyr. EFGH
ergo ſolidum X = Pyr. EFGH. §3 Ar.
adeoq;
Δ ABC : Δ EFG = Pyr. ABCD : EFGH. §10 Ar.
Q. E. D.

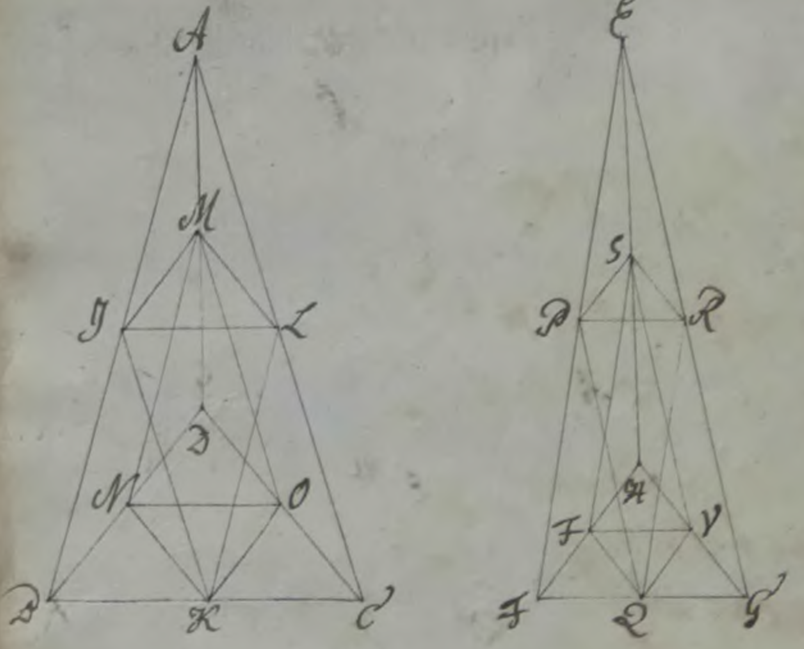
§488. Theorema 159.

Sub eadem Altitudine exiſtentes Pyramides ABCDEF et GHIKLM atq; Polygonas habentes Baſes ABCDE et GHIKL ſunt inter ſe ut Baſes. cf Fig. p. 404.

Demonſtratio.

Dantur duo Caſus aut enim ſub eadem Altitudine utraq; Pyramis
1) Multilateram numero æqualem
2) Multilateram numero tamen inæqualem Baſin habet.

Quare in Caſu 1ma divide Baſin multilateram per Diagonales in Triangula, ductis ꝑ AC, AD, itemq; GI, GK. §81 Hinc.



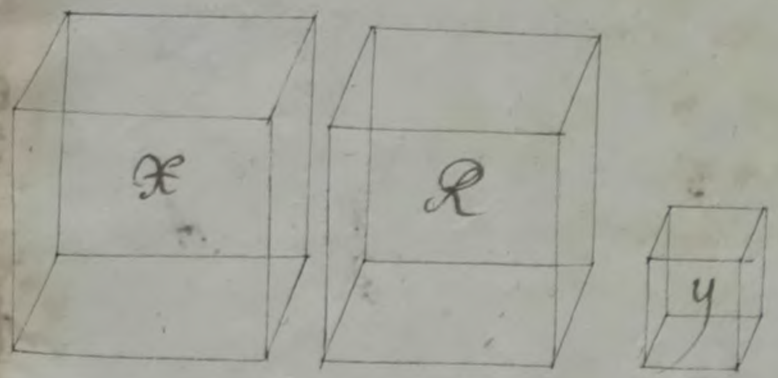
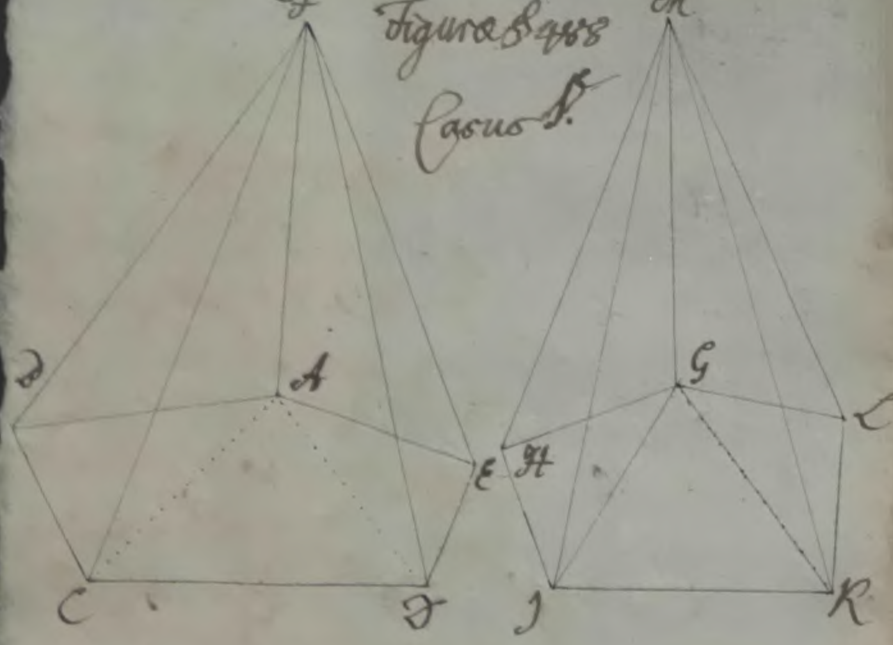
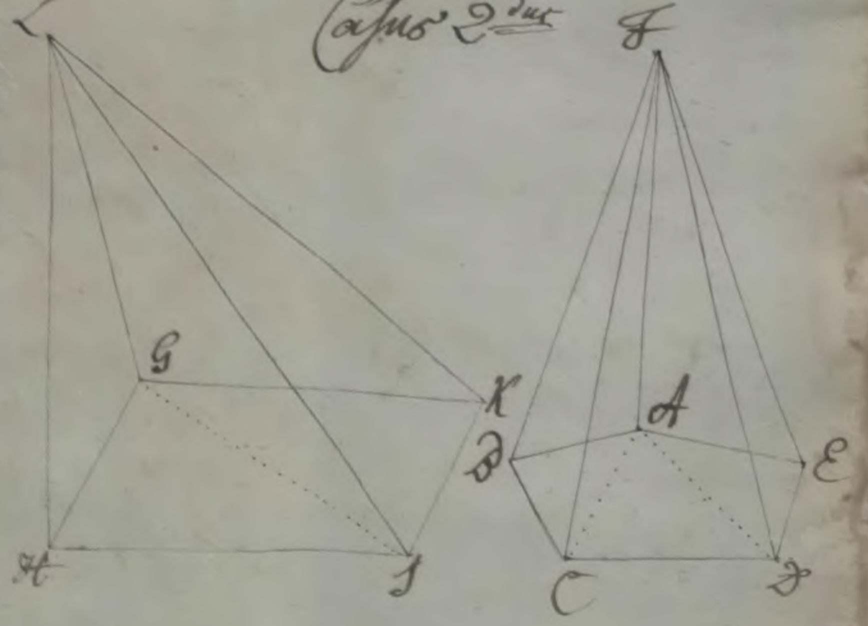


Figura §488

Casus 1.



Casus 2dus



Δ ABC : Δ ACD = Pyr ABCF : Pyr. ACDF. §487.

ΔΔ ABC + ACD : Δ ACD = Pyr ABCF + ACDF : Pyr. ACDF. §168 Ar.

h. e. ABCD : Δ ACD = Pyr. ABCDF : Pyr. ACDF. ſed et

Δ ACD : Δ ADE = Pyr. ACDF : Pyr. ADEF. §487.

ABCD : Δ ADE = Pyr. ABCDF : Pyr. ADEF. §172.

ſed ABCD + Δ ADE : Δ ADE = Pyr. ABCDF + ADEF : Pyr. ADEF. §168

h. e. ABCDE : Δ ADE = Pyr. ABCDF : Pyr. ADEF.

Idem ſimiliter in altera Pyramide oſtendam eſſe

GHIKL : Δ GLK = Pyr. GHIKLM : GKLM.

cumque utraque Pyramis ſit æqualis altitudinis

Δ ADE : Δ GLK = Pyr. ADEF : Pyr. GKLM. §487.

GHIKL : Δ ADE = Pyr. GHIKLM : Pyr. ADEF. §173 Ar.

ABCDE : GHIKL = Pyr. ABCDEF : Pyr. GHIKLM. §c.

Q. E. D.

Caſu 2do: Cum per modo demonſtrata ſit

ABCDE : Δ ADE = Pyr. ABCDEF : Pyr. ADEF.

atque per ſimiliter demonſtranda

GHIK : Δ GIK = Pyr. GHIKL : Pyr. GIKL.

adeoque ob eandem Altitudinem Pyramidum

Δ ADE : Δ GIK = Pyr. ADEF : Pyr. GIKL. §487.

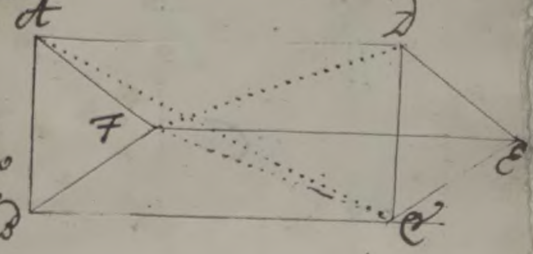
GHIK : Δ ADE = Pyr. GHIKL : Pyr. ADEF. §173 Ar.

ABCDE : GHIK = Pyr. ABCDEF : Pyr. GHIKL. §c

Q. E. D.

§489. Theorema 160.

Omne Prisma trigonum ABCDEF triangularem habens basin dividitur in tres Pyramides ACBF, ACDF, DCEF aequales inter se et triangulares bases habentes.



demonstratio.

Duc Pllgmorum BD, DE, EA diametros AC, CF, FD. §81.

demissa ergo ex Vertice F in quo Pyramides ABFC, et FCDA concurrunt in Planum baseos ipsum vel productum ⊥li, erit Altitudo Pyram. ABFC = Altit. Pyr. FCDA. §40 Ar.

Ergo

ΔABC : ΔACD = Pyr. ABFC : Pyr. FCDA §487.
Sed ΔABC = ΔACD. §169
Ergo Pyr. ABFC = Pyr. FCDA: §132. Ar.

Porro

demissa ex C ⊥li ad Planum AE
Altitudo Pyram. FCDA = Altit. Pyr. FCDE. §40 Ar.

Ergo

ΔFDA : ΔFDE = Pyr. FCDA : Pyr. FCDE. §487.
Sed ΔFDA = ΔFDE. §169.
Ergo Pyr. FCDA = Pyr. FCDE. §132 Ar.
Sed Pyr. ABFC = Pyr. FCDA. p. d.

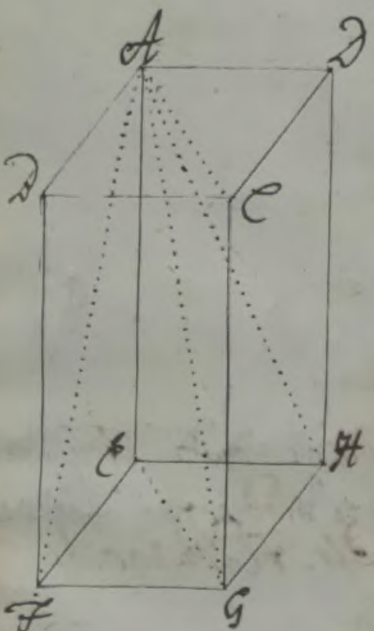
Pyramis ABFC = Pyr. FCDA = Pyr. FCDE. §41 Ar.

Q. E. D.

§490 Corollarium.

Inde quidem omnis Pyramis est tertia pars Prismatis eandem cum illa Basin et altitudinem habentis; s. qui. i. e. Prisma quodvis triplum est Pyramidis eandem cum ipso habentis et Basin et Altitudinem. Nam:

Resolve Prisma Polygonum in trigona et Pyramidem Polygonam in Trigonas ductis Diagonalib



§81.

Ergo:

Pyr. EFGH = $\frac{1}{3}$ Pris. ADCEFG §

Pyr. EFGHA = $\frac{1}{3}$ Pris. ACDEGH

Pyr. EFGHA = $\frac{1}{3}$ Pris. ADCEFG + $\frac{1}{3}$ Pris. ACDEGH. §42. A

= $\frac{1}{3}$ × Pris. ADCEFG + Pris. ACDEGH. §31 A

= $\frac{1}{3}$ Prismatis ADCDEFGH. §47. Ar.

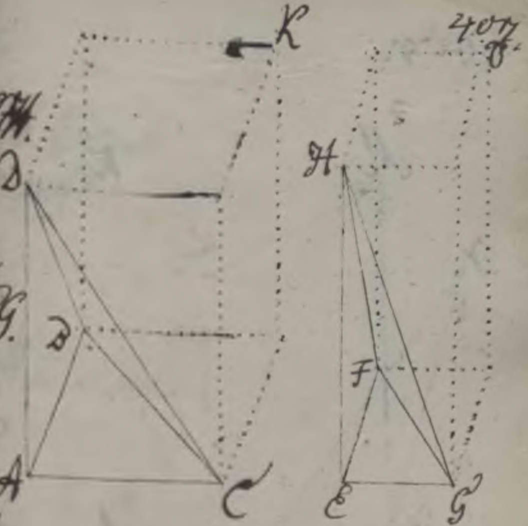
adeoque etiam

3 × Pyram. EFGHA = Prism. ADCDEFGH. §44

Q. E.

§ 491. Theorema 161.

Similes Pyramides ABCD et EFGH. Basium triangularium ABC atque EFG sunt in triplicata ratione Laterum homologorum AC et EG.



Demonstratio.

Perfice totum Prpdm AK item que Prpdm EO § 461.

Quia Pyr. ABCD ∼ Pyr. EFGH. p. H.

Ergo

Pyr. ABCD : Pyr. EFGH = AK : EO. § 440 Ar.

Sed AK : EO = $AC^3 : EG^3$ § 468.

Pyr. ABCD : Pyr EFGH = $AC^3 : EG^3$ § 410 Ar.

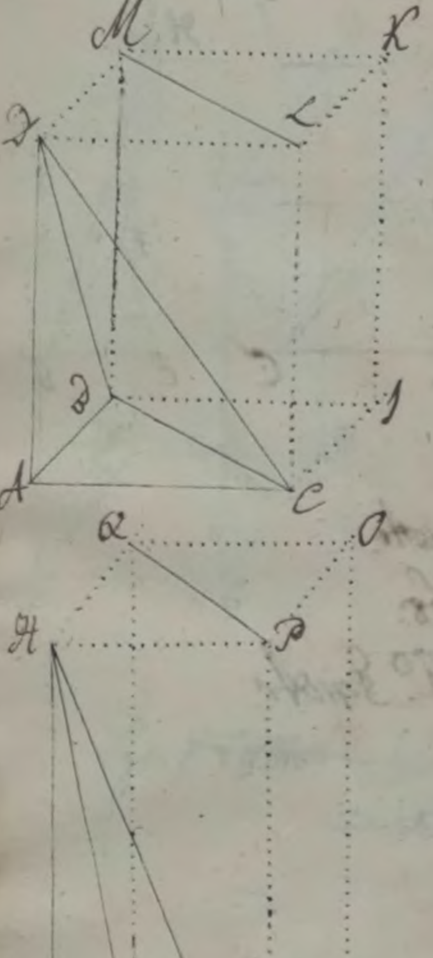
Q. E. D.

§ 492 Corollarium

Ergo etiam similes Pyramides Basium multilaterarum sunt in ratione triplicata homologorum Laterum, reduci enim possent ad Pyramides trilateras.

§ 493 Theorema 162.

Aequalium Pyramidum ABCD et EFGH atque triangulares Bases habentium ABC et EFG reciprocant Bases et Altitudines. Et contra:



Quarum Pyramidum Basium Triangularium reciprocantur Bases et Altitudines, illae sunt aequales. Demonstratio.

Perfectis Pppdis AK et EO. §461. duc Plana diagonalia ML, QG.

Quia Pyr. ABCD = $\frac{1}{3}$ ABCDLM §489

et ABCDLM = $\frac{1}{2}$ AK §462.

Ergo Pyr. ABCD = $\frac{1}{3} \times \frac{1}{2}$ AK §100 Ar.

= $\frac{1}{6}$ AK adeoque

6 × Pyr. ABCD = AK. §44. et 460 Ar.

similiter erit:

6 × Pyr. EFGH = EO.

Sed Pyr. ABCD = Pyr. EFGH. p. H.

Ergo 6 × Pyr. ABCD = 6 × Pyr. EFGH, §44 Ar.

h. e.

Pppdm. AK = Pppdo EO. §41 Ar.

Ergo:

AI : EN = EH : AD §470.

AI : EN = $\frac{1}{2}$ AI : $\frac{1}{2}$ EN. §160 Ar.

$\frac{1}{2}$ AI : $\frac{1}{2}$ EN = △ABC : △EFG. §145 Ar. et 169. G.

△ABC : △EFG = EH : AD. §144 Ar.

Q. E. D.

$\triangle ABC : \triangle EFG = EH : AD$. p. H.

$2\times\triangle ABC : 2\times\triangle EFG = \triangle ABC : \triangle EFG$. §15 Geotr.

$2\times\triangle ABC : 2\times\triangle EFG = AI : IM$ §170 Ar.

$AI : EN = EH : AD$. §14.

Ergo et $AK = EO$. §470.

$$\left.\begin{array}{l} AK = 6\times \text{Pyr. } ABCD \\ EO = 6\times \text{Pyr. } EFGH \end{array}\right\}\text{ p. d. ad M. 1.}$$

$6\times$ Pyr. $ABCD = 6\times$ Pyr. $EFGH$.

Ergo et Pyr. $ABCD =$ Pyram. $EFGH$.

Q. E. D.

§494 Corollarium 1.

Quae de Pyramidibus demonstrata sunt §488. 491. 493. conveniunt etiam quibuscunque Prismatibus polygonarum basium utpote triplis ipsarum Pyramidum eandem basin atque altitudinem habentium §489. 490. h. e.

1) Prismatum aeque altorum eadem est quae basium proportio.

2) Similium Prismatum Proportio est triplicata Proportionis homologorum laterum.

3) Aequalia Prismata reciprocant bases et Altitudines, et quae reciprocant, sunt aequalia.

§495. Corollarium 2

Quae §495. demonſtrata ſunt, valent etiam de Pyramidibus baſium multilaterarum, reduci enim poſſunt ad trilateras.

§496. Scholion.

Patet etiam ex hactenus demonſtratis Dimenſio Pyramidum atque Prismatum omnium.

Producitur enim ſoliditas

1) Prismatis ex Altitudine ducta in Baſin §494. N.1. §481.

2) Pyramidis ex tertia Altitudinis parte in Baſin §489. 490. vel ex tertia Baſis parte in altitudinem §186. Ar.

§497. Theorema 163.

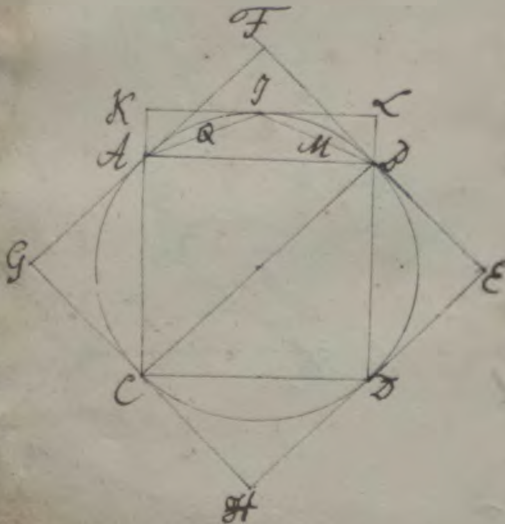
Polygona Circulo in infinitum inſcripta in Circulum deſinunt.

Demonſtratio.

Inſcribe et circumſcribe Circulo Quadratum §318. 319.

Cum ergo Latera Quadrati circumſcripti Circulum tangant §301. adeoque tota extra eundem cadant

Ergo Circulus = Parti Polygoni circumſcripti.



Ergo GFEH $>$ Circulo ABDC. §12 Ar.
Sed GFEH = 2x Quadr. ABCD. §321.

2x Quadratum ABCD $>$ Circulo,
Quadratum ABCD $>$ $\frac{1}{2}$ Circulo
Diſectis ergo Arcubus AB, BD, DC, CA. §281.
Inſcribe Circulo Octogonum §340.
ducta per I tangente §158. 268.
productisq CA et DB ad concurſum cum illa in K et L. §82
erit KL = AB. §284.
Ergo 2x $\triangle$ ABI = AL. §181.
2x $\triangle$ ABI = ABI + AKI + ILB
$\triangle$ ABI = ABI. §40 Ar.

$\triangle$ ABI = $\triangle$ AKI + $\triangle$ ILB
Sed $\triangle$ AKI $>$ Segmento AIR.
et $\triangle$ ILB $>$ Segmto IMB

$\triangle$ AKI + $\triangle$ ILB $>$ Sgmtis AIR + IMB.

$\triangle$ ABI $>$ Sgmtis AIR + IMB.
Similiter oſtendetur reliqua ſegmenta omnia ex Arcubus et ſubtenſis Octogoni circulo inſcripti facta eſſe minora Triangulis ex ſubtenſis illis productis. Pari argumentatione ſi in Arcuum Octogoni Diſectione pergatur.

atq; ita deinceps, Triangula inde facta majora erunt segmentis ad Triangula ista pertinentibus.

Quare, ablato Quadrato ABCD atq; Triangulis illis AEB &c. ea Circulo ABCD majus semper aufertur, quam dimidium,

Ergo, minus tandem relinquetur Quantitate minore §486.

h.e.

Polygonum inscriptum a Circulo deficiet Quantitate minore quacunq; data, s. q. i.e. in Circulum desinet. Q.E.D.

cf. And. Tacquet in Lemmate ad Propos. II. L. VIII. Eucl. p.m. 261.

§498. Corollarium I. Q.E.D.

Cum ergo Polygona in infinitum Circulo inscripta in Circulum desinant, Circulus pro Polygono ordinato infinitorum laterum habetur.

§499. Corollarium 2.

Quare, cum Polygona similia Circulis inscripta sint inter se ut Quadrata Diametrorum §481.

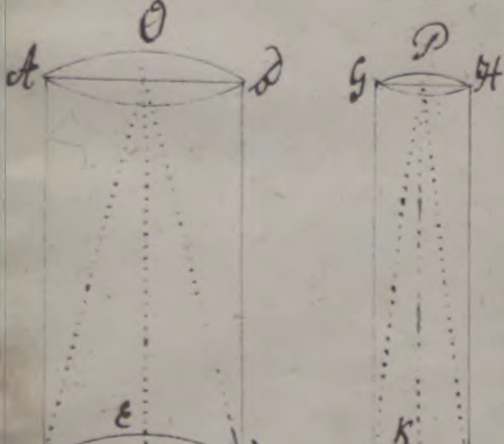
Omnes autem Circuli sint similes
§78. 79. 23. et Polygona laterum infi-
nitorum constituant. §498.
Ergo
Circuli sint inter se ut Quadrata
Diametrorum.

§500. Corollarium 3.
Ergo Cylindrus pro Prismate infi-
nitangulo §419. 497.
Conus autem pro Pyramide in-
finitangulo §422. 497. haberi po-
test atq; inde patet utriusq; dimen-
sionem fieri per §498.

§501. Corollarium 4.
Ergo Conus est tertia Pars Cylin-
dri ejusdem cum illo Baseos et
Altitudinis. §490.

§502. Theorema 164.
Cylindri atq; Coni sunt in Ratio-
ne composita Basium atq; Alti-
tudinum. Demonstratio.

Membrum I.
Quia Cylindrus pro Prismate
infinitangulo haberi potest §500.



Prisma vero componitur ex Basi ducta in Altitudinem. §496. Ergo Cylindrus componitur ex Basi ducta in Altitudinem; h.e. Cylindrus est in Ratione composita Basis atq; Altitudinis. §142. Ar.

Membrum 2.

Eadem fere Demonstratione liquet

§503. Corollarium:

Hinc sub eadem Altitudine existentes Cylindri sunt inter se ut Bases §494. N.1 cumq; Cylindrorum Bases sint Circuli §424 hi vero Rationem habeant eam, quam Quadrata a Diametris descripta §499. h.e duplicatam Diametrorum §147. Ar. Ergo sub eadem Altitudine existentes Cylindri sunt in duplicata Ratione Diametrorum Basium §144 Ar.

h.e. Esto AC = GI. Ergo

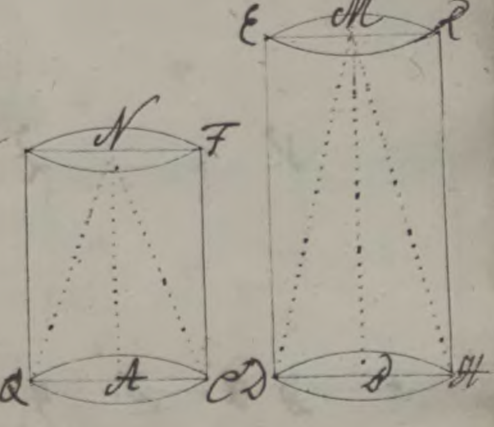
CB : IH = CEDF : IMLK. §494. et 500.

CEDF : IMLK = CD² : IL². §499.

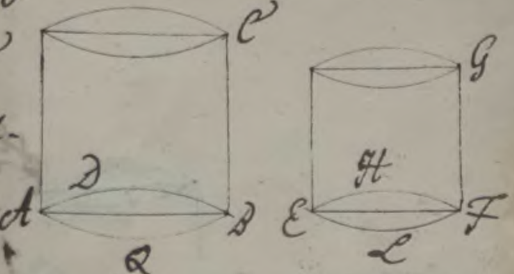
CB : IH = CD² : IL². §144 Ar.

§504 Corollarium 2.
Idem de Conis similibus eousque evincitur per §488.

§505 Corollarium 3.
Liquet etiam Cylindros atque Conos sub basibus aequalibus esse inter se ut Altitudines. Nam:
Sit A = B. p. H. Ergo HR.
$QF : DR = A \times QF : HR$. §160 Ar.
Idem similiter ostenditur de Conis QNC atque DMH.



§506 Corollarium 4.
Quodsi ergo Cylindris Altitudo Basium Diametris aequalis contigerit, erunt in triplicata ratione Diametrorum Basium. h. e.



Sit $CB = AB$ et $EF = GF$.
Ergo
$AC : EG = AQDB \times CB : ELFH \times FG$. §502
sed $AQDB : ELFH = AB^2 : EF^2$. §499.
et $CB : FG = AB : EF$ §100 Ar.
$AQDB \times CB : EFLH \times FG = AB^3 : EF^3$. §165 Ar.
$AC : EG = AB^3 : EF^3$. §144 Ar. Q.E.D.

§507. Theorema 165.

Aequalium Cylindrorum et Conorum reciprocantur Bases et Altitudines et contra:

Quorum Cylindrorum et Conorum Bases et Altitudines reciprocantur illi sunt inter se aequales.

Demonstratio.

Dantur duo Casus aut enim

I Altitudines sunt aequales, adeoque et Bases aequales erunt. Cum enim

$VD = ZY$ p. H.

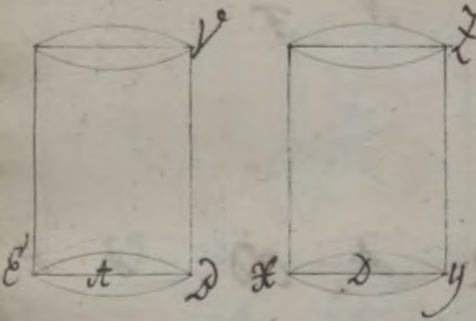
Ergo $EV : XZ = A : D$. §503.

Sed $EV = XZ$ p. H.

Ergo $A = D$. §132. Ar.

Ergo $A : D = ZY : DV$. §126 Ar.

Q. E. I.



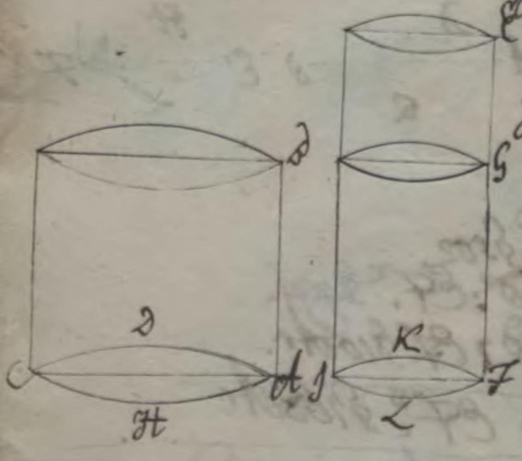
II. Altitudines sunt inaequales

Membr. 1.

Aufer ergo minorem AB a majore EF ut sit

$AB = GF$.

Quarecum per §40. Ar



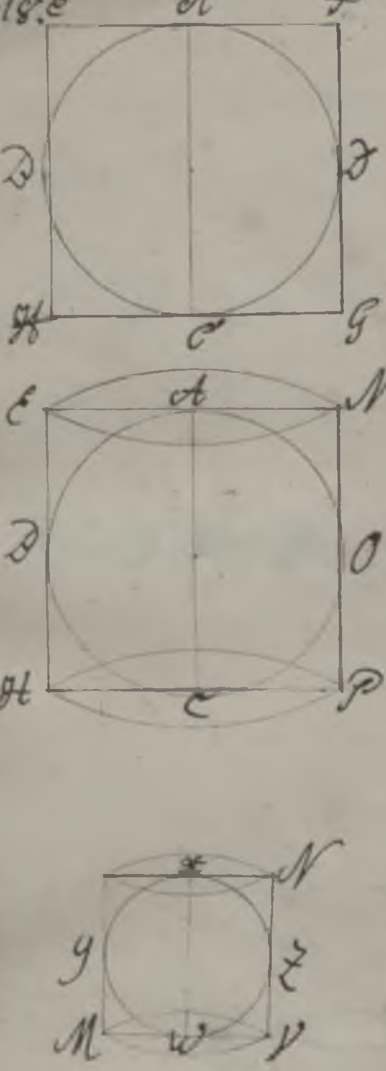
Basis IKLF = Basi IKLF.
IE : IG = EF : FG. §505.
Altit. FG = Altit. DA. p. C.
CD : IG = CDHA : IKLF. §503.
sed CD = IE. p. H.
EF : DA = CDHA : IKLF. §40 Ar. et 144 Ar.
Q. E. I.

Membrum 2.
∴ EF : DA = CDHA : IKLF. p. H.
DA = FG. p. C.
∴ CD : IG = CDHA : IKLF. §503.
IKLF = IKLF. §40 Ar.
∴ EI : IG = EF : FG. §505.

CD : IG = EI : IG. §144 Ar.
CD = EI. §152
Q. E. I. D.

Idem de conis evincetur, quippe qui tertiam cylindrorum suorum partem constituunt §501.
Q. E. D.

418.



§508. Theorema 166.

Sphæræ sunt in triplicata ratione diametrorum.

Demonstratio.

Sit circulo ABCD quadratum EFGH circumscriptum §319. Concipe semicirculum ADC cum dimidio quadrato EACH circum axem communem AC in orbem moveri; illa sphæram ABCD §420. hoc autem cylindrum ENPH §424. describet, ita ut altitudo sit basis diametro æqualis.

Pone alium semicirculum WYX et dimidium quadratum MWXY illi circumscriptum §319. sphæram §420 XYWZ atque cylindrum MNV §424. describere.

Quia HC : CA = 1 : 2 } p. C.
et MW : WX = 1 : 2 }

HC : CA = MW : WX §144 ar.
Ergo Rect. HCA ∾ Rect. MWX. §341.
Ergo et Cyl. HCN ∾ Cyl. MNV. §518. 519.
Pari ratiocinio cum omnes circuli

sint inter se similes §78. 79.
producti scil. ex Motu recto cir-
ca punctum fixum §23.
adeoque et semicirculi p. dem. ad §84.
Ergo Sphaera ADCO eodem modo
determinatur quo Sphaera XY
WZ §78. Ergo.

$ADCO : XYWZ = XN : MN$ §148 Ar.

$XN : MN = XP^3 : MV^3$ §80 b.

$ADCO : XYWZ = XP^3 : MV^3$ §144 Ar.

Sed $XP = XE = AC$ p. dem. ad §319.

Ergo $XP^3 = AC^3$ §4 Ar.

Similiter $\left.\begin{array}{l} MV = WX \\ MV^3 = WX^3 \end{array}\right\}$ §. § cc.

Sph. $ADCO$: Sph. $XYWZ = AC^3 : WX^3$ §10 Ar.

Q. E. D.

Finis Geometriae Theoreticae.

Theoreticæ Geometriæ Exercitaturis
Signum erit d vel θ.

Q. D. D. V.

Elementa Geometriæ Practicæ

Caput I.

De linearum et Angulorum Dimensione atque Constructione.

§.1. Definitio I.

Mensura Linearum est Recta longitudinis arbitrariæ in minores partes pro lubitu dividenda et subdividenda. Hodie a Practicis Geometris in 10 æquales partes, quæ pedes vocantur, unde integra illa Recta et Decempeda audit; Pes in 10 Partes, Digitos dictos; Digitus in 10 alias æquales, Linearum nomine notas, subdividitur.

§2. Scholion.

Definivimus §1. mensuram Geometricam, quam magno calculi compendio primus Simon Stevinus introduxit. Differt autem illa Mensura a plurimarum Gentium divisionibus. Sic enim quamlibet propoſitam menſuram v. c. Pariſinam aut Auſtricam in decem æquales Partes divideret, ſubdividere ſit longe expeditiſſimum. §364. & ut hoc modo tota Pariſina Longitudo aut Auſtriaca Geometricæ noſtræ ſit æqualis, ſubdiviſæ tamen Partes noſtræ decimales non æquabunt illas Partes cum illa quidem in 12 hæc in 16 etc. Minores particulas abeat, utrinque noſtris decimalibus minores. Quamobrem omnino conſulendi qui varias menſurarum differentias expoſuerunt. Quorum in Numerum referendi ſunt præter Auctores a Wolffio Sel. Geom. excitatos Doegen in Architectura Militari, Schwenterus in Geometria practica, Claudius Perault in Tractatu de Arte Libellandi gall. quem Wolffius in Tabula mox exhibenda ſequitur.

Simieno Wietz in Artelleria, Mallet in Geom. pract. Johan Georg Trutmann in Geometria repetita p. 279–288. Dantisc. 1739. 8. Andr. Celsius cujus Excerptum dant die Hamburgischen Beyträge zum Aufnehmen der Gelehrten Historie und Wissenschaften ad Ann. 1740. p. 607. seqq. aliiq. plures minore tamen adcuratione usi.

En potissimorum Europae Pedum Tabulam ex Wolfio l.c. Dublium Partium est.

Parisinus . . .	1440 talium
est Rhenanus . .	1391 $\frac{3}{10}$
Romanus . .	1320.
Londinensis . .	1350
Suecicus . . .	1320
Danicus . . .	1403 $\frac{2}{5}$
Venetus . .	1540
Cstanus . .	3140
Bononiensis . .	1682 $\frac{2}{5}$
Argentinensis . .	1282 $\frac{3}{4}$
Norimbergensis .	1340 $\frac{3}{4}$
Dantiscanus .	1271 $\frac{1}{2}$
Halensis . .	1320.

Hic quoque Diſcrimen notes velim
Perticam inter atque Decempeda
Perticam dico Menſuram civilem
maximam agrimenſoriam in 1
bus aequales partes diviſam et
ſubdiviſam: Decempedam autem
menſuram eandem quidem ma-
ximam agrimenſoriam, ſed ſe
cundum rationem decuplam di
viſam et ſubdiviſam. Sit v.c. Pes
Pariſinus in 12 partes aequales ſu
diviſus duodecim pedes hoc mod
diviſos dicam Perticam Pariſi-
nam. Sit autem idem pes Pari-
ſinus in 10 aequales partes ſubdivi
ſus decem pedes m. diviſos dica
Decempedam Pariſinam. Simili-
ter in reliquis.

§3. Hypothesis.

Perticas itemq; Decempedas
ſignificabimus
circello ſuperius adſcripto: °
Pedes commata uno, '
Digitos commatibus duobus. ''

Lineas tribus: ''' numerorum Apicibus et extrorsum adscriptis sic 7° 9' 3'' 2''' significat septem Perticas vel Decempedas, novem Pedes, tres Digitos atque duas Lineas.

§ 4. Problema 1.

A dato puncto ad datum Pctm rectam Lineam ducere.

Resolutio.

I In Charta

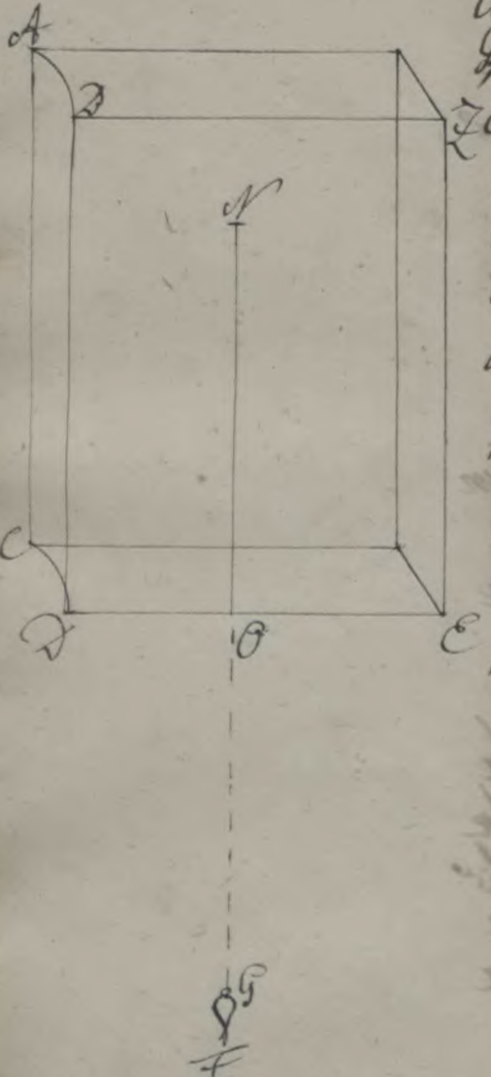
Recta linea ducitur secundum Regulam ad data duo Puncta applicatam Graphio, Penna aut Plumbagine.

II In ligno vel Saxo.

Funem vel filum Cerussa, Creta vel alio Colore delibutam in extremis Rectae describendae Punctis applicatum extende, medium illius digito prehensum eleva moxque demitte atque Cerussae aut Cretae Vestigia Lineam designabunt.

III In Campo.

Ducitur Recta Baculorum ope ad Horizontem normaliter



infixorum. Per Definitionem Recte Platonis §15. Θ. Quo vero Baculi normaliter infigi possent, utantur Cippo ligneo cujus alterum Planum ADCD secundum superficiem Baculorum excavatum est, alterum autem BDEZ normalem habet NO in cujus extremo N perpendiculum F suspenditur ope fili NG, id quod congruere debet normali NO cujus Operationis Ratio pendet a §440. Θ. Ut autem extremitates Rectarum longiorum videri possint, summitatibus Linteum album Vexillum vel Charta nitidior applicatur.

Vid. Malletum T. I. Geometr. pract. add. Pentherum et Leutmannum in Geometr. pract. qui Baculorum summitates vexillis versicoloribus ornant, quodquod et ipsos nigro alboque colore tinctos esse volunt.

§5. Problema II.

Scalam Geometricam construere.

Resolutio.

1) Duc Rectam infinitam AF, inq; illa abscinde 10 partes aequales decem harum partium intervallum h.e. AB transfer ex D in C et C in D quoties libuerit in Recta AF.

2) In A excita ∠lem §§158 G arbitrariae Longitudinis AH illamq; in decem aequales partes subdivide.

3) Per Divisionum Puncta 9, 8, 7, 6 ff age ∥las cum AF. §135 G.

4) In ultimam illarum HF transfer 10 partes partibus Rectae AB aequales.

5) Tandem Puncta: gA; 8, 9; 7, 8; 6, 7; 5, 6; ff junge transversis item q, CD, BC, CD, AF 9 ff §81 G. vel per §135 G. D. F.

Demonstratio

D1 = 1,2 = 2,3 = 3,4 ff = $\frac{1}{10}$ AB

D. C.

assumendo itaq; AB pro Decempeda erit

D1. pes unus

D2 pedes duo

D3 pedes tres ff §1.

Q. E. I.

$\alpha\beta \approx D1$. p. C. Ergo

$\angle C\alpha\beta = \angle CD1$. §132. G.

$\angle 1CD = \angle 1C\alpha$. §40. Ar.

$\triangle C\alpha\beta$ simil. $\triangle 1DC$. §155. 305. G.

Ergo

$CD : D1 = C\alpha : \alpha\beta$. §352. G.

$CD : C\alpha = D1 : \alpha\beta$ §150. Ar.

sed $CD : C\alpha = 1 : \frac{1}{10}$ p. C.

$D1 : \alpha\beta = 1 : \frac{1}{10}$ §144. Ar.

Similiter ostendetur.

$\varepsilon\zeta = \frac{4}{10}$ ipsius $D1$.

$\gamma\delta = \frac{9}{10}$ ejusdem $D1$. etc.

$\alpha\beta =$ uni
$\varepsilon\zeta =$ quatuor } Digitis §1.
$\gamma\delta =$ novem } Q. E. D.

§. 6. Scholion 1.

Quodsi et lineas desideres assumenda est AD pro pede, inde per modo demonstrata $D1, 4, 5, 2, 3$, etc. erunt Digiti: Lineae autem $\alpha\beta$, $\varepsilon\zeta$, $\gamma\delta$ atque decuplum ipsius AD hoc est AW decempedam constituet per §1.

§7. Scholion 2.

Similiter datis ſubdiviſionibus menſuræ cujusdam civilis conſtruetur Scala illius AB vel AH ſubdividendo in 12 aut 10 particulas pp æquales ut inde Scala Rhinlandica aut Norica fiat cf. §.2.

§8. Problema III.

Lineam rectam propoſitam metiri.

Reſolutio.

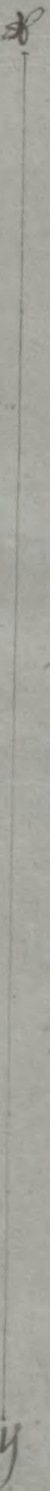
I In Charta. Eſto Recta data CM.

1) Detoccepta Circino Rectæ datæ extremum alterum C, ita applica in Punctis Rectæ ED, AC, ED, AH pp Scalæ §5 conſtructæ ut alterum Pctm M in Pcto quodam Linearum transverſarum E1; 1,2; 2,3; 3,4 pp terminetur h.e. ut data Recta uni illarum congruat.

2) Numeratis partibus iſtis.

D. 7. p 8 9 0

eg. ſunto AB = BC = CD pp Pedes applicato Pcto M in Recta ED



Puncto k, apparet alterum extremum pertingere usq; in i, Lineæ transverſæ 8, 4: erit ergo.

ik = LM. §90 et 81 θ.
ſed ik = 2' 3'' 8''' per Obſerv. et 83.
LM = 2' 3'' 8'''. §40 Ar.

Similiter
Recta xy altero ſui Extremo ſe condet in Recta ML Pctm q; altero in transverſa Linea 8, 9; Pctm o: cum ergo
qo = 5' 8'' 5''' Ergo
xy = 5' 8'' 5'''. §§ c.c.

II In Campo.

1) Erectis in utroq; Lineæ menſurandæ Extremo Baculis, plures intermedios ſi longior fuerit, ſtatue, in eadem Recta quod fiet, ita ſi collocaveris intermedios, ut ab extremorum alterutro obumbrentur §15. θ

2) Catenam ita quidem extende ut duos proximos Baculos aut plures intermedios ad ∠los R. ſecet, id quod applicando Perpendiculum §4. deſcriptum innoteſcet.

3 Numera Decempedas vel Perticas, Pedes, Digitos atque Lineas, factumque erit.

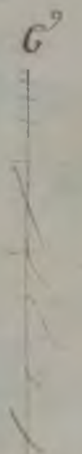
Ad manus tamen esse debet Pes ligneus vel metallicus secundum Rationem Decempedae vel Perticae in Digitos et Lineas subdivisus /: ein Zollstäbchen :/ cujus scili: illarumque ope Partes ultimo Pedes, si quae super fuerint, innotescant. Caeterum de Catenis omnia dicentes Mallet T. I. Geometr. pract. et Bion part I. des Mathematischen Werck Schul.

§9. Problema IV.

Data Longitudine Lineae in Mensura quadam v. c. Parisina invenire eandem in Mensura alia v. c. Londinensi cujus ad priorem datur, seu nota est ex §2 Ratio.

Resolutio

Esto Linea data = 200 Parisinorum quaeritur: quot Pedes Londinenses conficiet?



Quia
Pes Paris : Ped. Lond = 1440 : 1350 &c
= 16 : 15. Sibod.
h.e. Qualium Particularum est
Pes Parisinus 16 talium tantum
est Londinensis 15. Ergo Parisinus
Pes major Londinensi § 1 ctr.
Ergo per § 321 A
15 : 16 = 200 : Pedes Lond.

```
  16
 )3200(213 5/15 = 213 1/3 =
  30
  ---
   20
   15          Ped Lond.
   --
    50
    45
    --
     5
```

§10 Problema V.

Longitudinem Decempeda Rhenana Parisina Londinensi aut alia mensuratam in Pedes Rhenanos, parisinos, Londinenses, aut alios ordinarios seu Pertiva alius Gentis aut Civitatis convertere vel contra.

Resolutio.

Membrum I.

1) Quare Rationem Decempedæ ad pedes ordinarios s. q. i. e. civiles, Perticam conſtituentes § 2.

2) Infer: Ut Pedes Decempedæ ad Pedes Perticæ ejusdem Gentis aut Civitatis ita Longitudo, pedibus Decempedæ inveſtigata ad Longitudinem Pedibus Perticæ ordinariæ exhibendam § 314. A. S. F. C. A. P.

Schema Operationis.

Sit Longitudo Decempeda Rhenana menſurata = 105. Quaeritur: Quot Pedes Rhenanos Perticæ s. ordinarios Rhenanos conficiet.

Quia Ratio Decempedæ Rhenanæ ad Perticam Rhen = 10:12.

Ergo

$1\cancel{0} : 12 = 105$: Ped Rhen. ord

$\begin{array}{r} 12 \\ \hline 210 \\ 105 \\ \hline 126\cancel{0} \end{array}$ hoc eſt

126 Pedes Perticæ Rhenanæ.

✗ ✗

✗ ✗

Sit Longitudo Decempedæ Londinenſi menſurata $= 177\frac{7}{9}$: quot conficiet Pedes Londinenſis Perticæ. Quaere.

$10:12 = 177\frac{7}{9}$: Ped Lond Pert.

$10:12 = \frac{1600}{9}$: Ped Lond Pert.

$9\cancel{0} : 12 = 1600$ § 162 Ar.

$\begin{array}{r} 12 \\ \hline 3200 \\ 16 \\ \hline 1920\cancel{0} \\ 18.. \\ \hline 12. \\ 9. \\ \hline 30 \\ 27 \\ \hline 3 \end{array}$ | $213\frac{3}{9}$ h. e. $213\frac{1}{3}$ Pedes Lond Perticæ

Membrum 2.

Observatis, quæ ad Membr. 1. N. 1. dicta sunt, infer N. 2. sed invertendo 3140 ctr factumq erit per 3140

Schema Calculi

$12 : 10 = 126 :$ Ped Decemp. Rh.

$\frac{10}{\cancel{1260}}$ | 105 Ped Dec. Rh

Similiter Pedes Londinenses Perticæ $213\frac{1}{3}$ reducendæ esse ad Pedes Londinenses Decempedæ. Ergo per Auctores citatos

$12 : 10 = 213\frac{1}{3} :$ Ped L. Dec.

$12 : 10 = \frac{640}{3} :$ Ped Lond Dec.

$36 : 10 = 640 :$

$\frac{\cancel{3}0}{6400}$ | $177\frac{28}{36}$

36

280

252 $\quad 177\frac{7}{9}$ Ped

280

252

28

§11. Scholion.

De differentia inter Perticam atque Decempedam dictum est §2. Hoc tamen adhuc notandum, magnum nonnunquam discrimen intercedere inter decempedam alicujus Gentis atque Geometricam, hoc enim arbitrariæ omnino Longitudinis eſſe poteſt modo in decem partes et ita deinceps ſit ſubdiviſa §1. illa vero Longitudinem Legibus ſancitam æquare debet, licet et ipſa in decem partes æquales et ita deinceps ſubdividatur.

§12. Problema VI.

Menſuratam Decempeda v.c. Londinenſi Lineam ad Perticas alius ro Pariſinos Pedes reducere.

Reſolutio

1) Longitudinem datam decempedali Menſura h.l. Londinenſi expreſſam reduc ad Pedes ordinarios Londinenſes §10.

2) Inventos reduc ad Pariſinos §9.

Factumque erit.

16.

Schema Calculi.

Sit Linea Decempeda Londina
si expressa $= 177\frac{7}{9}$ Ergo per
Membrum 1.

$10 : 12 = 177\frac{7}{9}$: Lond. Pert. or
$9 : 12 = 1600$
$\underline{12}$
3200
16
$)19200(\ 213\frac{3}{9} = 213\frac{1}{3}$.
177
Ped. Lond. in ordin.
atq per Membr. 3.
$1440 : 1350 = 213\frac{1}{3}$: Ped. Par.
$16 : 15 = 213\frac{1}{3}$: Ped Par ord
$48 : 15 = 640$:
$\underline{15}$
3200
64
$)9600(\ 200$ Pedes
Parisin. ord.

§13 Problema VII.

Decempeda Geometrica me
ratam longitudinem in ordi
narios Perticas alicuius civi
datæ Pedes resolvere.

Resolutio.

1) Quaere Decempedae ad datam Perticam Rationem

2) Ad Rationis inventae Terminos et mensuratam Longitudinem, quaere quartum ppalem. § 314. Ar.

D. F.

Schema Operationis.

Ponamus v. e. Decempeda Geometrica quadam in Agro Londinensi mensuratam esse Longitudinem 36 Decempedarum: quaeritur; quot Perticas aut Pedes Londinenses ordinarias conficient illa 36 Decempedae Geometricae?

Ponamus duas Decempedas efficere tres Perticas Londinenses.

Ergo

Illius ad hanc Ratio = 2:3

Quare p. Memb. 2.

2:3 = 36 : Pert. Lond.

h. e.

1:3 = 18 : 54. § 162. 314. Ar.

Similiter in aliis.

§14. Problema VIII.

Ex dato in Recta AB puncto G normalem excitare.

Resolutio

I In Charta

Modus 1.

1) Circini crus utrumque super Recta AB utcunque constitue in D

2) Intervallo DG describe Circu §83 Th. secantem Rectam AB in R.

3) Applicata in D et R Regula interseca Pphiam in C.

4) Duc Rectam CG §81 Th.

D. F.

Demonstratio

Duc Rectam CR. §81. Th.

eritque RGC = $\frac{1}{2}$ Circulo §~~94~~. Th.

Ergo ∠ RGC = R. §288. Th.

Ergo CG $\perp$lis ad AB. §444 Th.

Q. E. D.

Demonstratio 2.

Eadem est quae §158 Th. si ducatur DG. §87 Th.

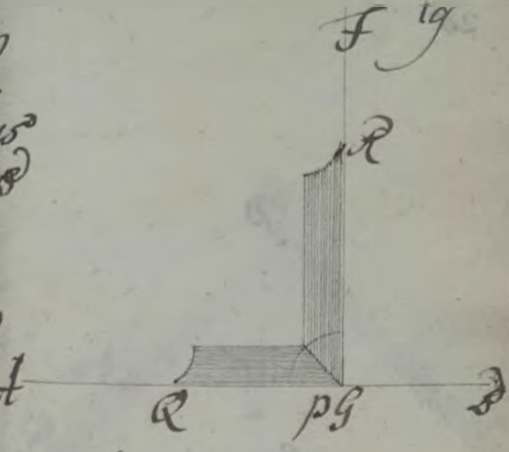
Q. E. D.

Modus 2dus.

1) Normae Angulum in Pcto G. Rectae AB ita applica ut crus alterum QG, Rectae GA vel GB coincidat.

2) Duc GF §81 secundum crus Normae alterum P.R

D. F.



Demonstratio

∠us Normae QGR = R. p. s. mechanicam

∠us Normae QGR congruit ∠lo AGF.

∠us AGF = R. §87.92. B.

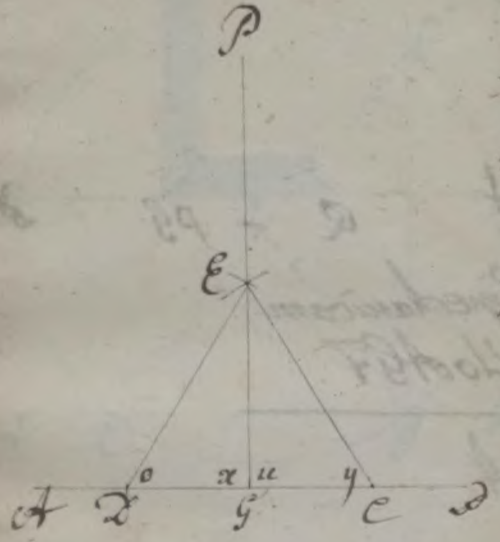
Ergo GF ⊥lis ad AB. §44.46 A.

Q.E.D.

II In campo

Modus 1.

Expeditissime Problemati satisfit adhibendo majorem normam, qua fabri lignarii utuntur, atque illius crus alterum QP juxta Lineam datam AB fune aut catena designatam in Pcto G applicando et juxta crus alterum P.R, funem. cf Fig prox. antec.



atium GF extendendo

Esse enim GF normalem ad AB patet ex Demonstratione proxime antecedente.

Modus 2dus.

1) Ex puncto dato G accipe Catena vel fune aequalia Intervalla GD et GC.

2) funiculi hujus vel Catenae extrema in Pctis D et C firma

3) Extensi deinde funiculi vel Catenae Dissectionis Pctm E ~~F~~ Gacatonata. D. F.

Demonstratio

Quia ED = EC p. C.

Ergo ∠o = ∠y. §100 G.

Sed DG = GC p. C.

∠x = ∠u. §99. G

EG $\perp$lis ad AD. §38. 44. G.

Aliter

DG = GC } p. C.

DE = EC }

EG = EG §40. G

∠x = ∠u. §106 G.

EG $\perp$lis ad AD. §38. 44. G.

Q. E. D.

§15. Scholion.

Normæ autem h. m. probatur:

1) Describe in Papyro bene extensa aut si majores fuerint in assere planissimo qua fieri potest Graphii aut Plumbaginis subtilitate super quavis Linea recta semicirculum §88. G. AXY.

2) Ex quolibet Peripheriæ puncto x assumto et externis Diametri A et y duc rectas Ax et xy §81. G. erit ergo.

$\angle x = R.$ §288 d.

3) Verticem Normæ Q applica in x et Crura QP et RQ in Cruribus Ax et xy.

Quod si Crus cruri utrumque utrique congruat Norma continebit ∠um Rectum, uti quidem liquet ex §§87. 92. G adeoque exacta erit.

Sufficit etiam alio quocunque modo Geometrico v. c. §119. 120 aut 158 G Angulum Rectum subtiliore Graphio aut Plumbagine

deſcripſiſſe, atq; dicta modo ra
ne Normam ad illam applicatam
examinaſſe

§16. Problema IX.

Ex dato ſuper Recta AB Pcto E li
neam normalem in campo dem
tere Reſolutio.

Modus 1.

1) Eadem ex E. Funiculi vel Cat
Extenſione nota Puncta D et
C in Recta AB.

2) Diſeca DC in G, quod fiet ſem
complicando funiculum DC,
atq; ex D verſus C aut v. v. ex
C, verſus D eidem Rectæ AB
applicando in G.

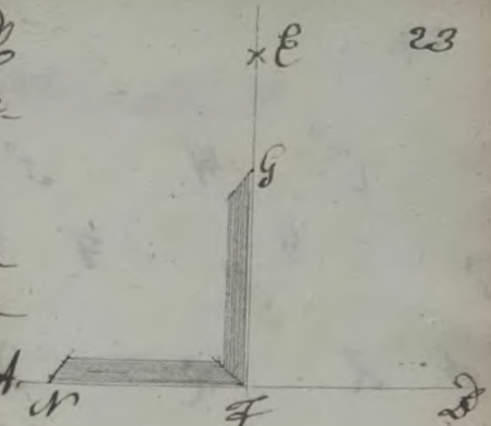
Dico EG normalem ad AB, uti
patet ex Demonſtrationibus
ultimis §i 14.

Modus 2dus

Normæ majoris crus alterum
NF ita ſtringat Rectam AB

ut Funiculus vel Catena ex Pcto C extensa Crus alterum FG stringat.

Dico CF esse normalem ad AD demissam; idquod patet ex demonstratione Modi 2di Resolutionis in Charta §14.



§17. Problema X.

Cum Recta AD per datum extra eam Pctm C ducere parallelam.

Resolutio.

I In Charta.

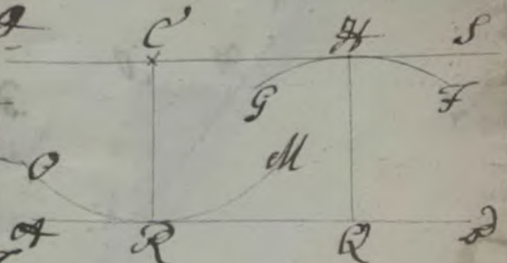
Modus 1.

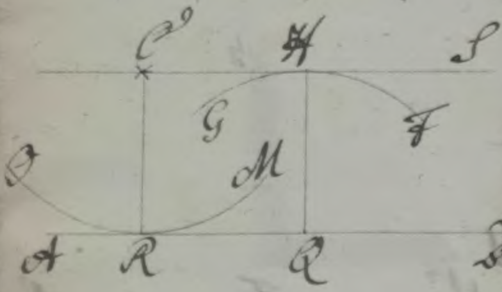
1) Ex C describe Arcum OM, qui rectam AD tangat in R. §850.

2) Ex quodlibet alio Pcto Rectae AD v.c. Q eodem radio CR describe alium Arcum GF. &c.

3) Applicata Regula in C et GF duc Rectam CS ita ut Arcum GF contingat in H.

D. F.





Demonstratio.

AD tangit OM. p. C.
Ergo CR ⊥lis ad AD. §241. G.
CS tangit GF. p. C.
Ergo HQ ⊥lis ad CS. §G.
Sed CR = HQ. §26. G.
et CR atq HQ sunt Distantiae
Rectarum CS et AD. §44.123
Ergo AD ∥ CS. §125 G.

Q. E. D.

Modus° 2.

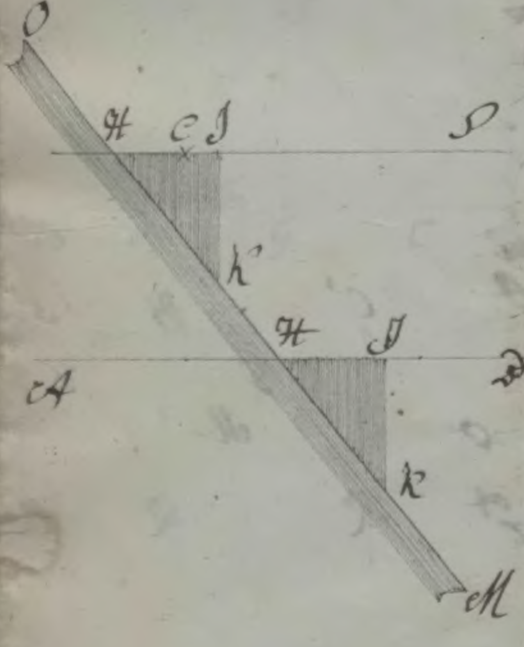
1) Trianguli Rectanguli HIK Cathetum HI applica Rectae AD vel Parti congruat, sic et

2) Applica Hypothenusam HK Regulam OM atq juxta hanc immotam promove Quan HIK donec eadem Cathetus per C transeat.

3. Duc secundum istam HI. per C Lineam CS. §81.

Dico CS ∥ AD.

Demonstratio



$\angle JHK = \angle IJK$. §70 Ar.

CM eſt Recta transverſa ſecans HB et AD pl.

Ergo

$CB \bumpeq AD$ §133.

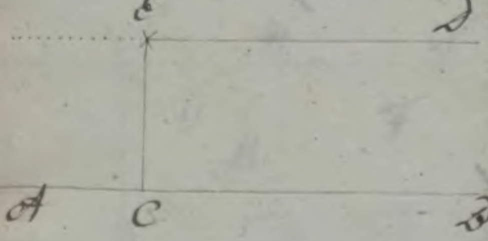
Q.E.D.

Optimæ notæ ſunt Regulæ et Triangula huic Praxi inſervientia, quæ fiunt ex Ebore aut durioribus Lignis Indicis.

Modus 2dus.

Utuntur in vulgus ita dicto Paralleliſmo, qui ex duabus Regulis ejuſdem ubique Latitudinis duplici et æquali inter ſe Retinaculo ita connexis paratur, ut Regulæ ipſæ pro variis intervallis datis varie diduci poſſunt.

Paralleliſmus iſte autem ſæpius adhibitus, vitioſus redditur, ob continuam enim Retinaculorum Frictionem ipſa laxeſcantur, plus juſto efforata. Neque etiam conſtans huic malo medela paratur, quomodocunque etiam Retinacula iſta ex Laminis orichalco duplicatis elasticis filum



II. In campo.

1) Ex puncto E ad Rectam AB duc normalem EC. §16.

2) Ex eodem Pcto E excita aliam normalem DE. §14.

Dico DE esse ∥lam ipsi AB, id quod liquet ex §134 et Q.E.F. et D.

§18. Hypothesis 2.

Gradus ut Perticas, Minuta ut Pedes, secunda ut Digitos, Tertia ut Lineas etc. sic commate uno, duobus etc. significabimus. v.c. 79 Gradus, 35 Minuta 47 secunda etc. h. m:

79° 35' 47''. etc.

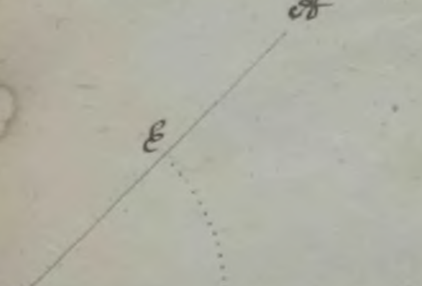
Peripheriae autem in 360 unius Gradus in 60 Minuta, uniusque Minuti in 60 Secunda, et ita deinceps secundum Proportionem sexagecuplam subdivisionem acceptam ferunt, veteribus Aegyptiis, prudenti sane Consilio adoptatam, cum per omnes

Digitos uno Septenario excepto exactam admittat Divisionem. Neq; tamen hoc omnibus ob difficiliorem fractionem sexagesimalium calculum arrisit Mathematicis. Fuerunt unde ceteri primus Oughtredus, Wallisius, alii subdivisionem Graduum per fractiones decimales, quatenus tam ardua non sit Reductio, suadentes, idq; consilii secuti Henricus Briggius, Joh. Neutonus et Nic. Mercator de quibus plura vide ap. Perill. Wolfium in Geom. lat. § 43. Nostris Temporibus Joh. Sam. Lippio A. M. aliam viam ingredi placuit, preter enim ea, quod decimales cum supracitatis Viris celeberrimis merito preferat sexagesimalibus, Pphio ipsius etiam in 360 Divisionem taxat, commodiorem in 400 particulas fore, ratus. Enimvero cum 400, neq; per 3 et multipla 6, 9, neq; per 7 exacte dividatur, meliorem antiqua, novam hanc Sphæræ ſectionem Viri haud cenſuerunt eruditiores. cf. Fr. illius de Transportatoriis curvilineo rectilineo et Arithmetico. Witteb. 1720. 4. Merito tamen in calculis ſervatur Hypotheſis Aegyptiaca, quam miſsa maximis iisq; accuratiſsimis laboribus eruti calculi ſummorum omnis aevi Virorum immutandi novoq; cuidam Hypotheſi accommodandi eſsent alii, ſummo, eq; ac aetatem Hominis ſuperante ſtudio. cf. de ſexageſimalibus Tabulas Prutenicas Eras. Reinholdi p. 1—14.

§19. Problema XI.

Angulum propoſitum metiri.

Reſolutio et Demonſtrati



Quoniam menſura ∠li ADC eſt Arcus radio prorſus arbitrario intra crura deſcriptus EF. §33. et universum eo redit negotium ut Quantitas Arcus EF in

Gradibus illorumq; Partibus Sexagesimis h. e. minutis Secundis etc. § 27. 28. determinetur in quod fit ope semicirculi in 180 Gradus eorumq; semisses etc. accuratissime subdivisi; quem Transportatorium dicunt.

Proinde

I In Charta

1) Centrum Instrumenti transportatorii in Vertice D ita colloca, ut ejus semidiameter crus DB alterum v. c. DE exactissime stringat.

2) Numera Gradus inter DE et DF interceptos; quod si autem DF minus fuerit transportatorii semidiametro producendum est § 28.

Q. E. F. per § 33. 28.

II In Campo.

1) Instrumento Goniometrico

1) Instrumentum Goniometricum, situ Horizonti parallelo ita colloca, ut Centrum ejus

Verticis Δli propoſiti exacte immineat, Id quod fit ope Perpendiculi fulcro appenſi; Beneficio autem Libellæ ſitus Inſtrumenti Geometrici examinatur.

2) Latus Regulam Dioptris immobilibus inſtructam ita dirige ut mediam Baculi in Extremo cruris alterius def xi collimando appareat.

3) Regula vero Dioptris mobilibus inſtructa, cis et ultra mota medium baculi alterius crus ſcil. alterum Δli dati conſtituentis determinet

4) Numera Gradus interceptos

D. F.

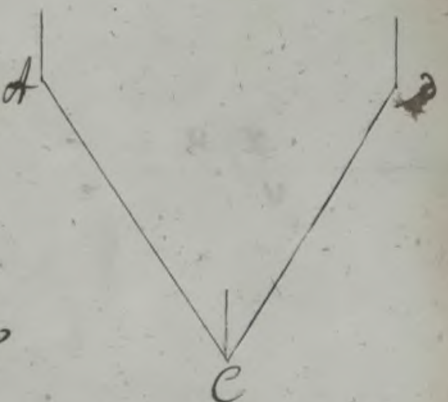
2) Menſula Prætoriana

1) ſuper Vertice Δli in menſ la ad Horizontem rite con tuta per Num. 1. anteced. Pu ctum determina ope ſubtiliſ me Acus, quod ipſum Vertic Δli propoſiti immineat;

2) Acui huic Regulam Dioptris instructam applica.

3) Juxta hanc facta ad Bacillos legitima collineatione duc Graphio aut Plumbagine Rectas in Puncto ab Acu facto coituras. §81. G.

4) Applicato Transportatorio Numera Gradus per §: hujus Resolut. 1mam.



D: F.

Demonstratio

AC in Campo ≈ ag in Mensula }
et BC in Campo ≈ bg in Mensula } per legitimam Mensulae constitutionem.

Proinde

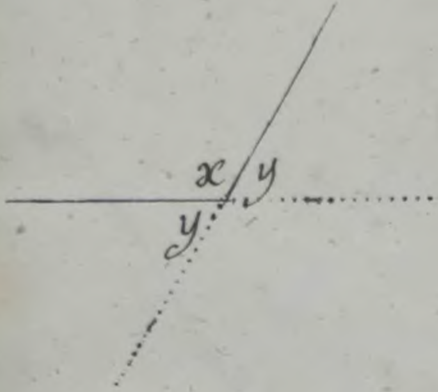
∠ ACB in Campo = ∠ agb in Mensula §442.

Q. E. D.

Eadem est Demonstratio si Instrumento Goniometrico Anguli propositi Quantitas investigetur.

§20. Scholion 1.

Quod si Quadrante Geometrico mensurandus est obtusus x, mensu deinceps positum acutum y cogni tumque a 2 Rectis aufer h.e. ex 180 dicto Residuum = x.



Nam

$\angle x + y = 2R.$ § 93 b.

$\angle y \ = \ y.$ § 40 Ar.

$\angle x = 2R. - y.$

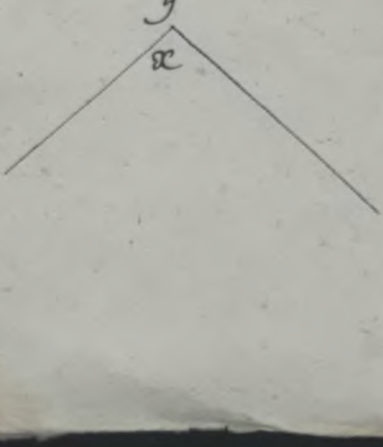
Esto in c. s. $y = 36^{\circ},, 12'$

Quia $\angle x + y = 179,, 60$

et $\angle y = 36,, 12$

Ergo $x = 143,, 48'$.

Designabis autem ∠lum deinceps positum y, defigendo cum alter utro Crure ∠li x, Baculum in eadem Recta; uti patet ex § 8.



§ 21. Scholion 2

Si Semicirculo Geometrico ∠lus major 2 Rectis investigandus est y, Quaere illius Complementum ad 4 R. x; inventumque ex illis aufer. Dic: Resid. ∠y.

Nam quia.

$x + y = 4 R.$ §90. G.

et $x = x.$ §40. Ar.

$y = 4R. - x.$ §48. Ar.

Sit in C. S. $x = 120°,, 33'.$

Quia $x + y = 360° = 359,, 60.$

$y = 239,, 27'.$

§22. Scholion 3.

Quodsi vero Quadrante Geometrico idem ∠lus y quaerendus esset, adde ∠li x deinceps positum z, duobus Rectis, quemadmodum ex eodem §90. G. constat.

§23. Problema XCII

Angulum propositum describere in Charta.

Resolutio.

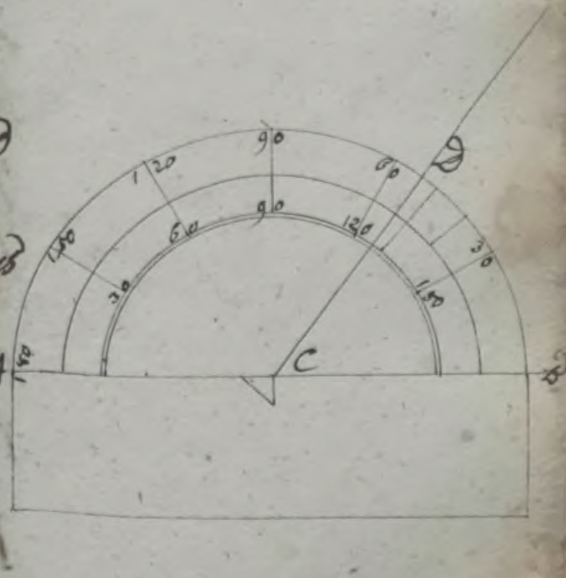
Modus 1.

1) Duc Rectam infinitam AB & in illa accipe Ptm C.

2) Applica Instrumentum Transportatorium, ut Diameter Recta AB et Centrum puncto C congruat.

3) Numeratos tot Gradus quot Angulus propositus conficit abscinde in D.

4) Duc Rectam DC. §81. G. D.F. §399. G.



Modus 2.

Si ∠us in Charta fuerit propositus, adeoq in aliam transferendus expeditissime utimur Circino tribus Cruribus instructo, cujus Operationis Ratio patet. ex §. 107. G.

Caeterum notandum Practicos praecipere, Transportatorio eandem cum Instrumento Goniometrico, quo usi sumus in determinanda ∠li Quantitate in Campo aut paullo tantum minorem esse debere Diametrum nec sine causa uti liquet ex § 399. G. Iidem auctores quoque Transportatorium Regula circa Centrum mobili instructum suadent, secundum quam Linea DC accuratius possit duci.

§ 24. Problema XIII.

Angulum propositum transferre in Campum.

Resolutio.

I Ope Instrumenti Goniometrici

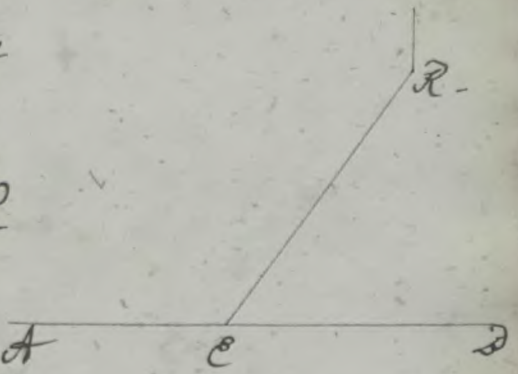
1) In Recta AD puncto dato vel aſſumto E colloca Centrum Inſtrumenti cum Horizonte ope Libellae ∥le conſtituendi.

2) Immobilem Regulam diſpone in ipſa AD

3) Mobili autem obſerva ∠li propoſiti Quantitatem. §19.

4) Secundum hanc Baculum in Recta ER infige, qui collineanti occurrat.

Dico ∠DER = dato. §442.



II Ope Menſulae Praetorianae

1) Angulum propoſitum deſcribe in Charta ſuper Menſula bene extenſa. §28.

2) Reliqua fac uti Reſolutione I Regula np. circa Axem mobili collineando et Crurum ED et ER inclinationem Baculis defixis determinando.

§250. Problema XIV.

Angulum in Campo datum ADC transferre in ejusdem

Campi Pctm. E Funiculi vel Catenae atque Baculorum adminiculo.

Resolutio et Demonstratio.

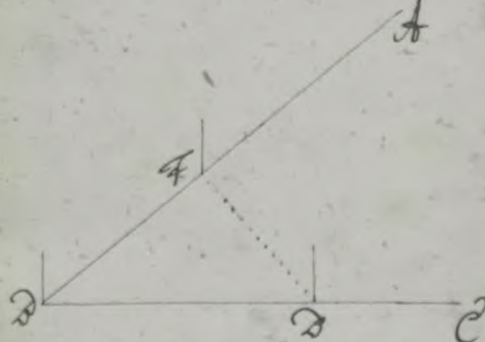
1) In cruribus BC et AB statue Baculos utrunque in D et F.

2) Fac EG = BD.

3) Transfer in Funiculum Longitudines BF et FD eumque vel Catenam in extremis Punctis ad E et G firmatam ita extende Baculo quodam tertio H,

ut HE = BF.
HG = FD.

Dico ∠E = B. §10b. θ.



§26. Problema XV.

Metiri Distantiam duorum Locorum A et B ex eodem tertio C accessorum.

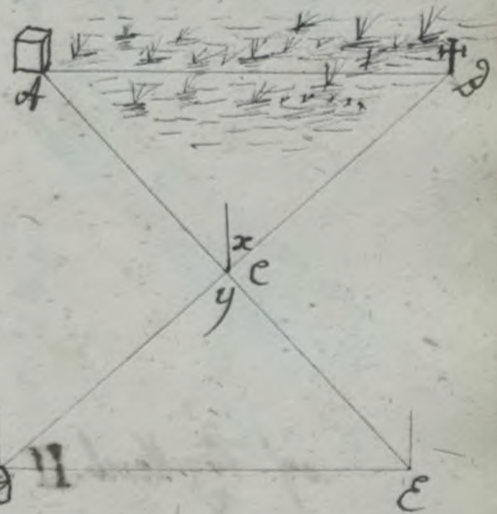
Resolutio.

I. Ope Funiculi vel Catenae et Baculorum

1) In C defige Baculum normaliter, id quod h. l. semper supponitur p. §4. Num 3.

2) Mensuratam §8. AC transfer ex lin C, ut AEC sint in eadem recta. §8. 24.

3) Simili Operatione Rectam CD transfer ex lin D in eadem Recta BD. §8. cc.

4) Mensura Longitudinem DE §8. Dico DE = AB.

Demonstratio.

$\angle x = \angle y$
$AC = CE$ } r. C.
$BC = CD$

$AB = DE$. §99. θ.
Q. E. D.

II Ope Mensulæ Prætorianæ

1) Fac ∠um DFE = ∠ ACB. §19.

2) Quantitates Rectarum AC et CD Catena vel Funiculo mensuratas §8. ope Scalæ §6 transfer ex F in D et C.

3) Duc DE. §81. ~~θ~~ et quære Rectæ DE. §81. Quantitatem ex eadem Scala. §5. 8.

D. F. Rectamque AB tot respondere Perticis, Pedibus

Digitis &c. ipsius Catenae vel funiculi quot Perticae aut Decempedae Pedes &c. respondent ipsi DE in scala accepta §58.

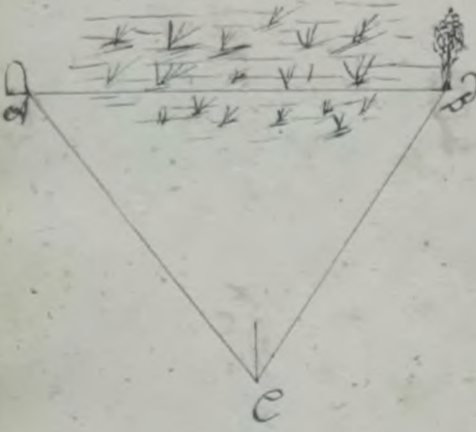
Demonstratio.

∠DFE = ∠ACB p. Obser.
et DF : FE = AC : CB. p. l.

Δ DFE est eq∠gl. Δ ABC. §256. A.

FD : DE = CA : AB
FE : ED = CB : BA } §352. θ.

Q. E. D.



cf. Fig. Memb. II

III Ope Instrumentorum Goniometricorum.

1) Observa Angulum C. §19.
2) Quantitates Rectarum AC et CB §8.
3) ∠lum C transfer in Chartam §23.
4) Fac Rectas ED et DF Rectis AC et CB ppales §8.5.
5) Duc EF. §81. θ.

D. F.

Demonstratio.

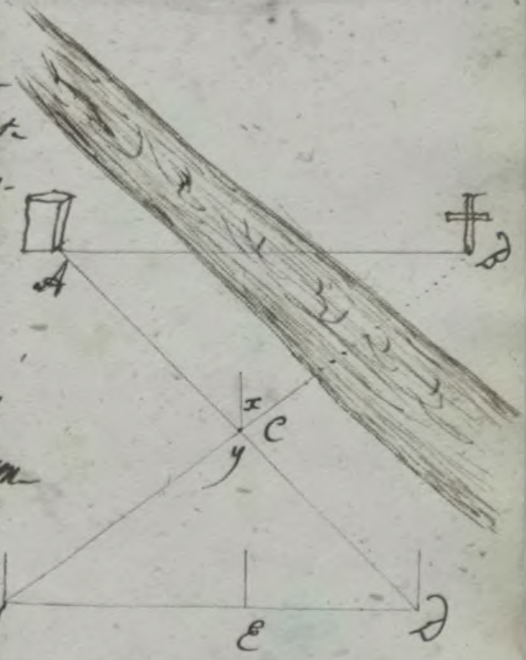
Coincidit cum proxime antecedente

§27. Scholion.

Si integrae Rectae AC et CD ope Baculorum transferri nequeant in D et E transfer quamcunque illarum Partem, secundam, tertiam, quartam, eritque DE secunda, tertia, quartaque Pars ipsius AB ut ex Demonstratione liquet. §26. N.2.

§28. Problema XVI.

Invenire Distantiam duorum locorum A et B ad quorum unum tantum Aditus patet ex assumta vel data statione C.



Resolutio

I Ope Funiculi vel Catenae et Baculorum.

Modus I

1) Mensuratam AC ex C transfer in D in eadem Recta §8.

2) Fac ∠lum CDE = ∠lo A. §25.

3) Defige in Recta continuata ED Baculum F, qui et cum ipsa ED et BC sint in eadem Recta §4.

DF

AC = CD p. demonstrat.

∠x = ∠y §94 G.

∠A = ∠D p. C.

AB = FD. §14 G.

Modus 2dus.

ſi Angulus A. Funiculo vel Catena et Baculis ob vicinam Ripam aut alias Agri circumſtantias menſurari nequeat.

1) Baculum D in directum ſtatue cum Baculis A et B

2) Menſuratas AC et CD. §8. transfer in directum utramque utrique in E et F uti §26. N.1.

3) Baculum J in directum ſtatue ipſi EF, qui ſit et ipſis C et B in directum poſitus. §4.

Dico EJ = AB.

Demonſtratio.

$\angle o = \angle y$. §94. Ө

$DC = CF$ } p. C.
$AC = CE$

$DA = EF$. §99. Ө.

$\angle D = \angle F$ §c

$\angle BCD = \angle JCF$. §94 Ө.

$DC = CF$. p. C.

$DB = JF$. §114 Ө.

$AB = JE$. §43. Ar.

Q. E. D.

Paullo aliter

$\angle o = \angle y$ §94 G.

$DC = CF$ }

$AC = CE$ }

$\angle DAC = \angle FEC$. §99. G.

Sed $\angle DAC + z = 2R$. } §95 G.

et $\angle FEC + \alpha = 2R$ }

$\angle DAC + z = \angle FEC + \alpha$. §41 Ar.

Ergo.

$\angle z = \angle \alpha$ §43 Ar.

Sed $\angle u = \angle \alpha$. §94 G.

et $AC = CE$. p. C.

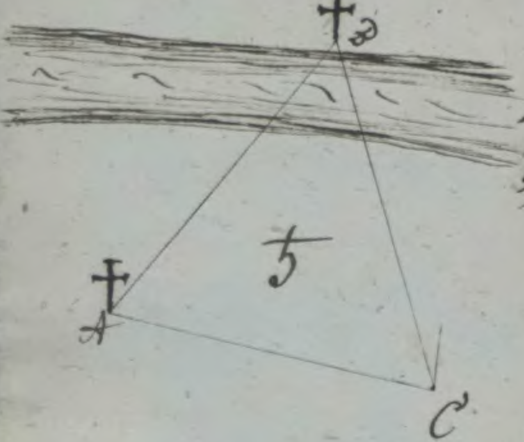
$AD = FE$. §99. G.

Q. E. D.

II Ope Menſuræ Prætorianæ

1) Menſula legitima collocata in A deſcribe ∠lum F = ∠lo A. §19.

2) Menſura Rectam AC eamq ope Scalæ transfer ex F in d §8.



3) Transfer Mensulam ex A in C defixo tamen in A Baculo ut punctum d ipsi C immineat et AC sit ∥la cum dF.

4) Describe ∠lum d = ∠ ACB §19.

5) Ope ejusdem Scalæ quære Quantitatem Rectæ FE. §8.

D. F. h. e fore

dF : FE = AC : AB

Demonstratio.

∠A = ∠F } p. Constructionem
∠C = ∠d }

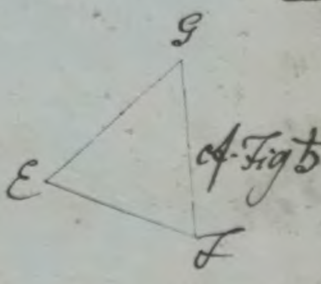
∠ACB æq. ∠gl. △lo FdE. §155. et 308. Θ.

Ergo

dF : FE = AC : AB. §352. Θ.

Q. E. D.

III Ope Instrumentorum Goniometricorum.



1) Observatis legitime ∠lis A et C. §19.

2) Atque Lineæ AC Quantitate §8.

3) Ope Scalæ §5. describe in Charta Lineam EF, ipsi AC proportionalem §8.

4) Deniq; translatis ∠lis A et lin. E et F. §23. D.F.

et eſse
EF : EG = AC : AB.

Demonſtratio:

Coincidit cum proxime antecedente.

§29. Problema XXVII

Metiri Diſtantiam duorum Locorum inaceſsorum A et B.

Reſolutio

I Ope Menſulæ

1) Electis duabus ſtationibus C et D.

2) Fac in menſula ∠los

ACD = ∠gei
BCD = ∠ieh } §19.
ACB = ∠geh

3) Quantitatem Rectæ CD quæſitam transfer ope Scalæ ex e in i. &c.

4) Transfer Menſulam ex C, relicto tamen baculo in D ut Pctm i immineat Pcto D et DC ſit ‖la ei ✗

✗ 5) Fac ∠los eih = ∠CDB } §
eig = ∠ADC } 19

6) Menſura Rectam gh. D.F. et eſse

ei : gh = CD : AB

Fig Q

Demonstratio

$\angle ACd = \angle gei$ } p. C.
$\angle AdC = \angle eig$ }

$\triangle ACd$ aeq∠gl. $\triangle gie$. §155. 305. Th.

$ei : eg = Cd : CA$. §352. Th.

$\angle CDd = \angle eih$ } p. C.
$\angle dCD = \angle ieh$ }

$\triangle dDC$ aeq∠gl. $\triangle eih$. §155.

$ei : eh = CD : Cd$. §352. Th.

$eg : eh = CA : CD$. §174. A.

$\angle geh = \angle ACd$. p. Obs.

$\triangle geh$ aeq∠gl. $\triangle ACd$. §356. Th.

$eg : gh = AC : Ad$. §352. Th.

sed

$ei : eg = CD : CA$ p. d.

$gh : ei = Ad : CD$. §175. A

h. e.

$ei : gh = CD : Ad$ §146. A.

Q. E. D.

II Ope Goniometricorum Instrumentorum.

1) Observa ∠los x, y, z etc. §19.

2) Mensura Rectam ED. §8.

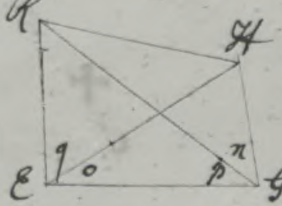
3) Ope Scalæ §5. fac EG ‖alem ED §8. in Charta atq

4) Ad Pctm E transfer ∠los q et o = ∠lis x et y. §23 et

5) Ad Pctm G transfer ∠los p et n = ∠lis z et n. utrumque utrique §c.

6) Duc RH. §81. C.

Dico AD ‖alem RH ut ex demonstratione antecedente liquet.

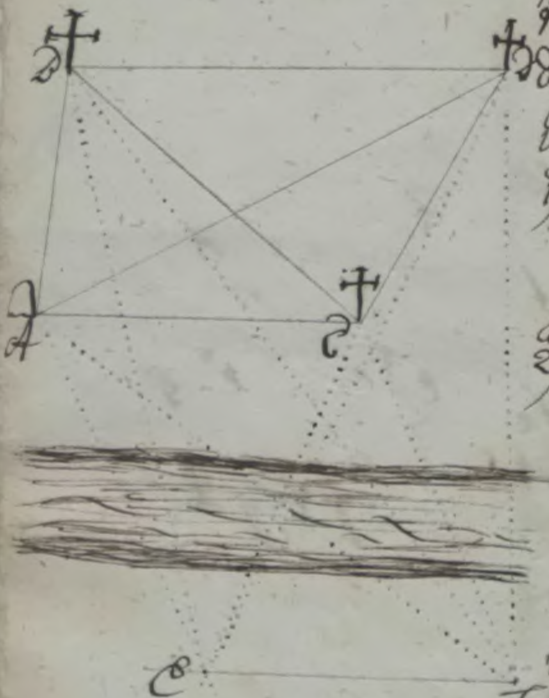


cf Fig 9

III Mitto Resolutionem Baculorum et Funiculi vel Catenæ adminiculo absolvendam quam inter primos dedit Schwenterus in Geometria Practica quippe quæ et minus expedita est nec satis Praxi accommodata.

§30. Scholion.

Facile adparet, simili omnis



Operatione vel Menſula vel Inſtrume
torum goniometricorum præſidio
diſtantias plurium locorum in-
veniri poſſe. Nam:

1) Aſſumtis duabus ſtationibus commodis ex quibus ſcil. verſus ſingula loca collineare licet.

2) Vel in Menſula deſcribantur vel ope aliorum Inſtrumentorum obſerventur omnium Angulorum a Punctis A, B, C, D, F, et A, B, C, D, E in Punctis E et F concurrentium Quantitates. Fig.

3) Menſuretur Linea EF, eidemque fi ~~pp~~alis in Menſula vel in Charta LM. §.5.6.

4) Deſcriptis ergo in eadem Charta ∠lis ad Pcta L et M, qui ſinguli ſingulis æquales ſint qui obſervati erant in Campo et

5) Junctis Triangulorum GLM, LPM, LHM verticibus GH, K, P, Rectis GH, GK, GP, HK, HP, PK.

1) $LM : GH = EF : AB$.
2) $LM : GK = EF : AD$
3) $LM : GP = EF : AC$.
4) $LM : HK = EF : BD$
5) $LM : HP = EF : BC$
6) $LM : PK = EF : CD$.

Demonstratio.

Sine negotio concinnatur ex iis quae ad § 29. evicta fuere sunt enim:

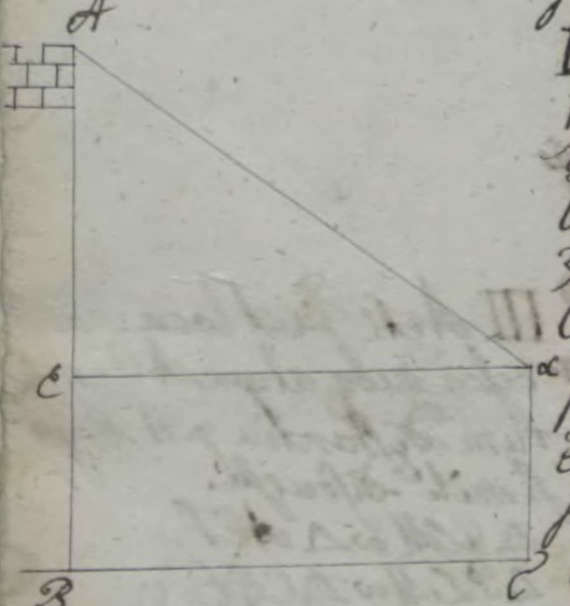
I. A et B loca duo inaccessa et AB eorum distantia p. H. Collata ergo &c. Demonstratione liquet
$\triangle GLM \sim \triangle AEF$.
$\triangle HLM \sim \triangle BEF$
$\triangle HLG \sim \triangle AEB$
Quare omnino
$LM : GH = EF : AB$ cf. &c.
Q.E.I.

II A et D sunt duo Loca inaccessa eorumque distantia AD p. H. simili ergo Ratiocinio evincam.
$\triangle GLM \sim \triangle AEF$
$\triangle LMK \sim \triangle EFD$
$\triangle GLK \sim \triangle AED$. adeoque
$LM : GK = EF : AD$.
Q.E.II.

III A et C sunt loca inaccessa duoque atque AC eorum distantia p. H. Ergo simili Discursu.
$\triangle GLM \sim \triangle AEF$.
$\triangle PLM \sim \triangle CEF$.
$\triangle GLP \sim \triangle AEC$. adeoque
$LM : GP = EF : AC$. &c.
Q.E.III.

Idem similiter de locis
$\left.\begin{matrix} B \text{ et } D \\ B \text{ et } C \\ C \text{ et } D \end{matrix}\right\}$ eorumque distantia $\left\{\begin{matrix} BD \\ BC \\ CD \end{matrix}\right.$

ostendit esse
$LM : HK = EF : BD$ etc



§31. Problema XVIII.

Metiri Altitudinem accessam AB ex Pcto C in eodem Plan sito.

Resolutio

I Ope Mensulæ

1) Reducta Mensula ex Horizontali situ in Verticalem ope Libellæ

2) Assume in illa Pctm α quod ipsi C immineat, id quod fit ope Perpendiculi ex α in C demissi.

3) Duc per illud α αβ cum Mensulæ Latere MQ vel DH. §16. aut §138. b.

4) Ex α collinea versus A, et duc Rectam αγ. §19.

5) Quære Quantitatem Rectæ BC. §8. eamque

6) Ope scalæ transfer ex α in Rectam αβ, quæ sit αδ §8.

7) Ex δ excita ⊥lem δε. §120 b. aut 148.

8) Mensuratæ δε adde Instrumenti h. e. Mensulæ Altitudinem αC.

Q. F.

Demonstratio.

∠ABC = R. p. H. et r. bat. q. 126 B.

∠αCB = R. p. C.

AB ∥ αC. §138. Θ. sed et

∠CαE = R. p. C. vel §136. Θ.

BC ∥ αE. §138. Θ.

Bα est Parallelogm. §72 Θ.

Ergo EB = αC } §167. Θ.
BC = αE }

Quarecum

∠B = ∠E. §132. Θ.

∠E = R. §92. Θ. sed

∠εδα = R. p. C.

∠E = ∠εδα §92 Θ. cumque

∠AαE = ∠εαδ. §40 A.

△ AαE aeqℓ. △lo εδα §155. et 305 A.
AE

Ergo

αδ : δε = αE : EA. §352. Θ.

sed αE = BC. p. d.

αδ : δε = BC : EA. §10 A.

Q. E. I.

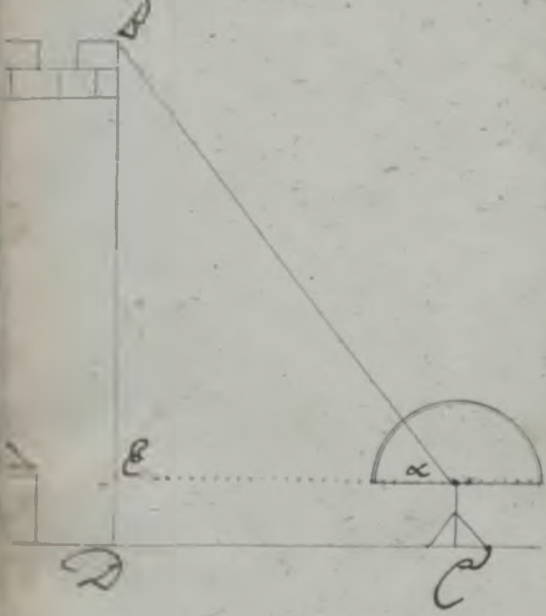
Quare

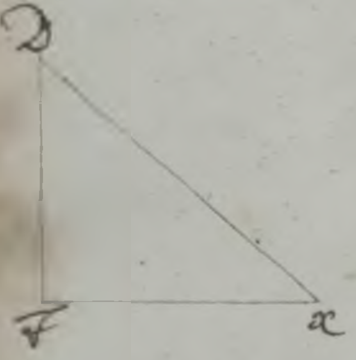
addendo αC = EB p. d.

AE + αC = AE + EB. §10 A

= AB. §47 A.

Q. E. D.





II Ope Instrumentorum Goniometricorum.

1) Instrumento verticaliter ita constituto ut Centrum α immineat assumto stationis Pcto C.
2) Observa Quantitatem ∠ Bα E §19.
3) Mensura Rectam DC §8.
4) Fac ∠ loα in Charta aequalem oc. §23.
5) Et Lineo DC ppalem αF. §8.
6) Excita ⊥lem ex F, FD. §158 aut §14.
7) Mensura DF et adde Instrumenti altitudinem αC.

D.F.

Demonstratio

Coincidit cum proxime antecedente

Q. E. D.

III Ope Baculi atq Funiculi vel Catenae ß

1) Defixo in Terram secundum $\angle R$. Baculo §4. N.3. C.D.

2) Humi jacens eo lineae ut Oculus in E constitutus cum summis Altitudinis et Baculi Punctis A et D sit in eadem Recta, id quod obtinetur vel revolvendo vel advolvendo Corpus ad DC.

3) Quaere Quantitates Baculi DC atq Rectarum CE et EB §8 C.D. ß

4) Ad inventas quartam ppalem §314. A. D.F. h.e. AB esse quartam ppalem

Demonstratio.

$\angle B = R$. §126. Θ.

$\angle C = R$. p. C.

$\angle B = \angle C$. §92. Θ.

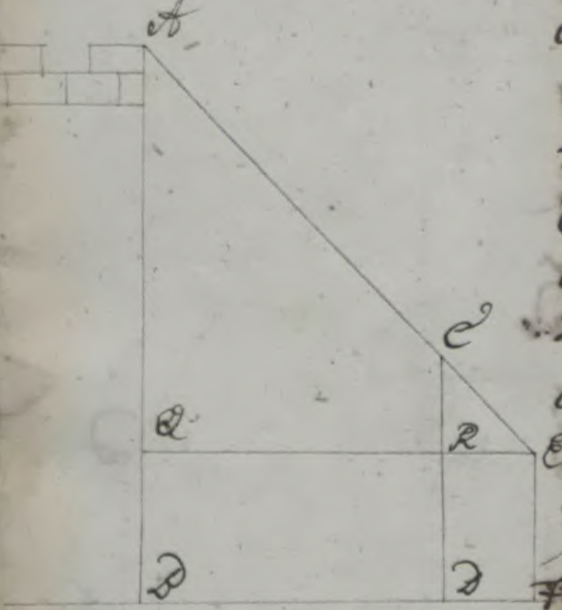
$\angle E = \angle E$. §40. A.

$\triangle ABE$ aeqlgl. $\triangle DCE$. §155. et 305 Θ.

Ergo

$CE : CD = EB : BA$. §352 Θ.

Q.E.D.



Aliter.

1) Baculos duos CD et EF Longitudinis inaequalis verticaliter eodem cum Altitudine AB Plano statue, ut inter illam et minorem major pateat minoris vero summam [illegible] E cum A et C sit in eadem Recta collineanti; id quod fiet Baculum EF in terram vel altius defigendo vel defixum nimius extrahendo.

2) Quaere Distantiam Baculorum CR. iteq.

3) Rectas FD et FB. [illegible]

4) Adde FD, FB, CR quartam proportionalem. §314. AQ.

5) Inventae huic adde Longitudinem minoris Baculi EF. D. F.

Demonstratio.

Quia EF, RD, QB ⊥les ad BF p. c.
Ergo EF = RD = BQ. §138, 134. t.
Sed AQ et BF ∥lis ad EF y. c.
Ergo QE = BF. &c.

Ergo
DE et QF ſunt ‖læ §72 G.
Ergo DQ = EF
EQ = DF } §164. G.
RE = DF

Porro: cum ∠ Q = R. §92. pf.
et ∠ E = ∠ E. §40 Ar.

△ AQE æqtgl. △ CRE §155. 305 G.
Ergo
ER : CR = EQ : AQ §352 G.
ſed ER = DF.
et EQ = DF. pr.d.
Ergo
DF : CR = DF : AQ §10 Ar.
Q.E.I.
Eſse vero AB = AQ + CF liquet
ex Demonſtratione Ima hujus
§i
Q.E.D.

§32. Problema XIX.
Metiri inacceſsam Altitudinem AB.
Reſolutio.
I Ope Menſulo.
Electis duabus ſtationibus

D et G, quo ſint in eadem Recta cum Altitudine D A. §. 9. i. e. in eod Plano.

2) Factisq omnibus ut Nbr. 1 – 48.

3) Menſura Rectam D C. §8. eamq

4) Ope Scalæ transfer ex α in β. §8.

5) Defixo in C Baculo menſulam ex C transfer in D, ut β immineat ipſi D et Menſula ſit in eodem Plano cum Baculo C et Altitudine A D.

6) Duc Lineam A β G. §19.

7) Ex δ demitte ⊥lem δγ in αβ §119. continuatam. §82. C

8) Quære illius Quantitatem in ſcala. §8.

9) Inventaq adde Altitudinem α C vel β D.

D. F. h. e.

G A æqualem ipſi γδ. h. e.

$\alpha\beta : \gamma\delta = DC : GA$

Demonſtratio

Gl et Dl esse Plgma et
$GD = \beta\delta = \alpha l$ atq.
$\beta\alpha = Dl$ patet ex Dem. §31.

Porro quia $G\alpha$ est Recta p. Hyp.

$\angle\angle A\beta G + A\beta\alpha = \angle\angle \delta\beta\gamma + \delta\beta\alpha$ §93 Th. §241 Ax.
$\angle A\beta G = \angle \delta\beta\gamma$. p. Obs.

$\angle A\beta\alpha = \angle \delta\beta\alpha$ §48 Ax. cumque
$\angle \alpha = \angle \alpha$. §40 Ax.

$\triangle A\beta\alpha$ aequiangl. $\triangle \delta\beta\alpha$. §155. 305 Th.

Ergo

$\alpha\beta : \beta\delta$ nempe in Mensula $= \alpha\beta : \beta A$ in Campo.
Sed $\alpha\beta$ in Campo $= Dl$ p. d.
$\alpha\beta : \beta\delta = Dl : \beta A$. §10 Ar.

Similiter quia

$\angle ADG = \angle \delta\beta\gamma$. p. Observ.
$\angle AG\alpha = \angle \delta\gamma\beta$. p. C. §92 Th.

$\triangle ADG$ aeqlgl. $\triangle \delta\gamma\beta$. §352. Th.

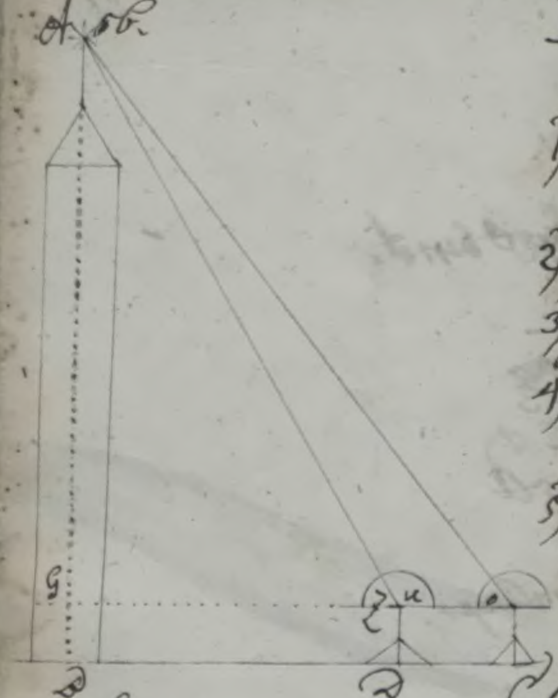
$\alpha\beta\delta : \delta\gamma = \beta A : AG$. §352. A.

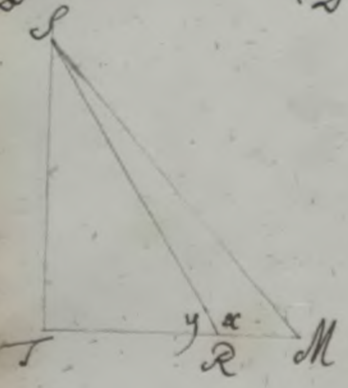
$\alpha\beta : \delta\gamma = Dl : AG$. §172. A.

Q. E. I.

Ergo et $AG + \alpha l = AG + GD$. §41 A et 162 A.
$= AD$ §17 Ar.

Q. E. II. D.





II Ope Instrumentorum Goniometricorum.

1) Factis omnibus Resolutionis primo Mbro 1mo

2) Observa ∠los o et z §19.

3) Itemque Quantitatem Rectae DC. §6.

4) Rectae DC describe pallelam in Charta §8.5.

5) In puncto M, fac ∠lum M = ∠lo o itemque in producta MR ∠lum y = ∠lo z §23.

6) Ex S demitte ⊥lem ST. §119 G.

7) Huic mensuratae in Scala §8. adde Altitudinem Instrumenti zD vel OC. Q.F.

Demonstratio.

$$\left.\begin{array}{l} \angle z + u = 2R \\ y + x = 2R \end{array}\right\} \text{§93 G.}$$

$z + u = y + x$. §41 Ar.

$z = y$ p. Observat.

$\angle u = \angle x$

$\angle o = M$. p. Obs et C.

$\angle u + o = x + M$. §40 Ar.

$\angle u + o = \angle 2R$. §144 G.

$x + m < 2R$. §46 Ar.

✕ ✕

✕ ✕

Convergent ergo SR et SM. §141 G.

Reliqua demonstrantur uti Membro I hujus §

Q.E.D.

§ 33. Scholion 1.

Ex Demonstratione § antecedentis liquet eandem in utraque statione esse debere Instrumenti Altitudinem id quod accurate observandum est.

§ 34. Scholion 2.

Supponimus etiam Altitudinem ipsam atque stationes in eodem Plano horizontali esse constitutas; id quod tamen evenire cum rarissime soleat, Altitudinibus mensurandis vel infra vel supra Horizontem consistentibus.

Ergo in

Casu I.

Non Instrumenti Altitudo, sed Recta QD, quae determinatur ex ∠to z §19. mensurato et Linea AQ producta §82. b addenda est ipsi AQ, quo sit Altitudo et Spatialis Recta in Scala mensurata, p. §8. np. Ms. cf. Fig. 1 pag 60.

× ex tota Altitudine AH

Casu II Linea AH quae determinatur ut AD sit spatialis Recta ab Horizonte arbitrario EH ∠y, et in Scala §8. accepta altitudine AH, auferenda est RG. cf. Fig. II p. 61

×

Demonstratio.

Casus I.

Supposita vel in Charta vel in Mensula debita Delineatione vi Resolutionis §32. Liquet ex ejusdem §phi Demonstratione esse:

$PG : M\Theta = DC : AQ$, atq. ob

$\triangle AQZ \sim \triangle M\Theta P$. p. dem. eand.

$M\Theta : \Theta P = AQ : QZ$ §352. Θ.

Porro quia

$\angle \alpha + \beta = 2R$ } §93 Θ.

$\angle \gamma + \delta = 2R$ }

$\angle \alpha + \beta = \angle \gamma + \delta$. §41 Ar

$\angle \alpha = \angle \gamma$. §92 Θ.

$\angle \beta = \angle \delta$ §43. Ar.

$\angle Z = \angle n$. p. Obs. et Constr.

$\triangle QZD$ aequiang. $\triangle \Theta PS$. §155. Θ.

$\Theta P : \Theta S = QZ : QD$. §352. Θ.

$M\Theta : \Theta S = AQ : QD$. §172. Ar.

Ergo

$M\Theta + \Theta S : M\Theta = AQ + QD : AQ$. §164. Ar

h.e. $MS : M\Theta = AD : AQ$. §47 Ar.

sed $PG : M\Theta = DC : AQ$ p. d.

$MS : PG = AD : DC$. §173 Ar.

Q. E. D.


A
Q
α
β
B
L
D
C
M
O
V
n
p
G
δ
S

Casus 2dus

Iisdem Delineationibus suppositis erit per §32. Demonstrationem;

$PS : RM = DC : AH$. atq. ob

$\triangle RMP \sim \triangle AHO$. y. demonstrand. &c.

$RM : MP = AH : HO$. §352 G.

Porro quia

$\angle H = \angle M$. §92. 126 G

$\angle y = \angle GPM$. p. obs. et Con.

$\triangle dHO$ aeq∠gl. $\triangle GPM$. §155. G.

$GM : MP = dH : HO$. §352 G.

$RM : GM = AH : dH$. §173 A.

$RM - GM : RM = AH - dH : AH$ §174 A

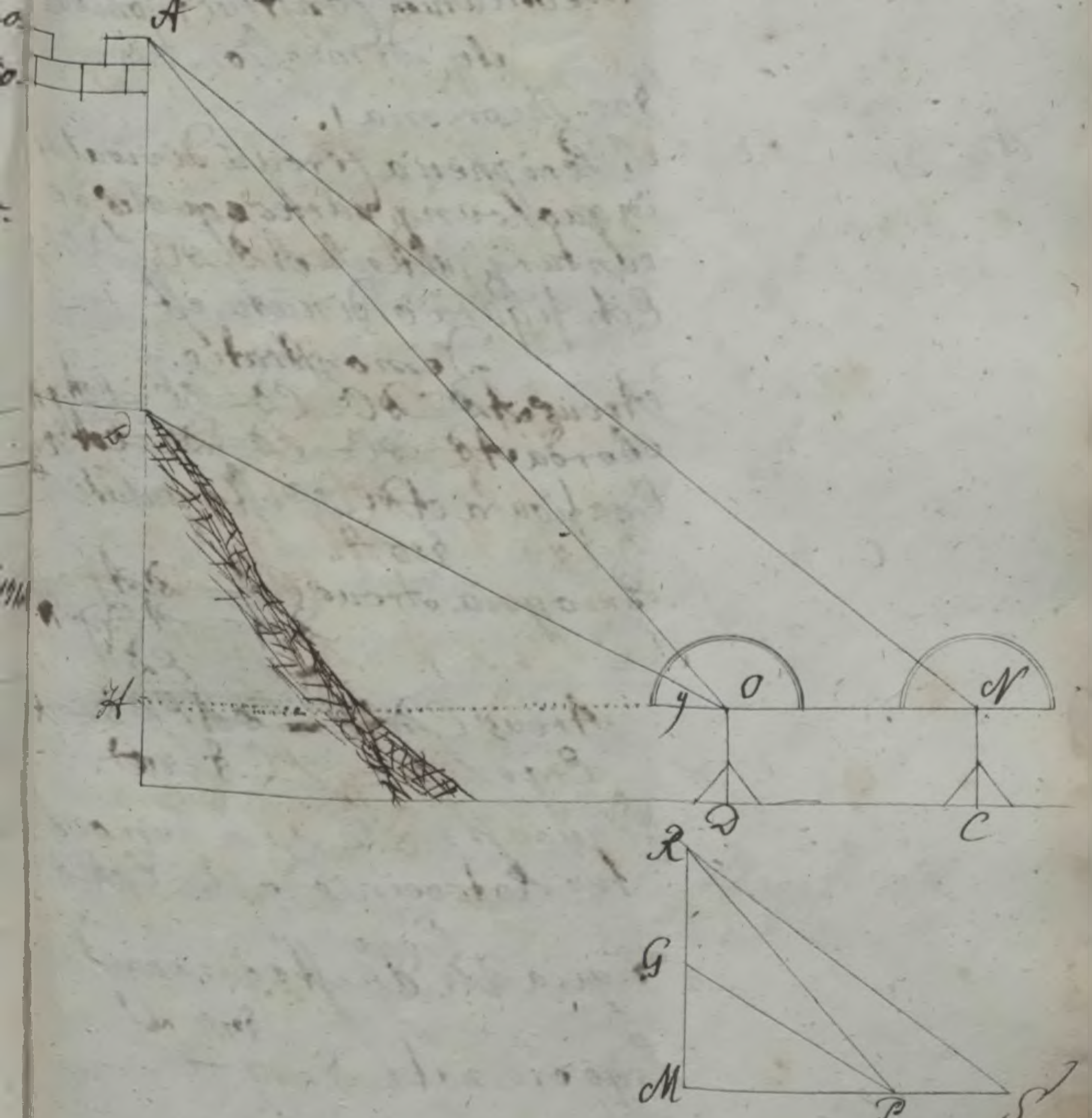
h.e. $PS : RM = DC : AH$ p. d

$GR : PS = Ad : DC$. §173. A.

adeoq. et

$PS : GR = DC : Ad$. §146 Ar.

Q. E. D.


A
m
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y
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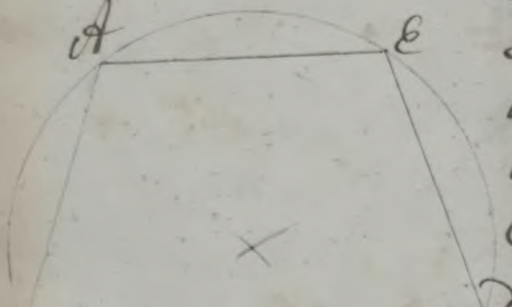
62

Caput II^{dum}

De Arearum Conſtructionibus atq; Dimenſionibus.

§35. Theorema 1.



Si Peripheria Circuli dividatu in quotcunq partes æquales, du cdnturq ſubtenſæ AB, BC, CD, DE, EA, figura ordinata eſt

Demonſtratio.

Arcus $AB = BC = CD = DE = EA$. p.

Chorda $AB = BC = CD = DE = EA$. §2

Ergo figura ABCDE eſt æquilater

§56 θ.

Porro quia Arcus $\left.\begin{array}{l} CB = BA \\ BA = AE \\ AE = ED \end{array}\right\}$ p.

Arcus $CBAE = BAED$ §42

Ergo $\angle D = \angle C$. §281 θ.

Id quod ſimili cum demonſ tur Ratiocinio de ∠lis E, A, B,

Ergo Figura ABCDE eſt æquiangula §47 θ.

Ergo ordinata. §299 θ.

Q. E. D

§36. Problema XX.

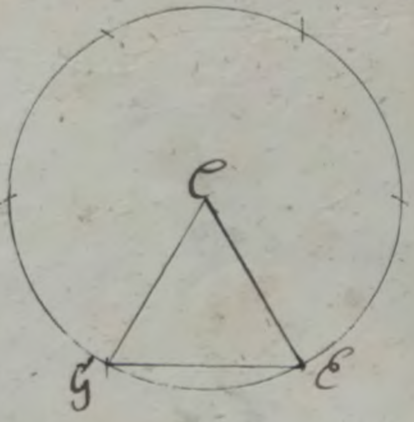
Circulo dato Polygonum ordinatum inscribere.

Resolutio et Demonstratio.

1) Divide 360° per Numerum laterum ut innotescat Quantitas ∠li C.

2) Construe inventum ad Centrum §23.

3) Chordam CG in Pphia circumfer §307. θ quoties fieri potest, atque Figuram descriptam esse ordinatam, liquet ex §35.



§37. Problema XXI.

Invenire Angulum A cujuslibet Figurae ordinatae ABCDEF.

1) Quaere Summam omnium Angulorum Polygoni dati §309. θ.

2) Inventum divide per Numerum Laterum. S.F.C.E. D.Q.P.

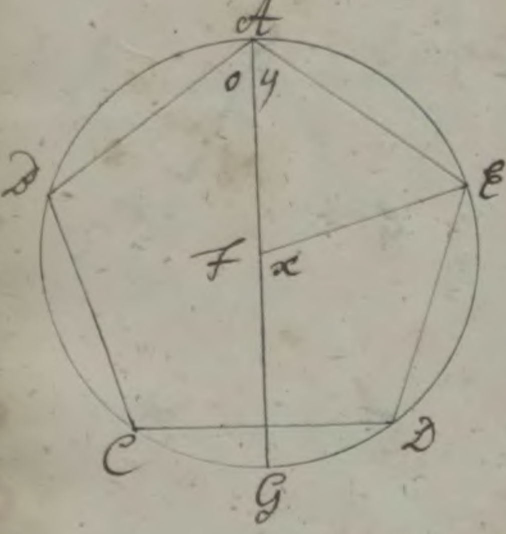
r.c in Hexagono.

1) Quia $\angle A = \angle B = \angle C = \angle D = \angle E = F$ §299 θ. et H.
adeoque $\angle A + B + C + D + E + F = 12R - 4R$. §309 θ.

2) Differentiam 8 Rectorum divide per Laterum Numerum Figurae propositae ordinatae h.l. per 6 Quotus est aequalis A.

Schema Calculi

$6 \times A = 8R = 720°$ Ergo
$A = \frac{8}{6} = 120°$ similiter in aliis



Aliter

1) Divide 360° per Numerum Later
2) Quotum aufer a 180°
Dico Differentiam esse ∠lum qu ſitum.

Demonſtratio.

Concipe Polygonum Circulo inſc ptum §36. Per Centrum F ex ∠li unius Vertice A duc Diametrum AG. §81. 82. θ.

Quare cum

Arc. ABCG = GDEA. §54. θ.
et Arcus AB = $\frac{1}{5}$ Pphiae } p. l.
et Arcus AE = $\frac{1}{6}$ Pphiae }

AB = AE. §41 A.

Arc. BCG = Arc. GDE §43 A.

∠o = ∠y §281. θ. Ergo
∠A = ∠o + y. §47 A.
∠A = 2×∠y. §10. A.
Duc FE. §81. θ. Ergo.
∠x = 2×∠y. §273. θ

∠A = ∠x. §41. A. Sed
Menſ. ∠x = Arc. GDE. §53. θ
Arc. GDE = GDEA − EA. h. e.
= 180° − $\frac{1}{5}$ Pphiae §10.

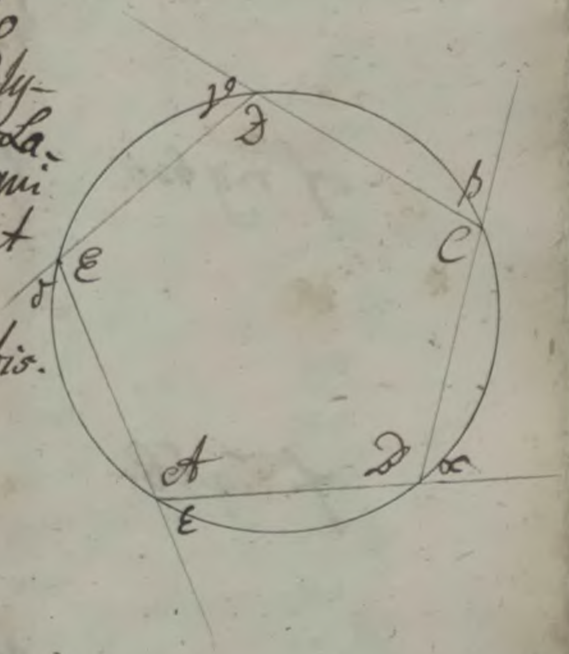
Proinde

∠A = 180° − $\frac{1}{5}$ Pphiae §41 A.

Q. E. D.

Demonstratio II

Longe est expeditior supposito Theoremate: Si cujuscunque Polygoni ordinati producantur Latera AB, BC, CD, DE, EA: ∠li qui oriuntur externi, α, β, γ, δ, ε, sunt inter se aequales.



Id quod h.m. demonstrabis.

$\angle A+\varepsilon = 2R.$ } § 93 θ.
$\angle B+\alpha = 2R$ }
$\angle C+\beta = 2R$
$\angle D+\gamma = 2R.$
$\angle E+\delta = 2R.$

$\angle A+\varepsilon = B+\alpha = C+\beta = \angle D+\gamma = \angle E+\delta$ § 41 A.

Verum.

$\angle A = B = C = D = E.$ § 299 et A.

$\angle \varepsilon = \angle\alpha = \angle\beta = \angle\gamma = \angle\delta.$ § 45. A.

Q. E. D.

Quo ergo demonstrato

$\angle \alpha+\beta+\gamma+\delta+\varepsilon = 4R.$ § 341 θ.

Ergo $5\times\alpha = 4R.$ p. Theor modo demonstratum.

Ergo $\alpha = \frac{4}{5}R.$ § 45 A.

Sed $B+\alpha = 2R.$ 93 θ.

$\angle B = 2R - \frac{4}{5}R.$ § 43 A.

Angulus h.e. Polygoni dati prodit, si a duobus Rectis auferas

quatuor Rectos divisos per Num
rum Laterum Polygoni ejusdem
Q.E.D.

§. 6.

cf Fig §37. Esto datum Hexagonum. Ergo

$$\angle A = 2R - \tfrac{4}{6}R.$$
$$= 180^\circ - \tfrac{360}{6}$$
$$= 180 - 60^\circ$$
$$= 120^\circ$$

cf Fig. §65 Esto datum Pentagonum. Ergo

$$\angle A = 2R - \tfrac{4}{5}R.$$
$$\angle A = 180 - \tfrac{360}{5}$$
$$= 180 - 72^\circ = 108\ \text{ps.}$$

§38. Scholion.

Non abs re erit Tabulam addere
in qua ∠li Polygonorum a III–XX
IV. Laterum continentur secun
da ejus series fit per continu
am duorum R. h.e. 180 additi-
onem.

Tertia oritur ex Divisione se-
riei 2dae per primam uti liquet
ex §37. Resolutione 1ma.

Quarta vel per continuam 360
Divisionem per Divisionem La-
terum Polygoni dati vel per sub-
ductionem seriei tertiae h.e. ∠li
ligonalis a 2 Rectis

Num. Laterum	Sūma ∠orum	∠lus Polyg.	∠lus ad Cent.
3	180	60	120.
4	360	90	90
5	540	108	72.
6	720	120	60
7	900	$128\frac{4}{7}$	$51\frac{3}{7}$
8	1080	135°	45°
9	1260	140	40
10	1440	144	36
11	1620	$147\frac{3}{11}$	$32\frac{8}{11}$
12	1800	150	30
13	1980	$152\frac{4}{13}$	$27\frac{9}{13}$
14	2160	$154\frac{2}{7}$	$25\frac{5}{7}$
15	2340	156	24
16	2520	$157\frac{1}{2}$	$22\frac{1}{2}$
17	2700	$158\frac{14}{17}$	$21\frac{3}{17}$
18	2880	160	20
19	3060	$161\frac{1}{19}$	$18\frac{18}{19}$
20	3240	162	18
21	3420	$162\frac{6}{7}$	$17\frac{1}{7}$
22	3600	$163\frac{7}{11}$	$16\frac{4}{11}$
23	3780	$164\frac{8}{23}$	$15\frac{15}{23}$
24	3960	165	15°
etc	in	infinitum	

ſeriem autem ſecundam conſtru per continuam duorum Rectorum Additionem, inde liquet, cum omnes ∠li Polygoni junctim ſumtis inaequales bis tot Rectis quot ſunt Latera demtis 4 R. § 309 b. Ergo accedente Latere uno accedunt duo R. ſub eadem ſempu 4. Rectorum differentia. Proinde quia

Suma ∠∠ Trigoni = 2 R § 309. b
erit Quadrati = 4 R §c. $\overline{2+2R}$
Pentagoni = 6 R. §c. $\overline{4+2R}$
Hexagoni = 8 R §c. $\overline{6+2R}$
Heptagoni = 10 R §c. $\overline{8+2R}$
etc.

Prior Seriei quartae Conſtructio liquet ex § 36.

Poſterior h. m. demonſtrabitur. Eſto Polygonum ordinatum Circulo inſcriptum ABCDEG. Ductis ex Centro F radiis ad ſingulos ∠lorum Vertices § 81. erit:

$DF = FB$ §26. Θ.
$FC = FC$ §39. A.
$DC = CB$ §299. Θ.

$\angle u = \angle y$. §106. Θ.
Sed $\angle C = \angle u + y$. §47. A.
Ergo $\angle C = 2 \times \angle u$. §10 et 42 A.
Porro $DF = FC$. §26. Θ.

$\angle o = \angle u$ §100. Θ.
Tandem $\angle x + o + u = 2R$. §143. Θ.
Sed $\angle o = \angle u$ p. d.

$\angle x + 2 \times u = 2R$. §10 A.
Sed $\angle C = 2 \times u$ p. d.

$\angle x + \angle C = 2R$. §41. A.
Ergo $\angle x = 2R - C$ h. e.
Angulus ad Centrum Polygoni ordinati equalis est differentia Anguli Polygoni ejusdem a duobus Angulis Rectis.

Q. E. D.

Fig.

§39. Problema XXII
Super data recta Linea CB Polygonum quodvis ordinatum Circulo inscribere.

Resolutio I.

1) Quære ∠ Aum Polygoni ordinati, describendi. §37.

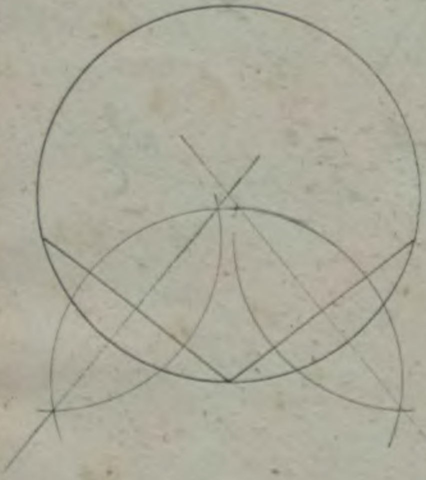
2) Eundem construe ad A vel B §25.

3) Fac $AC = AB$. §260.

4) Per Pcta A, B, C, describe Circulum §317. G.

5) In hoc coapta Rectam AB ad AC quoties fieri potest. §307. G.

D. F.



Demonstratio

Chorda $AB = AC = CD = DE = EB$ p.
Ergo Polygonum ABCDE est æquilaterum. §56. G.

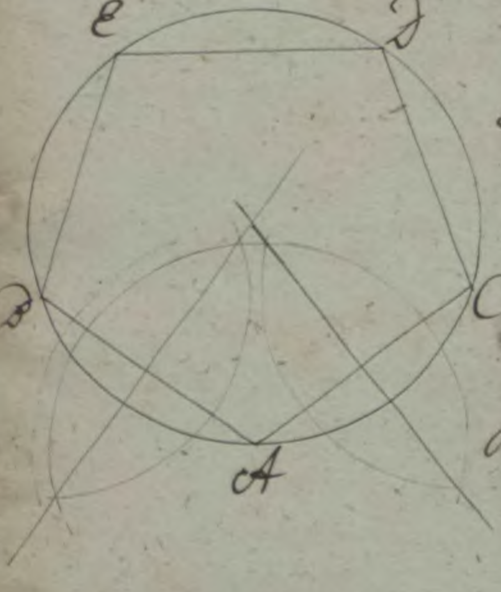
Sed et Arcus
$AB = AC = CD = DE = EB$. §285. G.

Quare cum Arc. $BA = DE$
$BE = ED$ p. d.
$ED = DC$

Arc. $ABED$ = Arc. $BEDC$ §41. A.

Ergo $\angle C = \angle A$. §282. G.
Idque simili cum Discursu de ∠lis reliquis demonstratur Erit Polygonum ABCDE et æquiangulum §77. G.
Ergo ordinatum §299. G.

Q. E. D.



Resolutio. 2.

1) Quaere ∠lum Polygoni construendi v.c. Pentagoni §37.
2) Construe semissem ad data Recta extrema puta A et B §25.
3) Productis deinde Cruribus ad Concursum in G. §82.
4) Centro G Radio GA vel GB describe Circulum §88. A.
5) In hoc coapta datum AB quoties fieri potest §307. D. F.

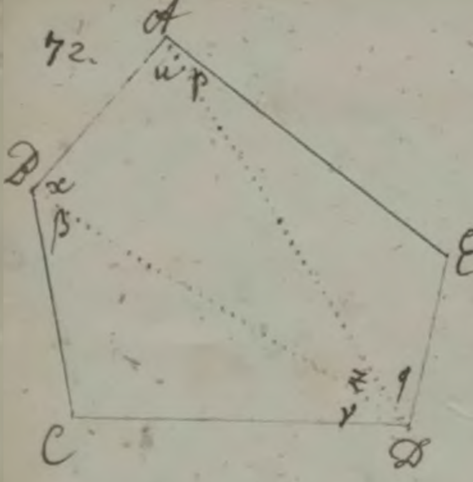
Demonstratio.

Esto ∠lus Polygoni $= \pi$
inde $\angle\beta = \angle\gamma = \frac{1}{2}\pi$.
$\angle\alpha+\beta+\gamma = 2R$. §143. A.
$\angle\alpha + \frac{1}{2}\pi + \frac{1}{2}\pi = 2R$. §10. A.
$\angle\alpha + \pi = 2R$ §47 A.
$\angle\alpha = 2R - \pi$. §43. A. h. e.
∠α = ∠lo ad Centrum Polygoni dati h. l. Pentagoni p. d. ad §38.

Atque inde
Arcus AB = $\frac{1}{5}$ Pphiae.
Sed Chorda AB = BC = CD &c. p. p.
Ergo Polygonum est aequilaterum. §56. A.

Verum et Arcus
BC = CD = DE. §285. A.
Quare cum
AE = ED
ED = DC } p. d.
DC = CB
Arcus AEDC = Arc: EDCB §42. A.
Ergo
∠A = ∠B. §282. A.
Idem cum et de reliquis ∠lis simili Ratione ostendatur.
Ergo Polygonum aequ∠lum §77. A.
Proinde
ABCDE est ordinatum §88. A.
Q. E. D.

72.



§40. Problema XXIII

Areæ cujusdam campeſtris liber permeabilis Ichnographiam perficere h. e. Figuram Areæ campeſtri ſimilem conſtruere.

Reſolutio.

1) Quære Quantitatem Rectarum AB, BC, CD, DE, EA, atque diagonalium BD, DA. §8.

2) Juxta Scalam modicam conſtrue Triangula FKI, FGI, GIH. §98. B. et p. §5 hujus Latera ppalia Lateribus Triangulorum AED, ABD, ~~BDC~~, homologa
D. F. h. e.
Figuram ABCDE ∼ Fig. FGHIK.

Demonſtratio

ƀ ƀ
AB : CD = FG : HI. §172. A.

ſed EA : AB = KF : FG. p. β.

CD : EA = HI : KF. §175. A.

DE : EA = IK : KF. p. d. α.

CD : DE = HI : IK. §173. A.
(ε)

DE : EA = IK : KF. p. c. (α)

EA : AD = FK : FI. p. c.

AD : AB = FI : FG. p. c.

EA : AB = FK : FG. §172. Ar. (β)

AB : BD = FG : GI } p. c.
CB : BD = GH : GI }

AB : CB = FG : GH. §173. A (γ)

ſed CB : CD = GH : HI. p. c. (δ

ƀ ƀ

Quarecum homologa ∠lorum horum Laterum ſint ppalia p. d. et c.
Ergo.

$\angle E = \angle K$
$\angle p = \angle o$
$\angle q = \angle r$
$\angle u = \angle s$ etc. } §355. θ. (a)

Quare cum $\angle p = \angle o$
$\angle u = \angle s$ } p.d.a.

$\angle A = \angle F$. §42. et 47. Ar. (b)

Similiter $\angle B = \angle G$ §§. cc. (c)
$\angle D = \angle I$. (d)

Proindequia

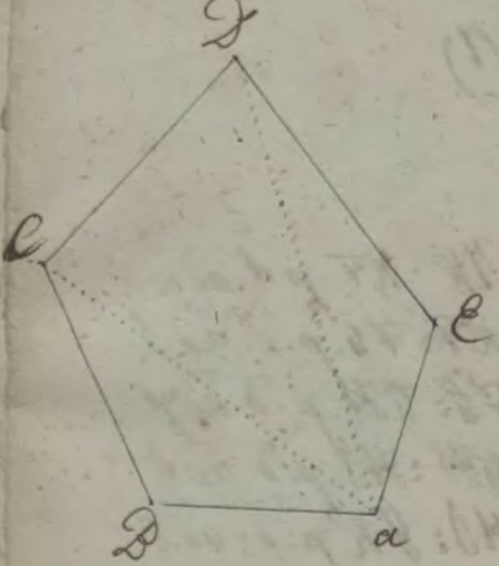
1) $\angle E = \angle K$ p.d. a. et $DE : AE = IK : KF$. p.d. ad α.
2) $\angle A = \angle F$. p.d. b. et $EA : AB = KF : FG$. p.d. ad β
3) $\angle B = \angle G$. p.d. c. et $AB : CB = FG : GH$. p.d. ad γ.
4) $\angle C = \angle H$ p.d. α et $CB : CD = GH : HI$. p.d. ad δ
5) $\angle D = \angle I$ p.d. d. et $CD : DE = HI : IK$ p.d. ad ε.

Ergo

Figura ABCDE ∼ figura FGHIK. §341 θ.
Q.E.D.

Aliter:

1) Pone Mensulam in unum figurae ∠lum Horizonti ∥lam ut punctum in illa acceptum a Vertici illius immineat



2) Collinea verſus ſingulos figu Baculos B, C, D, E atq. duc Re tas in Menſula determinatas: fac ∠los CaD, DaE. §19.

3) Quaere Longitudines Rec rum aB, aC, aD, aE. §8.

4) Hisq. juxta ſcalam modicam fac ppales in Menſula ab, ac, ad, ae. §8.

5) Duc bc, cd, de §81. Th. Dico fig ram aBCDE ∾ abcde.

Demonſtratio.

$\angle BaC = \angle bac$. p.c.

$Ba : aC = ba : ac$. p.c.

$\angle B = \angle cba$ et
$\angle BCa = \angle bca$ } §356. Th.

$Ba : BC = ba : bc$. §352. Th.
(α)

$\angle DaE = \angle dae$ $\angle CaD = \angle cad$.

$Da : aE = da : ae$. $Ca : aD = ca : ad$

$\angle aDE = \angle ade$. } §356 Th. $\angle aCD = \angle acd$. } §356. Th.
$\angle aED = \angle aed$ } $\angle CDa = \angle cda$ }

$aD : DE = ad : de$ } §352 Th. $Ca : CD = ca : cd$. §352. Th.
$DE : aE = de : ae$ }
(γ)

Q.E.D.

Quare cum

$\angle BCa = \angle bca$ p.d. α

$\angle aCD = \angle acd$ p.d. β.

$\angle C = \angle c$. §42. et 47. A. I

Cum $\angle CDa = \angle cda$ p.d. β

$\angle aDE = \angle ade$. p.d. γ.

$\angle D = \angle d$. §§. cc. II

Cum $\angle a = \angle a$ p. C. III

et $\angle B = \angle cba$ p.d. α IV.

et $\angle aED = \angle aed$. p.d. γ V.

aBCDE aeq/gla abcde. §305 C.

Q.E.I.

Porro quia

$aB : BC = ab : bc$. p.d. α I.

et $aB : aC = ab : ac$. p. C.

$aC : CD = ac : cd$ p.d. β.

$aB : CD = ab : cd$. §174. A. II

Sed et $aC : aD = ac : ad$ p. C.

$aC : CD = ac : cd$. p.d. β

$aD : CD = ab : cd$ §174 A.

$aD : DE = ad : de$. p.d. γ.

$CD : DE = cd : de$. §c. III

$DE : aE = \overset{et}{d}e : ae$. p.d. γ. IV

$\bar{b}$ $\bar{b}$

$\bar{b}$ $\bar{b}$

$CD : aE = cd : ae$ §172. A

$BC : CD = bc : cd$. p.d.

$aE : BC = ae : bc$ §.175 A.

$aB : BC = ab : bc$. p.d.

$aE : aB = ae : ab$. §173. A,

γ.

Ergo

Latera homologa utr. figura sunt ppalia

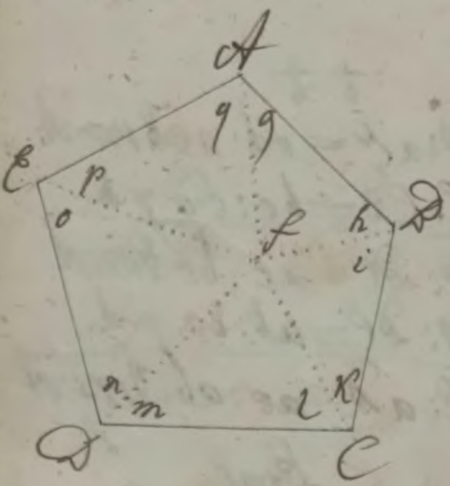
Q.E. II

Proinde

Fig aBCDE ∾ Fig abcde.

§341 C.

Q.E.D.



Aliter:

1) Assumto intra Figuram pcto f, statue Mensulam Horizontalam.

2) Collinea versus Baculos omnes A, B, C, D, E, Aduc Rectas in Mensu indeterminatas. §19.

3) Quaere Quantitates Rectarum A Bf, Cf, Df, Ef. §8.

4) Atq transfer ope scalae §5. ex f Rectarum Af, Bf, Cf puncta, a, b, c, d, e. §8.

5) Junge Rectas ab, bc, cd, de, ea, §81. Dico

Fig. ABCDE ~ abcde.

Demonstratio

$\angle AfB = \angle afb$ et
$Af : Bf = af : bf$. p.C.

$\angle g = \angle s$ }§356. θ.
$\angle h = \angle t$

$Bf : BA = bf : ba$.
$BA : Af = ba : af$. §352. θ. (α)

$\angle BfC = bfc$ et }p.C.
$Bf : fC = bf : fc$.

$\angle i = \angle u$ }§356. θ.
$\angle k = \angle w$

ƀ ƀ
$Bf : BC = bf : bc$ }§352 θ.
$BC : fC = bc : fc$ (β)

$\angle CfD = cfd$ et
$Cf : fD = cf : fd$. p.C.

$\angle l = \angle x$ }§356. θ.
$\angle m = \angle y$

$Cf : CD = cf : cd$ }§352 θ.
$fD : CD = fd : cd$ (γ)

$\angle DfE = \angle dfe$ et
$Df : fE = df : fe$. p.C.

$\angle n = \angle z$ }§356 θ
$\angle o = \angle \alpha$

$Df : DE = df : de$ }§352 θ
$fE : DE = fe : de$ (δ)

$\angle EfA = \angle lfa$ et
$Ef : fA = ef : fa$. p. l.
$\angle p = \angle \beta$ } §356. θ.
$\angle q = \angle r$ }
$Ef : EA = ef : ea$ et } §352. θ.
$EA : Af = ea : af$. } (ε).

Quare cum
$Ef : EA = ef : ea$. p. d. c.
$Ef : DE = ef : de$. p. d. δ
$EA : DE = ea : de$. §174 A. I

$Df : DE = df : de$. p. d. δ
$Df : CD = df : cd$ p. d. γ.
$DE : DC = de : dc$ §c. II

$fC : CD = fc : cd$. p. d. γ.
$fC : CB = fc : cb$. p. d. β
$CD : CB = cd : cb$. §c. III

$Bf : BC = bf : bc$. p. d. β
$Bf : BA = bf : ba$. p. d. α.
$BC : BA = bc : ba$. §c. IV

5

5

$DA : Af = ba : af$. p. d. α
$EA : Af = ea : af$. p. d. e
$DA : EA = ba : ea$. §173 A. V

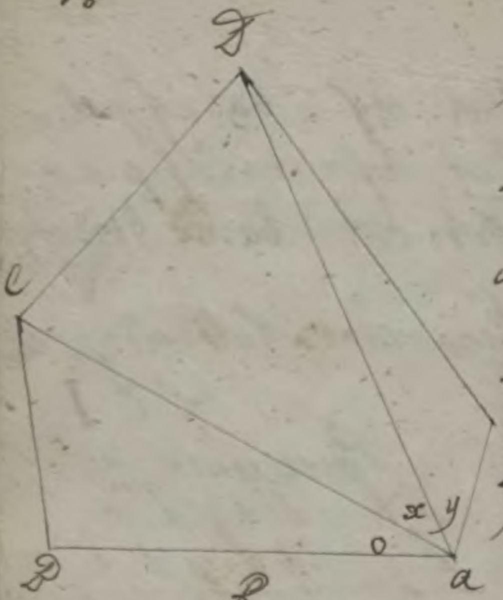
Ergo omnia latera st. ppalia
Q. E. I.

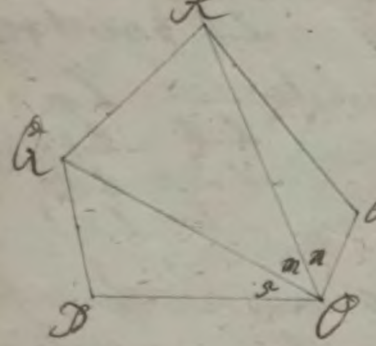
Porro quia
$\angle h = \angle t$ p. d. α
$\angle i = \angle u$. p. d. β.
$\angle B = \angle b$. §4e. et 42 not.

similiter $\angle A = \angle a$ }
$\angle E = \angle e$ } §§. cr
$\angle D = \angle d$ }
$\angle C = \angle c$ }

Ergo omnes anguli homologi aequales adeoque fig. ABCDE ∼ fig. abcde §341

Q. E. D.





Aliter:

1) Collocato Instrumento Goniom
trico cum Horizonte ∥le observ
∠los o, x, y. §19.

2) Quare Quantitates Rectarum
aB, aC, aD, aE. §8.

3) In Charta fac ∠los s = o
m = x } §23.
n = y

4) Atq OP, OQ, OR, ON ppales
Rectis aB, aC, aD, aE §8.

5) Duc Rectas PQ, QR, RN. §81 &
Q. F.

Demonstratio
coincidit cum Resolutionis
2da Demonstratione hujus
§phi

Aliter

1) Assumto intra Figuram
pcto f observa Instrument
Goniometrico legitime collo
cato ∠los AfB, BfC, CfD, DfE

EfA §19.

2) Atq. Rectas Af, Bf, Cf, Df, Ef. §8.

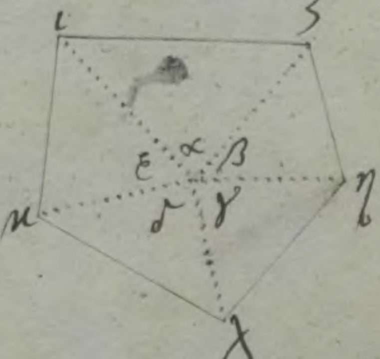
3) In Charta fac angulos
$\angle\alpha = \angle AfB$
$\angle\beta = \angle BfC$
$\angle\gamma = \angle CfD$ } §23.
$\angle\delta = \angle DfE$
$\angle\varepsilon =$ EfA.

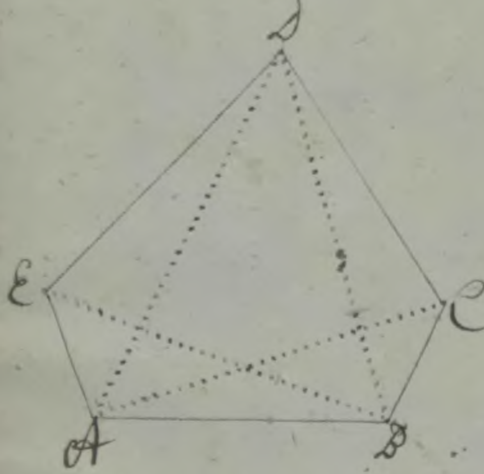
4) Itemq. αι ppalem Af
αμ - - - - Ef
αλ - - - - Df } §5. 8.
αη - - - - Cf
αζ - - - - Bf

5) Duc μι, ιζ, ζη, ηλ, λμ. §81. θ.
D. F.

Demonstratio.

coincidit cum Resolutionis tertiae Demonstratione hujus §phi.





§41. Problema XXIV

Areæ campeſtris Ichnographia
ex duabus ſtationibus perficere.

Reſolutio.

1) Collocata legitime Menſula e
aſſumto in illa puncto B, quod in
mineat pcto Areæ campeſtris B,
collinea verſus ſingula illius Pcta
E, D, C, atq; duc Rectas indetermi
natas, h. e. obſerva ∠los o, x, y. §1
2) Menſura Rectam AB. §8. et
3) Conſtitue huic juxta ſcalam
modicam ppalem ex B in R.
§c ejs.
4) Relicto in B baculo transfer in
A Menſulam ita ut R immine
ipſi A et RB in eodem cum AB
Plano & luq.
5) Ex R collinea verſus omnia fig
ræ puncta E, D, C, B, atq; duc Rect
priores interſecantes in F, G, H §80
6) Duc Rectas FG, GH. &c.

Q. F.

Demonstratio.

$\angle EAB = FRB$. p. l. }
$\angle EBA = \angle o$ p. l. } a.

$\angle AEB = \angle RFB$. §135. θ.
$AB : AE = RB : RF$ }
$AB : BE = RB : BF$ } §352 θ (β).
$\angle DAB = \angle GRB$ }
$\angle DBA = \angle o + x$ } p. ſ.

$\triangle DAB$ aeq$\angle$gl. $\triangle$lo GRB. §155. 301. θ.
$AB : AD = RB : GR$ }
$AB : BD = RB : BG$ } §352 θ.
$AB : BE = RB : BF$. p. d. (β.

$BD : BE = BG : BF$ §174. A
sed et $\angle DBE = \angle x$. p. ſ.

$\angle BED = \angle FBG$. §356.
$\angle AEB = \angle RFB$. p. d. a.

$\angle E = \angle RFG$. §42. 47. A.
sed $\angle EAD = \angle FRG$. p. c.

$\angle EDA = \angle FGR$. §155. θ.

Ergo

$AE : ED = RF : FG$. }
et $ED : DA = FG : GR$, } §352 θ. (e)

$$\begin{array}{l}
\angle CAD = \angle HRD \\
\angle D = \angle o + x + y \text{ p. C.} \\
\hline
\angle ACD = \angle RHD \text{. §155.}
\end{array}$$

Ergo

$$\left.\begin{array}{l}
CD : DA = HD : DR \\
CA : AD = HR : DR
\end{array}\right\} \text{§352 } \theta \text{ } (\gamma)$$

sed

$$\begin{array}{l}
AD : AD = RD : GR \text{. p. d. } \gamma. \\
\hline
CA : AD = HR : GR \text{. §172 A.} \\
\text{sed } \angle DAC = GRH \text{ p. C.} \\
\hline
\left.\begin{array}{l}
\angle ADC = \angle RGH \\
\angle ACD = RHG
\end{array}\right\} \text{§356 } \theta.
\end{array}$$

$$\left.\begin{array}{l}
AD : DC = GR : HG \text{. §352 } \theta. \\
ED : AD = FG : GR. \\
\hline
DC : ED = FG : FG \text{. §175. A} \\
ED : DC = FG : HG \text{. §146 A}
\end{array}\right\} 1.$$

$$\begin{array}{l}
\angle DCA = GHR \text{. p. d. } \delta. \\
\angle ACD = R[illegible]D \text{. p. d. } 3 \\
\hline
\angle C = \angle [illegible] \text{ §42. 47. A.} \\
\angle DCD = \angle y \text{. p. C.} \\
\hline
DC : CD = GH : HD \text{. §305.} \\
\qquad 352 \theta. \ (x)
\end{array}$$

Tandem.

$\angle ADC = \angle RGH$. p.d. δ.
$\angle EDA = \angle FGR$. p.d. δ

$\angle D = \angle G$. §42. 47. A.

Cum ergo.

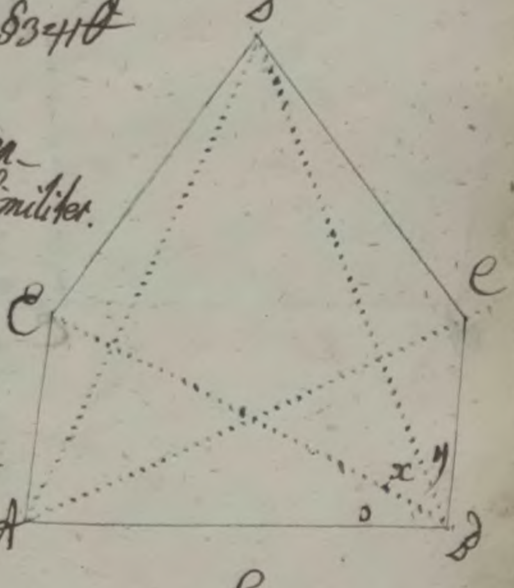
1) $\angle A = \angle R$. p. C. et $AB : AE = RB : RF$. p.d. α.
2) $\angle E = \angle F$ p. δ. et $AE : ED = RF : FG$. p.d. ε.
3) $\angle D = \angle G$ p. π. et $DE : CD = FG : HG$ p.d. κ
4) $\angle C = \angle H$ p. κ et $DC : CB = GH : HB$ p.d. κ
5) $\angle B = \angle o+x+y$. p. C. et $CB : BA = HB : BR$. p.d. η.

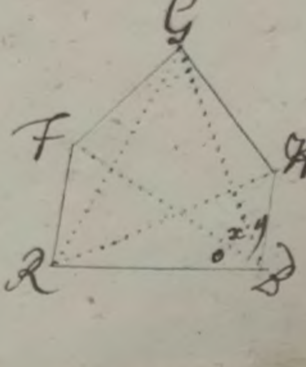
Proinde

Figura ABCDE ∼ RFGHB. §34 b

Aliter.

1) Ex statione B observa Instrumento goniometrico $\angle$los o, x, y §19. similiter.
2) Ex statione A $\angle$los EAB, DAC, CAB. &c
3) Mensura Rectam AB. §8.
4) In Charta ad AB ppalem constitue ipsi AB §8.5. et
5) Fac ad B $\angle$los o, x, y = $\angle$lis o, x, y. §23. Similiter ad punctum A $\angle$los FRG = $\angle$EAB
GRH = $\angle$DAC } &c.
HRB = $\angle$CAB





6) Junge Puncta concursuum F, G, H, Rectis FG, GH, § 81 G. D. F.

Demonstratio

Coincidit cum proxime antecedente; poterat vero et per § 374. G. erui.

§ 42. Problema XXV.

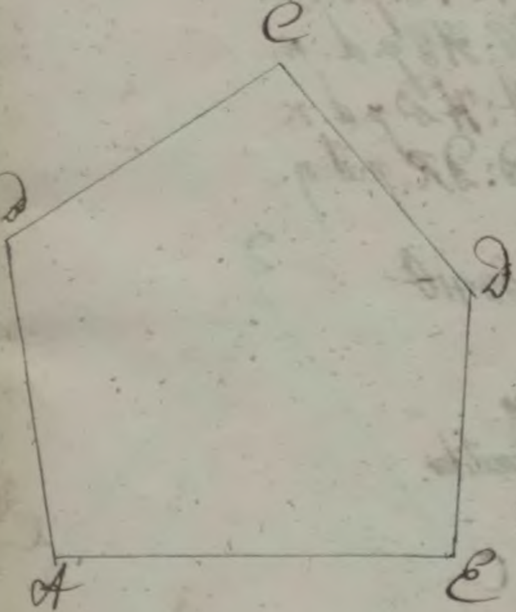
Campestris Areae Ichnographiam parare cujus integrum Perimetrum peragrare licet.

Resolutio

1) Mensula vel Instrumento Goniometrico legitime collocatis, in singulis ∠lorum Verticibus observa ∠los A, B, C, D, E. § 19.

2) Quaere Rectarum BA, AE, ED, DC, CB, Quantitates § 8.

3) Hisq. et Lineas ppales et ∠los aequales constitue homologos homologis § 8. 23. in charta. D. F.



Demonstratio

Nam omnes ∠li in Campo sunt aequales omnibus ∠lis homologis in Charta p. C. Praeter vero illos et Latera homologa ppalia

Ergo.

Figura in Campo ∾ Figura in Charta §321 G.
Q.E.D.

§42 Problema XXVI.

Figura in Charta delineatae similem in Campo perficere.

Resolutio.

1) Angulos in Charta constitue aequales homologos in Campo. §24.

2) Ex Verticibus ipsorum utrinq. in Cruribus designa ope Catenae vel Funiculi Quantitates Rectarum ppalium Rectis in Charta descriptis.
Q. F.

§44. Scholion.

Conversum hoc est Problema antecedentium, adeoq. et Casus eosdem cum Resolutionibus et Demonstrationibus admittit, quos attulimus a §40–42.

§45. Definitio 2.

Perticam quadratam aut Decempedam quadratam dicimus, cujus Latus est Pertica vel Decempeda
similiter Pedem, Digitum

et Lineam quadratam dici mu
cujus Latus est Pedi, Digito et Lin
equale.

§46. Hypotheſis 3.

Scribemus autem Perticas aut Decempedas quadratas illarum ſigno §3. dextrorsum addentes: q. v. c. duas Perticas quadratas $2^{\circ q}$. ſimili characteriſtica ſignificabimus pedes, digitos et lineas quadratas, ſignis §3. allatis addendo q. v. c

Septem pedes quadratos h. m: $7^{\prime q}$
ſex lineas qdtas — — — : $6^{\prime\prime\prime q}$

Addunt alii ſigna Perticarum aut Decempedarum, Pedum, Digitorum et Linearum Valorem quadrati cum expreſſuri ſignum $\square$. Novem ergo Perticas aut Decempedas ita ſcribunt: $9^{\circ\square}$ ſimiliter cum reliquis.

§4[illegible] Corollarium 1

Quia

$1^{\circ} = 10^{\prime} = 100^{\prime\prime} = 1000^{\prime\prime\prime}$ §31.

$1^{\circ} = 10^{\prime} = 100^{\prime\prime} = 1000^{\prime\prime\prime}$. §c.

$1^{\circ q} = 100^{\prime q} = 10000^{\prime\prime q} = 1000000^{\prime\prime\prime q}$ §45.46

et $1^{\prime} = 10^{\prime\prime} = 100^{\prime\prime\prime}$ §3.1.

$1^{\prime} = 10^{\prime\prime} = 100^{\prime\prime}$ §c

$1^{\prime q} = 100^{\prime\prime q} = 10000^{\prime\prime\prime q}$ §45.46.

tandem: $\left.\begin{array}{l} 1^{\prime\prime} = 10^{\prime\prime\prime} \\ 1^{\prime\prime} = 10^{\prime\prime\prime} \end{array}\right\}$ §3.1.

$1^{\prime\prime q} = 100^{\prime\prime\prime q}$. §45.46.

Proinde

Centum Lineæ quadratæ Digitum Quadratum, Centum digiti quadrati Pedem quadratum, centum pedes quadrati Decempedam quadratam conficiunt. ut h. m. Decempeda quadrata æquet centum Pedes, decem Digitorum millia Linearum Millionem.

§48 Corollarium 2.

Quare, data Area Rectilinea cujuslibet in Lineis expeditissima

est illius Resolutio in Digitos, Ped
et Decempedas, ternas abscindend
a dextra versus sinistram duas figu
numerica prima scilicet: duo dec
ma pro Lineis secunda duo pro Dig
tis, tertia duo pro Pedibus quadra
tis; quod reliquum est, exhibet De-
cempedas. §47.

§49. Corollarium.

Quia.

$$\left.\begin{aligned} 1^{\circ}\,\text{Rhl.} &= 12' = 144'' = 1728''' \\ 1^{\circ} &= 12' = 144'' = 1728''' \end{aligned}\right\}\ \S 2.$$

$$1\,\text{Rhl.} = 144'^{\square} = 20736''^{\square} = 2985984'''^{\square}.$$

§222. A

Porro quia.

$$\left.\begin{aligned} 1\,\text{Rhl. ml.} &= 12'' = 144''' \\ 1' &= 12'' = 144''' \end{aligned}\right\}\ \S 2.$$

$$1'^{\square}\,\text{Rhl.} = 144'^{\square} = 20736'''^{\square}.\ \S 222\,\text{A}.$$

Tandem quia.

$$\left.\begin{aligned} 1''\,\text{Rhinl.} &= 12''' \\ 1''\,\text{Rhinl.} &= 12''' \end{aligned}\right\}\ \S 2.$$

$$1''^{\square}\,\text{Rhinl.} = 144'''^{\square}\ \S 222.\,\text{A}.$$

Ergo,

144'''q Rhl. efficiunt digitum }
144''q — — — — pedem } quadratam
144'q — — — — perticam }

Quod si ergo resolvendæ fuerint v. c.
11766872'''q Rhl. in Digitos, Pedes
et Perticas divide Lineas per 144.
ut Digiti, Digitos per 144 ut Pedes,
Pedes per 144 ut tandem Perticæ
innotescant. Schema Calculi.

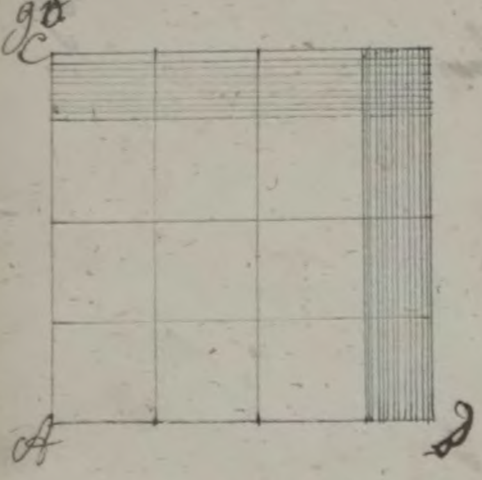
```
144) ~~11766872~~'''q | ~~81714~~''q | ~~567~~'q | 3°q .
     ~~3542056~~      | ~~39376~~     | ~~745~~
     ~~204687~~       | ~~1809~~      | 13
     ~~132225~~       | ~~136~~
       ~~1~~
```

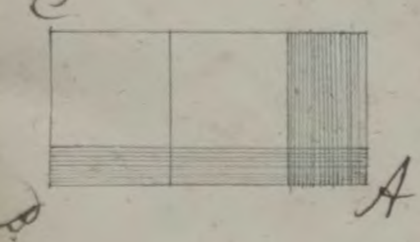
h. e.

11766872'''q Rhl. = 3°q, 135'q, 66''q, 56'''q.

Similiter in aliis

Inde quidem liquet ratio cur
Geometræ Perticam in Decempedam mutarint, ne scilicet tædia
repetitæ aliquoties divisionis
devorare cogerentur.





§50. Problema XXVII.

Aream Parallelogrammi producen

Casus I. si fuerit rectangulum.

Resolutio et Demonstratio.

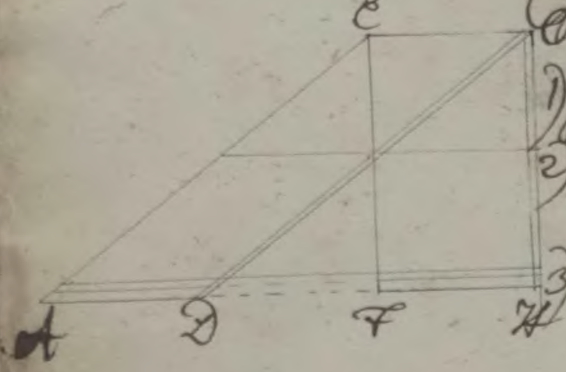
Quia Parallelogrammi rectanguli Area aequalis est facto ex Basi in Altitudinem §175 G. Hinc quaesitam §8. Basin atque Altitudinem duc in semet invicem, eritque factum Area quaesita. Q. E. R. et D.

Sit AB = AC = 35'. Ergo
AB×AC = AB² = 12,25'q §175. G.
Q. E. 48.
Esto AB = 26'. BC = 13' Ergo
AB×BC = 3,38'q §§. c.c.
similiter in aliis

Casus II si fuerit obliquangulum

Resolutio

1) In productam AB. §82. G.
2) Demitte ex C vel E ⊥lem §119 G.
EF vel CH.
3) Hanc multiplica in Basin AB aut EC. Q. F.



Demonstratio.

EH est Pllgm. rect∠glm. p. C.
AH ≃ EC p. H.

EH = AC. §174 G.
EH = EF × EC. §175. G.
EC = AD. §167. G.
EH = EF × AD. §10 Ar.

AC = EF × AD. §41 Ar. Q. E. D.

Schema Calculi.

Sit AD = EC = FH = 11.
et EF = 22.

22
22

AC = EF × AD = 2°, 42'. §46.

§57. Problema XXVIII.

Aream cuiuslibet Quadrilateri rectilinei GD producere.

Resolutio.

1) Duc Diametrum AC. §87. G.

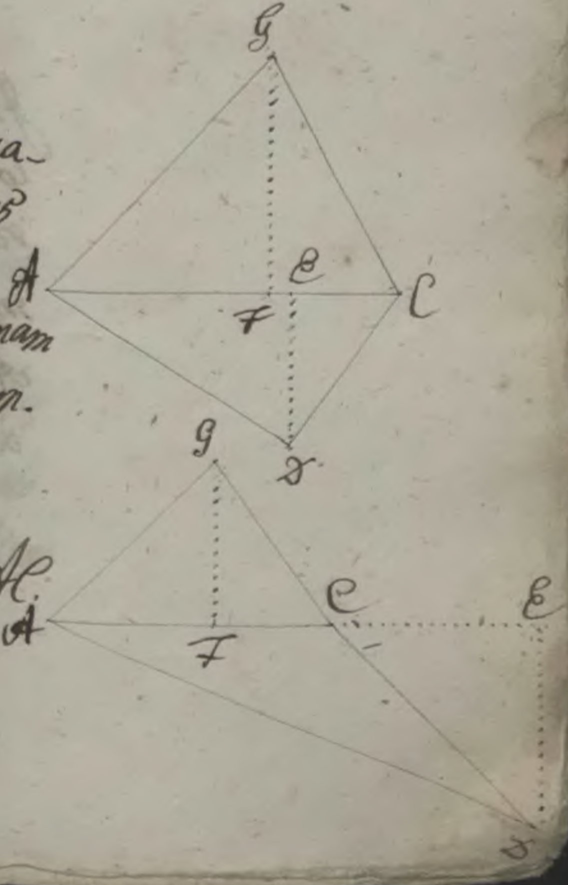
2) In hanc vel ipsam vel continuatam demitte ⊥les ex verticibus Triangulorum §119 G.

3) In Diametrum AC duc summam Altitudinum EF et ED bisectam.

vel

In summam Altitudinum duc Diametrum bisectam ½AC.

Q. F.



Demonstratio.

$AGCD = \Delta AGC + ADC$. §47. Ar.

sed $\Delta AGC = AC \times \tfrac{1}{2} FG$. $\Delta ADC = AC \times \tfrac{1}{2} ED$ } §182. G.

$\Delta AGC + ADC = AC \times \tfrac{1}{2} FG + AC \times \tfrac{1}{2} ED$ §42 A.

$AGCD = AC \times \tfrac{1}{2} FG + AC \times \tfrac{1}{2} ED$. §41.

$= AC \times \left(\tfrac{1}{2} FG + \tfrac{1}{2} ED\right)$ §31 A.

$= AC \times \dfrac{FG + ED}{2}$ §c

Vel.

$AGCD = AC \times \tfrac{1}{2}\left(FG + ED\right)$ §185. A.

$= \dfrac{AC}{2} \times \left(FG + ED\right)$ §c.

Q. E. D.

Indequidem compendio cuidam locus est, si vel Basis vel Altitudinum summa bisecari possit, si minus; tota Basis in summam Altitudinum ducitur hocq tandem Productum bisecatur.

Schema Praxis.

Sit $AC = 244''$,
$GF = 201''$
$DE = 117''$

Ergo $\frac{GF+CD}{2} = \frac{318}{2}$

$= 159$

$AC = 244$

636

636

318

$AC \times \frac{GF+CD}{2} = 3^{0}, 7\,8^{7'}, 9\,6'''\,9$ f §40.

$= AGCD$

Vel

$\frac{1}{2}AC = 122''$

$GF + DC = 318$

976

122

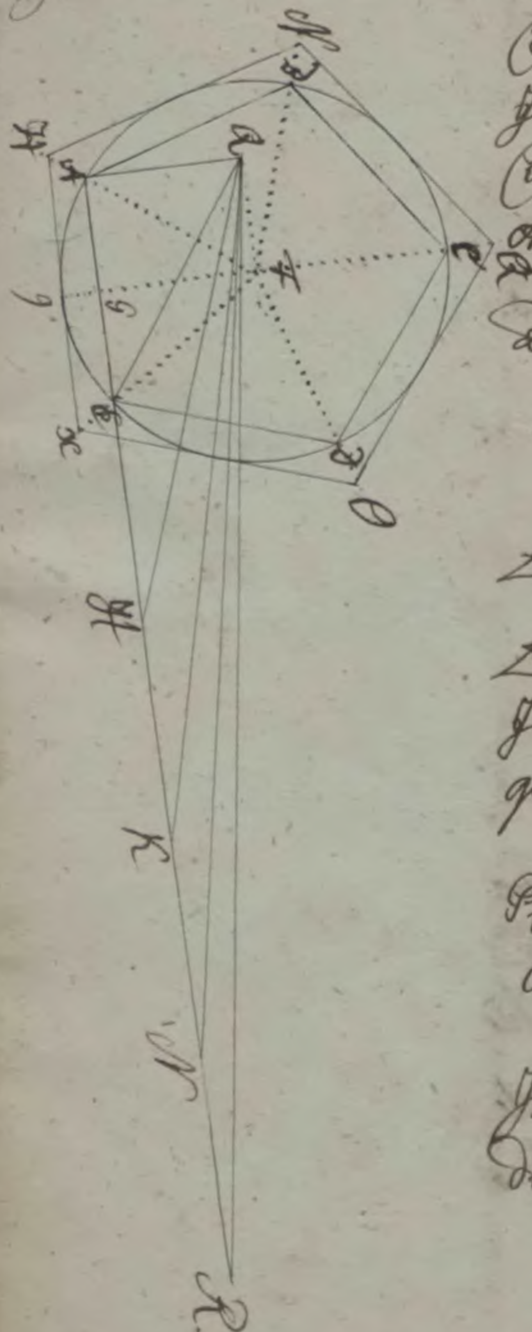
366

$\frac{1}{2}AC \times \overline{GF+DC} = 3^{0}, 7\,8\,7,'\,9\,6'''\,9$ &c.

$= AGCD$.

§52. Theorema 2.

Figura regularis ABCDE ex centro circuli circumſcripti F in Triangula æqualia et ſimilia reſolvitur. Area ejus æqualis eſt Triangulo cujus Baſis eſt Perimeter totius Figuræ AB × BC, + CD, + DE, + EA,



Altitudo autem normalis Fg ex Centro F in Latus unum demissa. Idem valet et de Area circumscrip Circulo Polygoni ordinati nisi quod Altitudo sit Radius Fg.

Demonstratio.

Duc Radios AF, FB, FC, FD, FE. §81 θ.

Quia $AB = AE$. p. H

$BF = AF$ } §26 θ.

$AF = FE$ }

$\triangle ABF$ congruit $\triangle AFE$. §106 θ.

Ergo

$\triangle ABF \sim$ et $= \triangle AFE$. §86 θ.

Idem simili modo liquet de reliquis Triangulis §.§. cc. 381 θ. et 44

Q. E. I.

Produc AE in R. §81. θ ut

$AR = AE + ED + DC + CB + BA$

$= 5 \times AE$.

In A fac ⊥lem $AQ = Fg$. §158 θ

duc QF, QE, QH, QK, QN, QR.

§81 θ.

Quia AQ = FB. p. C.
et AQ ∾ FB. §138 Th.

QF ∾ EA. §139. Th.
EA = EA. §40. Ax.

QEA = Δ FEA. §177. Th.

Porro;

EH = EA. p. H et C.
Δ EHQ = Δ QEA §178. Th.
Δ QEA = Δ EFA. p. d.
Δ EFA = Δ AFB. p. d. M. 1.

Δ EHQ = Δ AFB. §41 Ax. sic et
Δ QHK = Δ BFC
Δ KQN = Δ CFD
Δ NQR = Δ FDE

Δ QAR = Polygono ABCDE. §42. 47 Ar.

Q. E. II.

Hx est Tangens p. H. et §301. Th. et
Fg est Radius Circuli inscripti §. c.
inde Fg est Normalis ad ~~Hx~~. §241 Th.
adeoq. Fg altitudo Δli FHx. §126. Th.
Reliqua demonstrabis uti Mbr. 1 et
11. hujus §i

Q. E. III. D.

§53. Problema XXIX.

Aream cujuslibet Polygoni ordina
invenire.

Resolutio

Fig. 6 ph. 52. 1) Latus Polygoni duc in dimidium la
terum Numerum.

2) Factum duc in ⊥lem aut FG pro
Figura tua vel circumscripta ve
inscripta Circulo. D. F.

Demonstratio.

Polygonum ABCDE = Δ QAR. §5

Δ QAR = $\frac{AR}{2}$ × AQ. §182. θ.

Sed AR = Num. Lat. Polyg.

Ergo $\frac{AR}{2} = \frac{\text{Num. Lat. Polyg}}{2}$

et AQ = FG. p. d. ad §52.

Ergo

~~et AQ = FG. pd~~

Δ AQR = $\frac{\text{Num Lat Polyg} \times \text{F}}{2}$ §10 A.

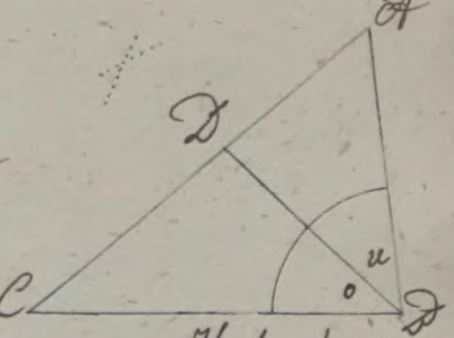
Ergo

Polyg. ABCDE = $\frac{\text{Num. Lat. P}}{2.}$ × FG p §41 A.

Q. E. D.

§54 Theorema 3.

Si in Triangulo ACB ∠lus B bisecetur Recta BD secante quoq[ue] Latus oppositum AC; erit summa Cruris et Baseos ∠lo bisecto adjacentium ad crus ipsi oppositum, uti Basis ad segmentum CD inter eam et secantem BD interceptum.



p. H dmdm

$AB+BC : AC = CB : DC$

Demonstratio.

Quia ∠o = ∠u. p. H et bisecans BD ∠lum, secat quoq[ue] basin AC p. H.

Ergo

$DA : DC = AB : BC$. §351 G.

Ergo

$AB : BC = DA : DC$.

Ergo

$AB + BC : BC = DA + DC : DC$. §168 A.

$AB + BC : BC = AC : DC$. §10 A.

Proinde

$AB + BC : AC = BC : DC$. §150 A.

Q. E. D.

§55. Theorema 4.

Circulus est aequalis Triangulo cujus Basis est Peripheria, altitudo autem Radius.

Demonstratio.

Polygona Circulo in infinitum inscripta in Circulum desinunt §49 r. 6. adeoq. et Latera Polygoni hujus et normales ex Centro ad illa demissae in Pphia terminantur, indeq. Circulus idem est cum Polygono h. m. inscripto. Enimvero Area Polygoni ordinati aequalis est Triangulo cujus Basis est Perimeter, Altitudo autem ⊥lis ex Centro ad latus unum demissa. §52. Quare Area Circuli aequalis est Triangulo cujus Basis est Pphia, Altitudo autem Radius. §40 A.

Q. E. D.

§56 Theorema 5.

Polygonum Circulo inscriptum minus, circumscriptum autem majus est Circulo.

Demonstratio

Polygoni inscripti Latera sunt Chordae Arcuum cognominum §300 θ sed Arcus sunt Chordis majores. §118 θ.

Ergo Polygonum inscriptum aequatur Parti Circuli.

Ergo Polygonum inscriptum minus est Circulo. §12. A.

Q. E. 1.

Latera Polygoni circumscripti tangunt Circulum §301 θ

Ergo tota ex Circulo~~m~~ cadunt §241. θ

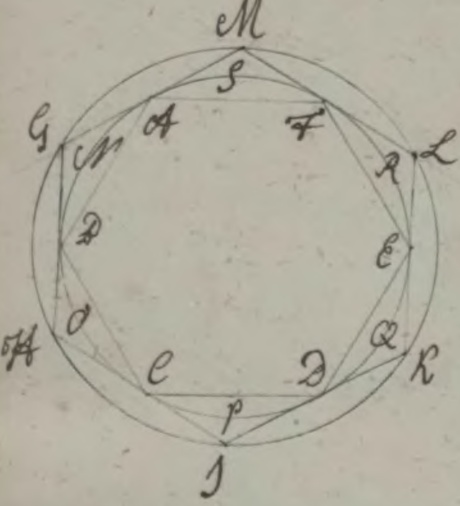
Ergo Circulus aequat Partem Polygoni circumscripti

Ergo Circulus minor Polygono circumscripto. §12. A.

Q. E. II. D.

§57. Theorema 6.

Perimeter ordinati Polygoni Circulo inscripti minor, circumscripti autem major est ejusdem Peripheria.



Demonstratio.

Quia BA, BC, CD, DE, EF, FA sunt latera Polygoni Circulo inscripta p. H. Ergo tota intra Circulum cadunt. §300. 252. θ. Ergo Chorda

$$\left.\begin{array}{lll} BA & < \text{Arc.} & ANB \\ BC & < & BOC \\ CD & < & CPD \\ DE & < & DQE \\ EF & < & ERF \\ FA & < & FSA \end{array}\right\} \text{§118 θ.}$$

$BA + AF + FE + ED + DC + CB < ANBOCPDQERFSA.$

§42. 47. A. Q. E. 1.

Area Polygoni circumscripti = Perimetro × ½ Rad §182 θ.

Area Circuli = Pphia × ½ Radium §182. θ. et §55.

Ergo

Area Polyg: Ar. Circ = Perim × ½ Rad : Pph × ½ Rad §145 A.

Area Polyg: Ar. Circuli = Perim : Pphiam. §160 A.

sed Area Polygoni > Ar Circuli §56.

Ergo Perim. Polyg. circumscr. > Pphia Circ, inscripti §132 A.

Q. E. II. D.

§58. Theorema 7.

Circuli Pphia habet ad ſuam Diametrum Proportionem minorem quam $3\frac{1}{7}$ ſive $3\frac{10}{70} : 1$. Et majorem quam $3\frac{10}{71} : 1$. h.e. dmdm.

Cf Fig pag 108.

Pphia : Diametr. $< 3\frac{10}{70} : 1$.
Pphia : Diametr. $> 3\frac{10}{71} : 1$.

Demonſtratio

Circulo circumſcribe Hexagonum ordinatum §329. G. atq biſeca ∠lum A. §108. G.

Quare

∠DlA = R. et } §111 G.
Dl = $\frac{1}{2}$ DZ }
ſed DZ = DA. §55. G.
Dl = $\frac{1}{2}$ DA. §10 A.
cumq ∠A = $\frac{2}{3}$ R. §115. G.
Ergo $\frac{1}{2}$ ∠A = $\frac{1}{3}$ R. §45. A.
h.e. ∠DAl = $\frac{1}{3}$ R. §10 A.

Quodſi ergo ponatur
DA = 1000 erit
Dl = 500

$AD^2 = 1000000$
$DC^2 = 250000$

$AD^2 - DC^2 = 750000 \,(866+$

64
11 | 0.0
(1 | 6)
9 | 96
1 | 04 | 0.0
17 | 2
1 | 03 | 56
44ff.

Hinc
$\sqrt{AD^2 - DC^2} = 866+$
$\sqrt{AD^2 - DC^2} = AC$ p.d.

$AC = 866+$ §41 A.

$AC = 1866$
$AC = 1866$

11196
11196
14928
1866

$AC^2 = 3481956$
$CG^2 = 250000$

$AC^2 + CG^2 = 3731956$

Proinde
$\sqrt{AD^2 - DC^2} = AC$. §195 G.
h.e. $866+ = AC$.
Ergo $866 < AC$
Sed $500 = DC$.
Ergo
$AC : DC > 866 : 500$ §181 A.
Quare
bisecto ∠lo DAC §108 G. Recta GA
erit
$CA + AD : DC = CA : CG$. §54.
Sed $CA > 866$ p.d.
$AD = 1000$ p. H.

$CA + AD > 1866$. §42 A.
Ergo
$CA + AD : DC > 1866 : 500$
§181 A.

proinde
$CA : CG > 1866 : 500$. §46 A.
h.e.
qualium CA est 1866 et paullo
+ talium CG est 500

Est autem

$\triangle$ GCA habet ∠ glm p. d. Ergo

$\sqrt{CA^2+CG^2}$ = AG. §195 b. h. e.

1931, 81 = AG. adeoq.

1931, 8 < AG. hinc

AG : CG > 1931, 8 : 500. §181. A.

Dissecto rursum ∠lo GCA recta Linea

AH. §108 b.

GA + CA : CG = CA : CH. §57.

Sed AG > 1931, 8 p. d.

CA = 1866 + p. d.

GA + CA > 3797, 8. §42 A.

GA + CA : CG > 3797, [illegible] 8 : 500 §181 A.

CA : CH > 3797, 8 : 500. §46. A.

Ergo

talium CA est 3797, 8

qualium CH est 500.

$AC^2+CG^2 = 3731956\ (1931,8+$

```
   1
  ---
  2|73
  (2
 2|61
  ---
  12|19
  (3|8)
  11|49
  -----
    |70|56
    |38|61
    ------
    31|95|00
    (3|86|2)
    30|90|247
```

$CA = 3797,8+$

$CA = 3797,8+$

3038 24
265846
341802
265846
113934

$CA^2 = 14423284,84$

$CH^2 = \quad 250000$

$CA^2 + CH^2 = 14673284,84$

104.

$CA^2 + CH^2 = 14673284,84$ [illegible] 0,5+

```
      9
    5|67
    (6)
    5|44
     23|32
     (7|6)
     22|89
        43|84
        76|6
     43|84|84.
     (7|68|0)
     38|30|25
```

$CA = 7628,3$+

$CA = 7628,3+$

```
   228849
  610264
 152566
457698
533981
```

$CA^2 = 58190960,89$.

$CK^2 = 250000$

$CA^2 + CK^2 = 58440960,89$

Porro.

ob ∠lum ad C Rectum p.d.

$\sqrt{CA^2 + CH^2} = HA$. §195. G.

h.e. 3830,5+ = HA. ergo

$3830,5 < HA$. §47 A.

cumq. $500 = CH$. p.d.

$HA : HC > 3830,5 : 500$. §181. Ar.

Quare

bisecto rursus ∠lo HAC Recta KA §108. G. erit

$HA + AC : CA = AC : CK$. §57.

sed $HA > 3830,5$

$CA = 3797,8$

$HA + CA > 7628,3$

sed $CH = 500$. p.d.

$HA + AC : CH > 7628,3 : 500$ §181. Ar.

h.e.

$AC : CK > 7628,3 : 500$.

Proinde

qualium CK est 500.

talium AC erit 7628,3 et

paullo +

Porro:

ob $\angle$lum l = R. p. d. erit

$\sqrt{CK^2 + lA^2} = KA$. §195. G.

h. e. $7644,6+ = KA$.

hinc $7644,6 < KA$. §477 Ar.

cumque $CK = 500$ p. d.

Ergo

$KA : CK > 7644,6 : 500$ §181 A.

Inde quidem

bisecto rursus $\angle$lo KAl Recta

LA. §108. G. erit:

$KA + Al : lK = Al : lL$. §54.

Sed $KA > 7644,6$ p. d.

$Al = 7628.3+$

$KA + Al > 15272,9+$

atque ob $lK = 500$. p. d.

$KA + Al : lK > 15272,9 : 500$ §181 A.

adeoque et

$Al : lL > 15272,9 : 500$. §45 A.

h. e.

qualium lL est 500.

talium Al est 15272,9. et

paullo +

Quare tandem ob ∠los
DAC, GAC, HAC, KAC bisectos
p. C. erit:

CG dimidium Latus Polyg. circumscr. 12
HC dimid. Lat. Polyg. circumsc. 24
CK dimid. Lat. Polyg. circumsc. 48
LC dimid. Lat. Polyg. circumsc. 96

p. d. ad §329. 328. 340. θ.

adeoq.

96 × Ll dabit dimidiam Perimetrum Polygoni ordinati Circulo circumscripti.

cumq. $Ll = 500$

$\beta\, 96 = 96$

$96 \times Ll = 48000$

et $AL = 15272,9+$

erit $AL > 15272,9$.

Proinde.

$\frac{1}{2}$ Perim. Polyg. 96 Lat. $\frac{1}{2}$ Diam $< 48000 : ~~15~~$
$15272,9$

h. e.

Perim. Polyg. 96 Lat. : Diam. $< 48000 : 15272,9$

§159 et 46

Perim. Polyg. 96 Lat. Diam. $< 48000,0 : 15272,9$.

adeoq.

Perim. Polyg. 96 Lat. Diam. $< \frac{48000,0}{15272,9} : 1$. §160 A.

h. e.

Perim. Polyg. 96 Lat : Diam $< 3,\frac{2181,3}{15272,9} : 1$.

Sed $\frac{2181,3}{15272,9} = \frac{1}{7}$ fere.

cum $7 \times 2181,3 = 15269,1$.

ergo

Perim. Polyg. 96. Lat. Diam. $< 3\frac{1}{7} : 1$.

Est autem Peripheria Circuli minor Perimetro Polygoni circumscripti §87.

ergo a fortiori.

Pphia Circuli : Diametr. $< 3\frac{1}{7} : 1$.

cumq. $3\frac{1}{7} = 3\frac{10}{70}$ §203 A.

ergo tandem

Pphia Circuli : Diametr. $< 3\frac{10}{70} : 1$.

Inſcribe Circulo Triangulum aeq
laterum §337. Θ. ADE.
Biſeca Arcum DE §287. Θ. Rect
CA, et duc DC. §81 Θ.
Quarecum DE = $\frac{1}{3}$ Pphiæ p. C.
Ergo $\frac{DE}{2} = \frac{1}{6}$ Pphia.
adeoq. ſubtenſa DC eſt latus
Hexagoni ordinati §286. 34

Similiter
biſectis arcubus CD in G
CG in H
HC in K
KC in L
} §287 Θ.

erit
GC Latus Polyg. ordin 12.
HC - - - - - - - 24
KC - - - - - - - 48
LC - - - - - - - 96
} Late §2 340

Ductis ergo GA, HA, KA, La
∠li DAC, GAC, HAC, KAC biſecan
§282 Θ.

Nam.

Arcus $DG = GC$. p. C.

Ergo $\angle DAG = \angle lo\ GAC$. §282. θ.

Sed $\angle DAC = \angle DAG + GAC$. §42 A.

$= \angle DAG + DAG$. §100 A

$= 2 \times DAG$.

$\frac{1}{2} \angle DAC = \angle DAG$. §45. Arg.

Quia vero Recta CA arcum DE biſecans et Chordam DE biſecat per Reſolut. ad §287. θ. Ergo.

CA tranſit per Centrum §254. θ.

CA ergo eſt Diameter §250. θ.

et CAD ſemicirculus §84. θ.

Ergo $\angle D = R$. §288.

Similiter

CGA
CHA } ſunt ſemicirculi §84 θ.
CKA

atq;

$\angle$li G, H, K atq; $\angle$ Recti §288. θ.

Quare ponendo.

$AC^2 = 4000000$
$-CD^2 = 1000000$
$AC^2 - CD^2 = 3,000000 \,(1732,0+$

1
2|00
(27)
1 89
11.00
(34)
10 29
71.00
(346)
69 24
1 76|00
346|4 ff.

h. e.

$\sqrt{AC^2 - CD^2} = 1732,0+$
$\sqrt{AC^2 - CD^2} = DA$ p. d
$DA = 1732,0+$ §41. Ar.

Ergo.

AC h. e. Diametrum = 2000
quia $CD = \frac{1}{2} AC$. §336. erit
Latus Hexagoni = 1000
Ergo
$\sqrt{CA^2 - CD^2} = DA$. §195. Ө. h. e.
$1732,0+ = DA$
$1732,1 > DA$ sed
$1000 = CD$ p. d.
$1732,1 : 1000 > DA : DC$. §181. A.
Per primam ergo Arcus DC biſec
onem in G p. C.
Quia $GD = GC$ p. C.
$\angle GCD = \angle GAC$. §282. Ө.
et $\angle G = \angle G$. §40. A.
Δ GFC aeqiangl. Δlo GAC. §155. 305
ergo
$CA : CF = GA : CG$. §353. Ө sed
$AD + AC : CD = CA : CF$. §354.
$AD + AC : CD = GA : CG$. §144. A.
cumque $AD < 1732,1$. p. d.
et $AC = 2000$. p. H.
$AD + AC < 3732,1$. §42 Ar.
sed $CD = 1000$. p. d
$AD + AC : CD < 3732,1 : 1000$. §181
$GA : GC < 3732,1 : 1000$ §46

h. e.
qualium Particularum CG eſt 1000
talium eſt. AG 3732, 1 et paulo —

Eſt autem CG = R. p. d.
Ergo $\sqrt{GA^2 + GC^2} = CA$. §195. Θ.
h. e. 3863,7 + = CA.
Ergo 3863,8 > CA
cumq; CG = 1000. p. d.
Ergo
3863,8 : 1000 > CA : CG. §181. A.

Porro
per Arcus GH biſectionem erit
GH = HC. p. C.
adeoq; ∠HCG = ∠HAC. §282. Θ.
cumq; ∠H = ∠H. §40. A.
△HCM. aeqlgl. △HAC. §155. 305. Θ.
Ergo
CA : CM = AH : HC. §353. Θ.
GA + AC : CG = CA : CM. §54.
GA + AC : CG = AH : HC. §144. A.
Sed GA < 3732,1. p. d.
AC < 3863,8 p. d.
GA + AC < 7595,9. §47. A.
cumq; CG = 1000. p. d.
7595,9 : 1000 > GA + AC : CG. §181. A.
et
AH : HC < 7595,9 : 1000 §46. A.

GA = 3732,1
GA = 3732,1
37321
74642
111963
261247
111963
$GA^2 = 13928570,41$
A. $GC^2 = 1000000$
$GA^2 + GC^2 = 14928570,41$ | 3863,74 $= \sqrt{GA^2 + GC^2}$
9
5 92
(6)
5 44
48. 85
(76)
45 96
2 89 70
(772)
2 31 69
58 01 41
(7 72 6)
54 08 69

112.

$HA = 7595,9$

$HA = 7595,9$

683631

379795

683631

379795

131713

$57697696,81$

$HA^2 =$

$CH^2\ 1000000$

$HA^2 + CH^2 = 58697696,81 \,|\, 7661,4+$

$= \sqrt{HA^2 + CH^2}$

49

9 69

6 4)

8 76

93 76

(15 2)

91 56

2 20 96

1 53 21

67 75 81

(15 32 2)

61 28 96 71

$AK = 15257,4$

$AK = 15257,4.$

610296

1068018

762870

305148

762870

152574

$AK^2 = 232788254,76$

$CK^2 = 1000000$

$AK^2 + CK^2 = 233788254,76.$

qualium HC est 1000

talium AH est 7595,9 et paullo — h.e.

Est autem:

$2H = \angle R.$ p.d.

Ergo $\sqrt{HA^2 + CH^2} = AC.$ §195. Th.

h.e. $7661,4+ = AC.$

Ergo $7661,5 > AC.$

cumque $1000 = HC.$ p.d.

$7661,5 : 1000 > AC : HC.$ §181. A.

Porro ob

Arcum HC bisectum in K erit

Arcus $KC = KH$ p.c.

Ergo $\angle KCH = \angle KAC.$ §282. Th.

Sed $\angle K = \angle K.$ §40 A.

ΔKCT aequiangul. $\Delta KCA.$ §155. 305. Th.

Ergo

$CA : CS = AK : KC.$ §353. Th.

$AH + AC : CH = CA : CT.$ §54.

$AH + AC : CH = AK : KC.$ §144 A.

Sed $AH < 7595,9$ } p.d.

$AC < 7661,5$ }

$AH + AC < 15257,4.$ §42. Ar.

cumque $CH = 1000$ p.d.

$AH + AC : CH < 15257,4 : 1000$ §181 A.

h.e.

$AK : KC < 15257,4 : 1000.$ §46. A.

h. e.
qualium KC est 1000
talium AK est 15°25°7,4 et paullo —
Tandem et

∠K = R. p. d.

$\sqrt{KA^2 + CK^2}$ = CA. §195. Θ. h. e.
15°29,0,1+ = CA ergo
15°29,0,2 > CA. sed
1000 = CK. p. d.

15°290,2 : 1000 > CA : CK. §181. A.

Quare ob
Arcum KC bisectum in L, erit
Arc. KL = LC. adeoq;
∠LCK = ∠CAL. §282. Θ.
sed ∠L = ∠L. §40 A.

Δ LCB aeqangl. Δ LCA §155. 305. Θ.
Ergo
CA : CB = LA : LC. §353. Θ.
KA+AC : CK = CA : CB §54.

KA+AC : CK = LA : LC. §144. A.
sed KA < 15°25°7,4 } p. d.
AC < 15290,2 }

KA+AC < 30547,6. §42. A.
sed CK = 1000 p. d.

30547,6 : 1000 > KA+AC : CK. §181. A.
Ergo.
LA : LC < 30547,6 : 1000 §45. A.

AK² + CK² = 2|33|78|82|54,|76 | 15°290,4+ = $\sqrt{AK^2+CK^2}$
1
1|33
(2)
1|25
8|78
(3|8)
6|04
2|74|82
(30|4)
2|74|41
41|54|76
30|58|0 //

LA = 30°547,6
LA = 30547,6
1832856
2138332
1221904
1527380
916428

LA² = 933155865,76
CA² = 1000000

LA² + CA² = 9|34|15|58|65,|76 | 30563,9+ = $\sqrt{LA^2+CA^2}$
9
34
6
34|15
(6|0)
3025
3|90|58
(61|6)
3|66|36
24|2.2.|6.8
(6|11|2)
14|33|69
5|80|96|76
(61|12|6)
5|56|14|21 /.

h. e.
qualium LL est 1000
talium LA est 30547$\frac{3}{5}$, b et paulo —
Deniq; quia
$\angle L = R.$ p. d.
$\sqrt{LL^2 + LA^2} = lA.$ h. e
$30563,97 = lA$ adeoq;
$30564 > lA$
Proinde
Perimeter Polyg. 96. Lat. Circulo inscr. $= LL \times 96$
h. e.
Perim. Polyg. 96. Lat. Circ. inscr. $= 96000$
Ergo
$96000 : 30564 <$ Perim. Pol. 96. Lat. : Diam. AC. §182
$\frac{96000}{30564} : 1 = 96000 : 30564.$ §160. A.

$\frac{96000}{30564} : 1 <$ Perim. Polyg. 96 Lat. : Diam AC. §46 A.

$3\frac{4308}{30564} : 1 <$ Perim. Polyg. 96 Lat. : Diam AC

$3\frac{2154}{15282} : 1 <$ Perim. Polyg. 96. Lat. : Diam. AC

Sed $\frac{15282}{71} = 215\frac{17}{71}$ Ergo

$15282 = 71 \times 215\frac{17}{71}$

et $2152\frac{28}{71} = 10 \times 215\frac{17}{71}$ nam

$$10 \times 215\frac{17}{71} = \left(215 + \frac{17}{71}\right) \times 10$$
$$= 2150 + \frac{170}{71}$$
$$= \frac{2150 \times 71}{71} + \frac{170}{71}$$
$$= \frac{152650}{71} + \frac{170}{71}$$
$$= \frac{152820}{71}$$
$$= 2152\frac{28}{21}$$

Ergo

$10 \times \left(215 + \frac{17}{71}\right) : 71 \times \left(215 + \frac{17}{71}\right) = 2152\frac{28}{71} : 15282$. §145. A.

Ergo $10 : 71 = 2152\frac{28}{71} : 15282$. §160 A.

Ergo $\frac{10}{71} = \frac{2152\frac{28}{71}}{152.82}$ §132. A. missoq $\frac{28}{71}$ ergo.

$\frac{10}{71} = \frac{2152}{15282}$ fere

Ergo a fortiori per §10 A.

$3\frac{10}{71} : 1 <$ Perim: Polyg. 96. Latt. ∴ Diam.

Sed Perim: Circ. Inscr. < Peripheria Circuli §57.

Ergo denuo a fortiori

$3\frac{10}{71} : 1 <$ Peripheria Circuli : Diametrum

Q. E. D.

§ 559 Scholion. 1.

Proportio haec Diametri ad Pphiam Circuli inter duos Terminos ade angustos continetur qui non nisi $\frac{1}{497}$ aut $\frac{10}{4970}$ mis abinvicem distant.

Nam.

$$\frac{1}{7}-\frac{10}{71}=\frac{71}{497}-\frac{70}{497}.\ \text{§207. A.}$$

$$=\frac{1}{497}.\ \text{§210 A.}$$

$$=\frac{10}{4970}\ \text{§293. A.}$$

Aut.

$$3\frac{1}{7}=3+\frac{1}{7}=\frac{21}{7}+\frac{1}{7}=\frac{22}{7}$$

$$3\frac{10}{71}=3+\frac{10}{71}=\frac{213}{71}+\frac{10}{71}=\frac{223}{71}.$$

$$=\frac{1562}{496}-\frac{1561}{497}$$

$$=\frac{1}{497}\ \text{adeoq et §.203. A.}$$

$$=\frac{10}{4970}.$$

§60. Scholion 2.

Methodus ista determinandi Rationem Diametri ad Peripheriam ope Polygonorum Circulo in et circumscriptorum, ita ut illa ad hanc sit fere uti 7:22. §59. Archimedi debetur. Quae tamen cum in majoribus Circulis minus accurata sit a Ptolomaeo usq; ad nostra tempora summi Geometrae ut Vieta Christianus Hugenius Andr. Metius Snellius Landsbergius in adcuratiore Diametri ad Peripheriam Determinatione desudarunt. Quorum omnium Diligentiam superavit Lud. a Ceulen in Tr. de Circulo et adscriptis duplici Proportione data, cujus prioris termini Notis numericis 21, posterioris autem 36, absolvuntur. Quia vero Numeri adeo prolixi minus praxi respondent, communiter resectis reliquis a dextra, tres primas tantum sinistimas, aut ad summum sex primas adhibemus.

ita ut ex Mente Ceulenii sit Diam: Pphiam = 100:314. aut
= 1000000:314159.
Qua et nos, aliquando tamen et Metiana utemur, quae ponit.
Diam: Pphiam = 113:355.

§61. Scholion 3.

Differentia utriusque Circumferentiae inter quam vera existit ex Ceulenii Calculo est Particula Diametri una denominata a Numero qui constat unitate et 35 Cyphris quae Particula ad Diametrum minorem habet Proportionem, quam Arenula una ad Orbem Terrarum. Non enim constat orbis Terrae tot Arenulis quot continentur Particulae tales in Diametro. Ita Andr. Tacquet in Selectis ex Archimede Theorematibus p. m. 296 f. Wolf. Geom. Lat. §425 sqq. Joh. Chr. Sturmius in Mathesi enucleata p. m. 178. Chr. Clavius in Geometr. pract. L. 4. c. 6.

§62. Theorema 8.

Area Circuli æqualis est Facto ex Diametro in Pphiam diviso per 4.

Demonstratio.

Area Circuli $= \frac{\text{Radio} \times \text{Pphiam}}{2}$ §55. hujus et §182. A.

$= \frac{\text{Diametro} \times \text{Pphiam}}{2}$ §25 A. 10 A.

$= \frac{\text{Diametro} \times \text{Pphiam}}{4}$ §212. A.

Q. E. D.

§63. Theorema 9.

Area Circuli est ad Quadratum Diametri fere uti 785 : 1000

Demonstratio.

Esto Diameter = 100. Ergo
Peripheria = 314. §60
Ergo Quadratum Diametri = 10000 §222. A
atq Area Circuli $= \frac{100 \times 314}{4}$ §62
$= 7850$

Proinde

Area Circuli : Quadrat. Diam = 7850 : 10000 §145. A.
= 785 : 1000. fere §160 A

Q. E. D.

§67. Problema XXX

Data Diametro invenire Circuli Pphiam.

Resolutio

Ad 100, 314 et Diametrum datam quaere quartum ppalem §314 A.

D. T. per §60.

Schema Calculi

Sit Diameter = 56, Ergo

$100 : 314 = 56 :$ Pphiam

$$\begin{array}{r} 314 \\ \hline 224 \\ 56 \\ 168 \\ \hline)17584(\end{array} \quad 175\frac{84'}{100} = \text{Pphia}$$

$$\text{h. e.} = 175' + \frac{80'}{100} + \frac{4'}{1000} \quad \S 47.\ A.$$

$$= 175' + \frac{8}{10} + \frac{4'}{100}$$

$$= 175'\ 8''\ 4''' \quad \S 13.$$

Esto Diameter Terrae = 1720. Milliar. Germ.

$10\cancel{0} : 314 = 172\cancel{0} :$ Pphiam.

$$\begin{array}{r} 628 \\ \hline 2198 \\ \hline)54008(\end{array} \quad 5400\tfrac{4}{5} \text{ Mill. Germ}$$

= Pphia Telluris

§61 Problema XXXI.

Data Sphia invenire Circuli Diametrum. Resolutio.

Ad 314, 100, datamq; Sphiam quære quartum ppalem § 314. A.

D. T. per §60

Schema Calculi

Esto Sphia = 17584'' Ergo

314 : 100 = 17584 : Diam

)175 8400 (5600''' = 56 = Diametro
2/4/2
1/8

Esto Sphia Telluris = 5400 $\frac{4}{5}$ Milliar German.

Ergo

314 : 100 = 5400 $\frac{4}{5}$: Diam. Telluris

314 : 100 = $\frac{27004}{5}$: Diam. Tell.

157~~0~~ : 10~~0~~ = 27004 : D. T. §162 A.

)270040 (1720. Milliar. Germ = Diam. Telluris
1/2/3/1
1/4
4/3
1

Similiter in aliis

§66. Problema XXXII.

Data Diametro vel Peripheria invenire Circuli Aream.

Resolutio.

1) Quaere vel data Diametro Pphiam vel data Pphia Diametrum. §64. 65.

2) Inventam duc in Quartam Diametri partem D. F. p. §62.

Aliter.

Ad 1000, 785 et datae Diametri Quadratum quaere quartam ppalem §314. Ar. D. F. p. §63.

Schema Operationis

I Esto data Diameter = 56
Ergo Pphia = $17584'''$ §64
et $\frac{1}{4}$ Diam = 14 = $1400'''$

$$\begin{array}{r} 703600 \\ 17584 \\ \hline \end{array}$$

Area Circ = $24617600''$
= $24^{\circ}, 96', 76''$ §47. 4

II Eſto data Pphia = 17584'''

Ergo ¼ ~~Diameter~~ = 1400''865.

Ergo Area Circ = 24617600'''98185

= 246176''9 ut ante.

Ad Reſolutionem 2dam.

Data ſit Diameter = 56'

56

336

280

Quadr. Diam: = 3136''9

Ergo

100 : 785 = 3136 : Ar. Circuli.

785

15680

25088

21952

)2461760 ÷ 24617 $\frac{60}{100}$ =

24617 6''9 847.48 = Area Circuli

Eſto in altero caſu ſpeciali

I Data Diameter = 1720. Germ. Milliar.

II

Quia Sphia = 5400 $\frac{4}{5}$ 864.

= $\frac{27004}{5}$

et $\frac{1}{4}$ Diam = 130

Sphia × $\frac{1}{4}$ Diam = $\frac{27004}{5}$ × 430

= 2322344 Milliar

German. quadra

II sit data Sphia = 5400 $\frac{4}{6}$ M. G.

quia $\frac{1}{4}$ Diam = 430. M. G.

$\frac{1}{4}$ Diam × Sph. = 2322344. M. G.

quadrata, uti ante.

Pro resolutione secunda

Quia DT = 1720. M. G.

1720

34400

1204

172

Ergo Quadr D. T. = 2958400

Ergo

1~~000~~ : 785 = 2958400 : Ar. Circ.

785

14792000

236672

207088

232234000 M. G. quadrat

= Ar. Circ.

§67 Problema XXXIII.

Data Area Circuli invenire Diametrum. Reſolutio.

1) Ad 785, 1000 atq. data Circuli Aream quaere quartam ppalem §314. A. §63.

2) Ex invento extrahe Radicem quadraticam. §256. A. D. F. §63.

Schema Operationis:

Data sit Area Circuli = 246176‴9

Ergo

785 : 1000 = 246176‴9 : Quadr. Diam.

2461760000 ∫ 313600″ = Q. D.
3262
10882
347
245

31,36,00″ ∫ 56″ = 56′.847.48.
25
6 | 36
(1 | 8)
6 | 36 | 00

= Diam Circuli

126.

Data sit Area Circuli =
2322344 Milliar. Quad. Germ.
Ergo
785 : 1000 = 2322344 : Quad. Diam.

2322344000 | 2958400 M.G.Q = Diametri Quadratum
1570 . . .
7523 . . .
7065 . . .
4584 . .
3925 . .
6594 .
6280 .
3140
3140
00

2|95|84|00 | 1720 Mill. Germ. = Diam. Circ.
1
1|95
(2)
1|89
6|84
(34)
6|84
00

Similiter in aliis

§ 68. Scholion.

Peripheriam autem data Circuli Area invenies per § 64. cognita prius per § 61. Diametro.

§ 69. Theorema 10.

Area Sectoris aequalis est Triangulo cujus Basis est Arcus Circuli seu Sectoris, Altitudo autem Radius.

Demonstratio.

Mutatis mutandis coincidit cum Demonstratione § 55.

§ 70. Problema XXXIV

Dato Radio AC et Quantitate Arcus AB invenire Aream Sectoris ABC.

Resolutio.

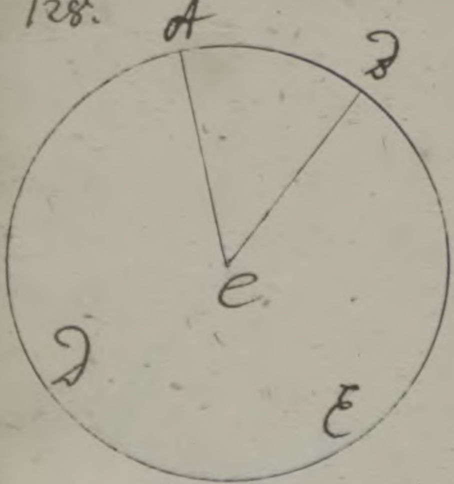
Casus I. Si Sector semicirculo minor fuerit

1) Quaere ad 100, 314 et Radium datum, quartum ppalem § 314. A qui est semi Sphia § 60 A § 64.

2) Quaere ad 180, quantitatem Arcus AB et semi Sphiam N. 1. notam denuo quartum ppalem § 314 A ut Arcus AB innotescat in Lineis $\frac{1}{5}$

3) Inventum duc in Radii semissem. D Z per § 182 et § 69.

128.



Casus 2. Si Sector Semi Circulo major

1) Quaere Aream Circuli §66.

2) Itemq; Sectorem minorem Cas. 1

3) Hunc ab illo aufer D. F.

Vel.

1) Quaere ad 100, 314, et datam Diametrum Pphiam §64.

2) Ad 360° Arcum datum ADE et Pphiam quartum pplem §314

3) Reliqua absolve uti Cas. 1. D. F.

Schema Operationis

Esto Radius AC = 6 } In Casu I
Arcus AB = 6° }

Ergo

1.) $100 : 314 = 6 : \frac{1}{2}$ Pph.
$\overline{1884'''} = \frac{1}{2}$ Pphiae.

2) $180^\circ : 6^\circ = 1884''' :$ Arc. AB
$11304 \mid 62\frac{144}{180} =$ AB
$62\frac{4}{5}''' =$ Arc AB

3) $\left(62''' + \frac{4'''}{5}\right) \times 3' =$ Ar. Sect. ABC
$\left(\frac{310'''}{5} + \frac{4'''}{5}\right) \times 300'''' =$ A. S. ABC
$\frac{93000'''' + 1200''''}{5} =$ A. S. ABC
$\frac{94200''''}{5} = 18840'''$
= Ar. Sect ABC

In Casu III !!

Esto Radius AC = 6'

Ergo Diameter = 12'

Arcus ADEB = 354°

Quare pro Resolutione I

1) $1000 : 785^{\circ} = 144^{\prime q} :$ Ar: Circ:

$$\begin{array}{r} 3140 \\ \hline 3140 \\ \hline 113040^{\prime\prime\prime\prime q} \end{array}$$ = Ar. Circuli. h. e.

Ar. Circ $= 1^{\circ q}_{\prime\prime}, 13^{\prime q}_{\prime\prime}, 04^{\prime\prime q} - {}^{\prime\prime\prime q}. 847.$

2) Ar: Sect ACB = — „ 1„ 88„ 40. Cas. I.

Ar. Sect ADEB $= 1^{\circ q}_{\prime\prime}, 11^{\prime q}_{\prime\prime}, 15^{\prime\prime q}_{\prime\prime}, 60^{\prime\prime\prime q}\ 847.$

Pro Resolutione II

1) $100 : 314 = 12 :$ Peripheriam.

$$\begin{array}{r} 628 \\ \hline 3768^{\prime\prime\prime} \end{array}$$ = Peripheria.

2) $\frac{360^{\circ}}{60} : 354^{\circ} = 3768^{\prime\prime\prime} :$ ADEB in

$\frac{60}{10} : 59 = \underline{3768} :$ Lineis:

$10 : 59 = 628 :$

$$\begin{array}{r} 59 \\ \hline 5652 \\ 3140 \\ \hline 37052 \end{array}$$ h. e.

Tandem

3) $3705\tfrac{1}{5}^{\prime\prime\prime} \times 300^{\prime\prime\prime} =$

$18526^{\prime\prime\prime} \times 300^{\prime\prime\prime} = \frac{5557800^{\prime\prime\prime\prime q}}{5}$

$= 1111560^{\prime\prime\prime\prime q}$ h. e.

Arcus ADEB in Lin $= 3705\tfrac{1}{5} = 1^{\circ q}_{\prime\prime}\ 11^{\prime q}_{\prime\prime}\ 60^{\prime\prime\prime q}. 847.$

5

uti paullo ante

Similiter in aliis

§71. Problema XXXV

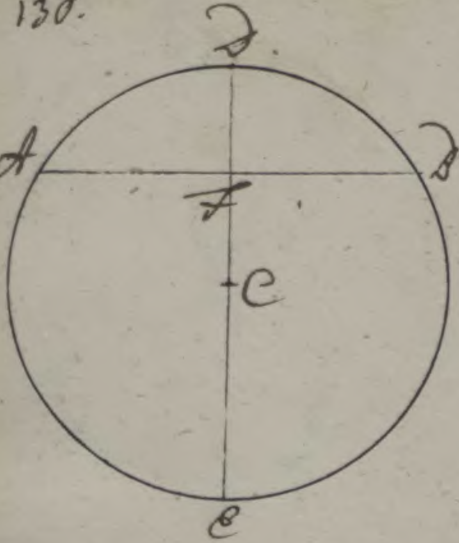
Data Chorda AB et Altitudine D
Arcus ADB h. e. Normalis ex Bis
ctionis Puncto Chorda AB nempe
invenire Diametrum et Circuli Ce
trum C.

Resolutio.

1) Ad datam Altitudinem DF et Cho
dam bisectam FB, quaere tertiam
proportionalem §314 A.
2) Inventae adde Altitudinem data
3) Summam biseca
DF.

Demonstratio.

Quia AB bisecta per DF. p. H.
Ergo DF producta transit per Centrum §254 C.
Ergo DF : FB = FB : FE §368 C.
Ergo DF + FE = DE = Diametro &c.
adeoque $\frac{DF + FE}{2}$ = CE = DE. §45. A.
Q. E. D.



Schema Calculi.

Sit DF = 64''. AB = 448'' erit
BF = $\frac{AB}{2}$ = 224'' Hinc
64'' : 224 = 224'' : FE.

224
896
448
448

$\sqrt{}$ 50176 | 784'' = FE cum
8 1 3 4
5 1 1
2

64 = DF p. H.
848 = DE adeoque

848'' = 424'' = $\frac{DE}{2}$ = Radio
CE.

§72. Problema XXXVI.

Datis Arcu ADB, Chorda AB ejusq; Altitudine DF invenire Aream Segmenti ADBFA.

Resolutio et Demonstratio.

1) Quaere Radium DC. §71.
2) Atq; ex hoc Aream Sectoris AC DB. §70.
3) Ex DC aufer DF, quae erit Altitudo Trianguli ACB. §254. 44. et 126 G.
4) Quaere Aream Trianguli ACB. §182. G.
5) Hancq; aufer ex Sectoris Area ACB inventa
B D.T. et D.Q.P.

Schema Operationis

Sit AB = 6' = 600''' §1.
DF = 80'''
Arcus ADB = 60°. Ergo
AF = FB = 300'''

Quare p. M. br. 1.

$80''' : 300''' = 300''' : FE$

$$\frac{\cancel{3}00}{9000} \Big|\; 1125''' = FE$$
80''' = DF
1205''' = DE
2) 602'''½ f = DC

f ½''' ob expeditiorem Calculum negligimus, id quod et de reliquis observandum est fractionibus.

per Mbr. 2.

$100 : 314 = 602''' : \frac{1}{2}$ Pphiam

$$\begin{array}{r} 602 \\ \hline 628 \\ 1884 \\ \hline 1890.|28 \end{array}$$

Ergo $\frac{1}{2}$ Pphia $= 1890'''$

Porro

$\frac{180^{\circ}}{3} : \frac{60^{\circ}}{1} = 1890''' :$ Arc. ADB

$= \cancel{1}1890''' :$ Arc. ADB

$630''' :$ Arc. ADB.

porro: $301 = \frac{1}{2}$ Diam. DE.

$189630'''' =$ Ar. Sctis ACBDA

Tandem per Mbr. 3.

$602''' =$ DC

$80''' =$ DF

$522''' =$ FC

$300''' =$ FB

$156600'''' =$ Ar. ΔACB p. M. IV.

$156600 =$ Ar. Δli ACB.

$33030'''' =$ Ar. Segmenti ADBFA p. Mbr. V.

§73. Scholion.

Quodsi Arcus non simul datur in-
vento prius per Mbr. 1. Radio

Circulus ipse describendus, Chorda coaptanda §807. & atq Quantitas Arcus per Instrumentum Transportatorium §19 investiganda.

§74. Problema XXXVII

Aream campestrem rectilineam ABCDE invenire.

Resolutio et Demonstratio.

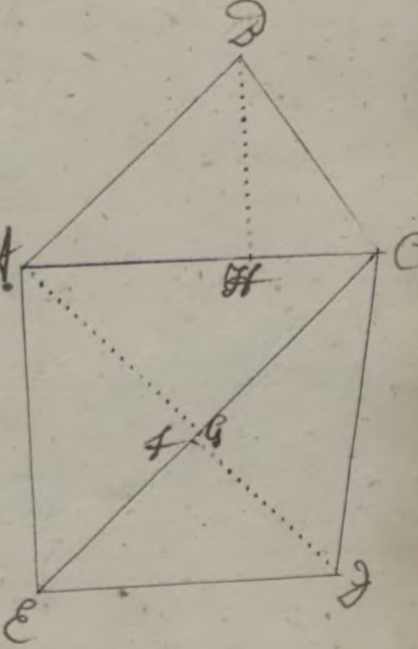
1) Describtam Areæ ABCDE Ichnographiam §40—42.

2) Resolve per Diagonales in Triangula §81. &.

3) Inventas eorum Areas §182 &.

4) Adde D.F.p.847.A.

Locum autem esse Compendio §51 allato per se liquet



Schema Calculi

Esto $AC = 268'''$
$DH = 204'''$
$CE = 368'''$
$AG = 217'''$
$FD = 102'''$

$\Delta ABC = \frac{AC \times BC}{2}$

$= 268'''$

$102'''$

536

268

$\therefore 2,73,36''''$

$ACDE = \frac{CE}{2} \times AB + FD$ §51.

$= 184'''$

$319'''$

1656

184

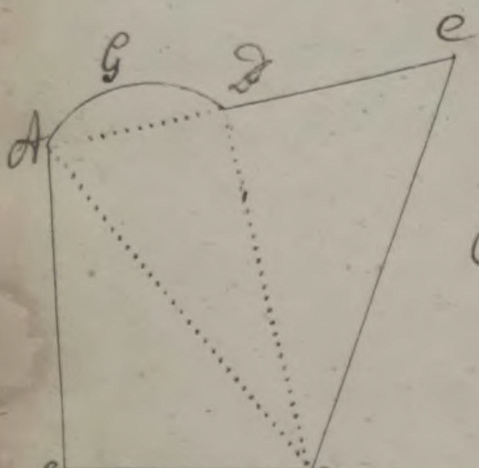
552

$\therefore 5,86,96''''$

Area $ABCDE = 8,60,32$

Similiter in aliis

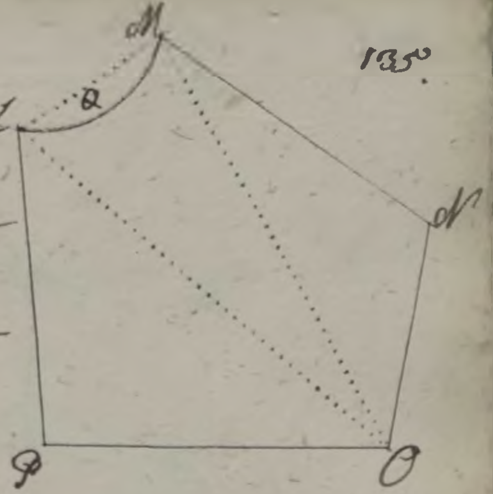
§75. Scholion 1.



Si Pars Perimetri Figurae ABCDEA data fuerit convexus, Circuli Arcus AGB, Aream segmenti inventam per 72. § adde Triangulorum reliquorum Aggregato.

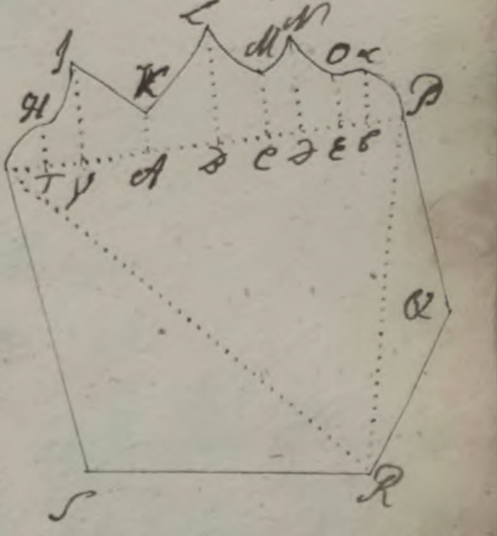
§76. Scholion 2.

Si vero Pars Perimetri figuræ datæ L, QMNOP fuerit concavus, Circuli Arcus LQM ducta Chorda LM Aream Rectilinei LMNOPL quære per §74 ab inventa aufer inventam Segmenti LQML per §. 72. Aream et est q3 Factum.



§77. Scholion 3.

Tandem ſi Pars Perimetri figuræ datæ GHIKLMNOP. Q RSG fuerit curva quæcunq; alia ducta ſubtenſa GP, quære Aream GPQRSG. §74. deinde a quovis ad quodvis Curvaturæ Punctum notabile H, IK, LM, N, O duc Rectas GH, HI, IK, KL, LM, MN, NO, OP. §81 quæ curvæ coincident, et in ipſa GM aſſume Puncta commoda T, V, A, B, C, D, E atq; h.m. innoteſcent lineæ GH, GT, HT, TV, HT, TV, per §8 adeoq; et Areæ §182. et per §81 quaſi Rectilineo GPQRS addideris factum erit q.p. per §41 A.



§78. Problema XXXIIX

Datis Area et Basi Trianguli invenire illius altitudinem.

Resolutio.

Aream datam divide per Basin bisectam.

Q. F.

Demonstratio.

Area Trianguli $= \frac{1}{2} b \times a$ §182. G.

$\frac{\text{Area Trianguli}}{\frac{1}{2} b} = a$ §44. 100. A.

Q. E. D.

§79. Problema XXXIX

Figuram rectilineam quamcunque ABCDE in partes imperatas aequales dividere.

Resolutio.

1) Quaere Aream Figurae §75.
2) Inventam divide in imperatas partes v. c. tres
3) Aream tertiae partis hujus divide ulterius bifariam.
4) Aream Trianguli ACD aufer a tertia Areae parte
5) Residuum divide per ½ AD ut innotescat Altitudo Trigoni

AD § 78. Trianguli priori AED
addendi, quo AED sit Figuræ
tertia pars

6) Intervallo itaq. Altitudinis inventæ duc =lam cum AD § 135 b aut § 1 b, quæ secabit Latus AB in I, jungeq. DI § 81. b Figuræ tertiam partem quæ determinabit

7) Partem Figuræ sextam M. III. inventam divide per ½ DI et innotescet Altitudo Trianguli DIK. § 78. Sextam Figuræ Partem constituentis.

8) Hujus altitudinis intervallo cum ID duc =lam secantem AB in K.

9) Eandem M. 3 inventam sextam partem divide per ½ DK et determinabitur Altitudo Trianguli DKL, sextæ etiam Figuræ Parti æqualis § 78.

10) Quare et hujus Altitudinis Intervallo duc cum DK =lam §§. ut determinetur pctm L, junctis ergo Punctis L et K Recta KL,

5

5

et Spatium DKL tertiam Figuræ partem efficiet, ut tandem LKB sit ultima determinata

Quod si Figura data in plures æquales partes fuerit subdividenda mutatis mutandis simili modo absolvetur Operatio.

Schema Calculi.

Esto AD = 247''' DG = 180''

AC = 363'' DF = 208

EH = 118.

Quare per M:1.

$\triangle DEA = \frac{AD \times EH}{2} = 247$

$\quad\quad 59$

$\quad 2223$

$\; 1235$

$1,45,73$

Area DCADC $= \frac{CA}{2} \times \overline{DG + DF}.$

$= 363 : 2$

$\quad 388$

$\quad 2904$

$\; 2904$

1089

$2)140844$ $\quad 7,04,22$

$8,49,95$

Area ADCDE =

Per Mbr. 2.

$\frac{1}{3}$ Area ADCDE = $\quad 2,83,31\frac{2}{3}$ quas tamen $\frac{2}{3}$

Per Mbr. 3.

$\frac{1}{6}$ Area ADCDE = $\quad 1,41,65.$

Per Mbr. 4.

$\frac{1}{3}$ ADCDE − △DEA = $\quad 1,37,58$

Per Mbr. 5.

$\frac{\frac{1}{3} ABCDE - \Delta DCA}{\frac{1}{2} AD} = \frac{13758''''}{\frac{247}{2}} = 13758'''' \times \frac{2}{247}$ 8Ar

$= 247 \,|\, 27516 \,|\, 111'''$ = TAG $\frac{99}{247}$ omittimus

3849
240
39
1

Ergo DCAY = Tertiæ Figuræ Parti

Per Mbr. VIII.

$\frac{\frac{1}{6} \text{Area } ABCDE}{\frac{1}{2} DY} = \frac{14165''''}{\frac{299}{2}} = 14165 \times \frac{2}{299}$

$= 299 \,|\, 28330''' \,|\, 94'' \frac{224}{299}$ quam fractionem adhærentem compensando sine sensibili errore assumimus $= 1'''$.

Ergo Altitudo sextæ Partis = 95''.

Per Mb. IX.

$\frac{\frac{1}{6} \text{Ar. } ABCDE}{\frac{1}{2} DK} = \frac{14165''}{\frac{376}{2}} = 188 \,|\, 14165'' \,|\, 75''$ = Altit. Δli alicuius sextam figuræ ps.

785
105
5
1

880 Scholion.

Quodsi Area Trianguli ACD minor fuerit tertia ~~totius Areæ parte~~ subtrahe hanc ab illa, ut innotescat Area Trianguli alicujus auferenda ab Area Trianguli ACD quo tertiæ Figuræ Parti fiat æqualis. Absoluta autem

5

5

divisione in Charta puncta Y, RL, in campo facile determinantur per Rectas AY, VK, ZL &c.

Caput III^{tium}

De Solidorum Dimensionibus.

§81. Definitio 3.

Pertica itemq Decempeda cubic
est, cujus Latus Perticam aut Decem
pedam æquat.
Similiter Pes, Digitus et Linea
ca est, cujus Latus est Pes, Digitu
et Linea. Et in genere: Mensura
di est Cubus cujus Latus vel Pertic
aut Decempedam vel Pedem vel
Digitum vel Lineam ~~adæquat~~ vel lo
gitudinem quamvis assumtam ad
quat.

§82. Hypothesis 4.

Perticas aut Decempedas signific
bimus signo §3 allato adjiciendo c;
v. c. 29 Decempedas cubicas expri
memus 29^{oc}
Simili scribendi modo utemur
digitos et Lineas cubicas signific
turi v. c.
35 Pedes cubicos : 35^{Ic}
39 Digitos . . . . 39^{IIc}
88. Lineas cubicas 88^{IIIc}
Aliis placet signum ⊞ scribunt
ergo 36^{oc} h. m. 36^{o} ⊞

83. Corollarium 1.

inc Decempeda cubica mille Pedes ubicos, Pes cubicus mille Digitos cubicos, Digitus cubicus mille Lineas ubicas continet §81.

hoc eſt

Decempeda cubica $= 1000'^{c}$

$= 1000000''^{c}$

$= 1000000000'''^{c}$

Nam

$1^{0q} = 100'^{q} = 10000''^{q} = 1000000'''^{q}$ §47.

$1^{0} = 10' = 100'' = 1000'''$ §1.

$1^{0c} = 1000'^{c} = 1000000''^{c} = 1000000000'''^{c}$ §81.

84. Corollarium

nde quidem expeditiſsima elucet olidi in Lineis dati Reductio ad Digitos, Pedes atq; Decempedas, er ſcilicet in Numero propoſito tria ſigna a Dextra ſiniſtram verſus abſcindendo np. tria prima pro Lineis; tria ſecunda pro Digitis, tria tertia pro Pedibus Cubicis atq; ita Reſiduum exhibebit Decempedas, uti quidem liquet ex §83.

$r.c. \; 99473207652609'''$

$= 99473^{0c}, 207'^{c}, 652''^{c}, 609'''^{c}$.

§85. Scholion.

Quæ §83. 84. dicta ſunt, de Menſur
Geometrica tantummodo vale
per ſe liquet. Quodſi enim ſolidi
Lineis cubicis menſuræ cujusda
Civilis dati Reductio abſolven
eſſet in Perticis, Pedibus etc. ejusd
Menſuræ, nota ex §2. eſſe debet
Pertica ad Partes ſubdiviſas Pedes,
Digitos atq Lineas

Eſto ſolidum datum =
205708249689 Rhinland.

Quia Pertica Rhinl = 12' }
Pes Rhl. = 12'' } §2.
Digitus Rhl. = 12''' }

Ergo

Pertica } cubic. Rhl. 1728'c }
Pes } cubic. Rhl. 1728''c } §83.
Digitus } cubic. Rhl. 1728''' }

Quare.

Numerum propoſitum divi-dendo per 1728. Digitos, Dividendo Digitos per 1728. Pedes, tandem et hos dividendo per 1728
Perticas cubicas Rhl. invenie
h. m.

Rhl. ″c Rhl. ʼc Rhl. °c Rhl.

1728) 205.708249 63‴c ƒ 119044127 ƒ 6889 ƒ 3

1728
3290
1728
15628
15552
7624
6912
7129
6912
2176
1728
4483
3456
1027‴c Rhl.

10368
15364
13824
15401
13824
15772
15552
220″c Rhl.

5184
1705ʼc Rhl.

Erit ergo data ſoliditas in Lineis Cubicis = 3°c, 1705ʼc, 220″c, 1027‴c Rhl.

§86. Problema XI.

Superficiem atque ſoliditatem Cubi determinare.

Reſolutio.

1) Quia Cubus ſex Quadratis equalibus terminatur §426.

1) Latus Cubi p. §.8 notum duc in ſemetipſum.

2) Factum duc in 6. et cognita erit ſuperficies

Q. E. I. R. et D.

II Idem Quadrati Producta
duc denuo in Latus et inve
erit Cubi ſoliditas.

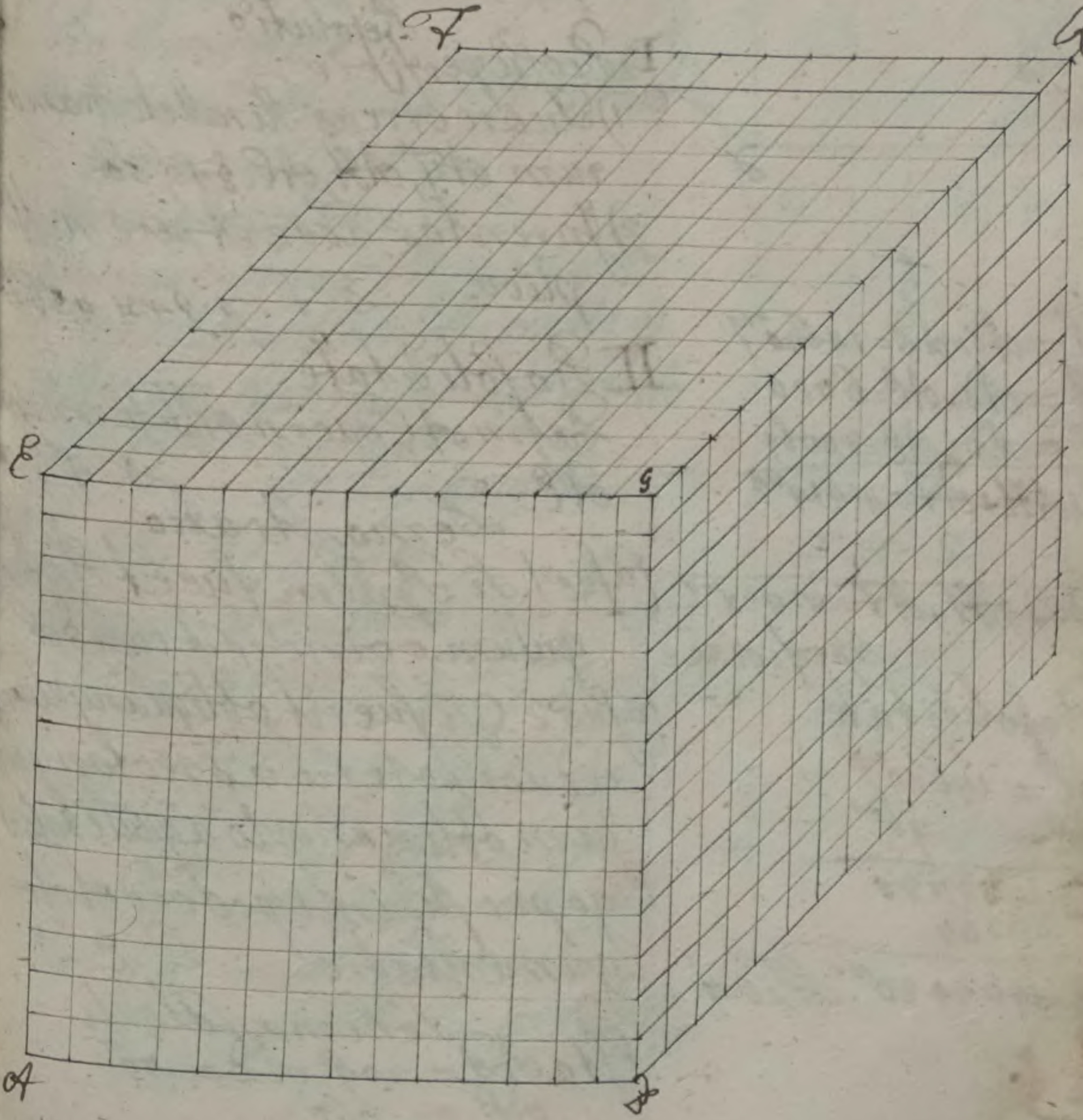
Demonſtratio

Quoniam Solidorum Menſu ſunt Cubi, quorum Latera ſunt cempedæ, Pedique æqualia §81. determinanda Cubi ſoliditat quæritur quot Decempedæ, Pe Digitique cubici contineantur i Cubo AK. Quare cognito La AD intelligitur, quot Cuboru ordines in ipſo, adeoque et ipſa AB diſponi poſſint. Hinc Baſi ducta in Altitudinem h.e. La FE, prodit multiplum Cubor minorum maiorem Cubum efficientium. Q.E.D.

Porro
$AD^2 = 196^{\prime\prime q}$
$EF = 14$
784
196
$AK = 1^{\circ c},744^{\prime\prime\prime c}$ = Soliditati Cubi.

Schema Calculi.

Sit $AD = 14$. Ergo
$AB = AD^2 = 196^{\prime\prime q}$
$6 = 6$
$6 AD^2 = 1176^{\prime\prime q}$
= ſuperficiei Cub


F
R
E
G
A
B

§87. Problema XLI

Superficiem atq. ſoliditatem Parallelepipedi determinare.

Reſolutio.

I Pro ſuperficie
1) Quære Areas Parallelogrammorum AF, AH, AC. §49.50.
2) Inventas adde et per 2 multiplica D.F. p. §431.456.

II Pro ſoliditate
Baſin AC duc in Altitudinem AE. D.F.

Demonſtratio.

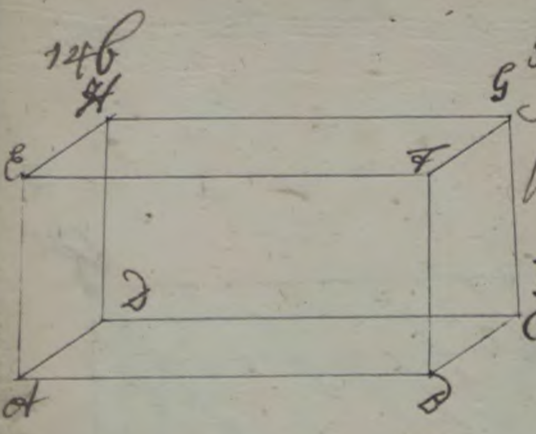
Caſus 1. Si ~~Ppdm~~ fuerit Rectangulum coincidit cum §86.

Caſus 2. Si fuerit obliquangulum reduci poterit ad rectangulum obliquangulo æquale §486.

Ergo per caſum 1 procedet Argumentatio. Q.E.D.

Schema Calculi

Eſto $AB = 209''$ $AD = 70'''$
$AE = 96'''$



1) $AC = AB \times AD = 14630^{'''q}$
$AH = AD \times AE = 6720$
$AF = AB \times AE = 20064$
$AC + AH + AF = 41414^{'''q}$
$2 = 2$
$2 \times AC + AH + AF = 82828^{'''q} =$ Superficiei

2) Pro ſoliditate
$AC = 14630^{'''q}$
$AE = 96^{'''}$
87780
13167
$AG = 1404480^{'''c}$. Solidit.

§88. Problema XLII

Superficiem atq; ſoliditatem Priſmatis invenire.

Reſolutio et Demonſtratio.

I Pro ſuperficie.

1) Quære baſin per §182 aut §53. 74. hujus.

2) Inventam duc in 2.

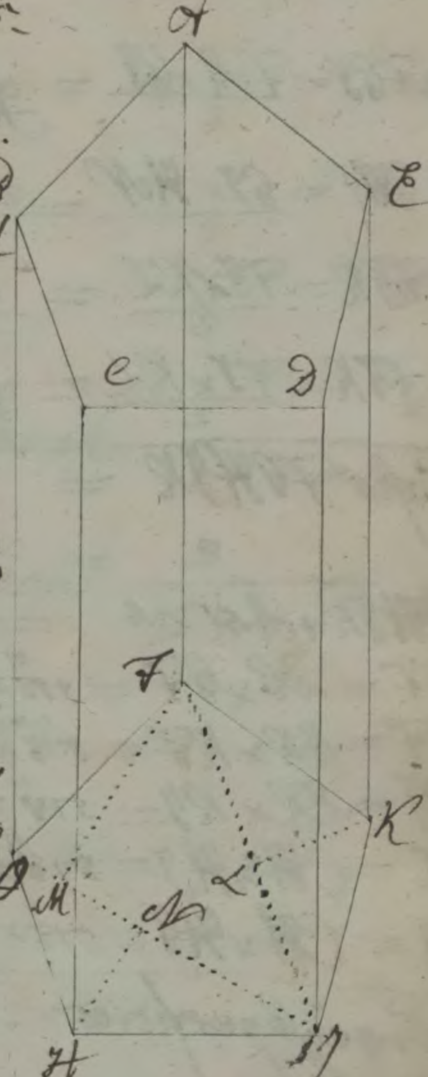
3) Quære Areas Pllgmorum determinantium Priſmata 49.

4) Horum ſummam adde facto per Nbr. invento.

D. F. p. §419. Q.

II. Pro ſoliditate

Baſin inventam duc in altitudinem D. F. §496. Q.



Schema calculi

Eſto $FG = 125'''$ $HN = 65''$

$GH = 97'''$ $KL = 75''$

$HI = 159'''$ $FI = 168'''$

$IK = 98'''$ $GI = 215'''$

$KF = 128'''$ $HC = 518 = DI$

$FM = 95$

Quare

I Pro ſuperficie

$$\Delta FGI = \frac{FM \times GI}{2} = \frac{95''' \times 215'''}{2} = \frac{20425'''q}{2}$$

$$\Delta GHI = \frac{GI \times HN}{2} = \frac{215 \times 65}{2} = \frac{13975'''q}{2}$$

$$\Delta FIK = \frac{FI \times KL}{2} = \frac{168''' \times 75}{2} = \frac{13975'''q}{2}$$

$$\Delta FIK = \frac{FI \times KL}{2} = \frac{168''' \times 75}{2} = \frac{12600'''q}{2}$$

$$\text{Baſis } FGHIK = \frac{47000'''q}{2}$$

$$2 = 2$$

Petro:

$$FGHIK + ABCDE = 47000'''q$$

$$DF = DG \times GF = 518''' \times 125''' = 64750$$

$$EF = EK \times KF = 518''' \times 128 = 66304$$

$$EI = EK \times KI = 518''' \times 98''' = 50764$$

$$IC = CH \times HI = 518''' \times 159''' = 82362$$

$$CG = CH \times HG = 518 \times 97 = 50246$$

Ergo ſuperficies = 361426'''q

Poterat vero longe expeditius Area omnium Rectangulorum horum inveniri, ducendo ſummam omnium Laterum FG, GH, HI &c in comunem Altitudinem quo ipſa Methodus prioris

Examen constitut v. c.

$FG+GH+HI+IK+KF = 607'''$

$CH \quad = 518'''$

$$
\begin{array}{r}
4856 \\
607 \\
3035 \\
\hline
\end{array}
$$

Suma Rglorum $= 314426'''q$

additis ergo Basibus $47000'''q$

Ergo superficies $= 361426'''q$

uti paullo ante

II Pro soliditate

Basis $FGHIK = \frac{47000'''q}{2} = 23500'''q$

Altitudo $CH =$

$$
\begin{array}{r}
518 \\
\hline
188000 \\
235 \\
1175 \\
\hline
\end{array}
$$

Soliditas Prismatis $= 12173000'''c$

Similiter in aliis.

§ 89. Problema XLIII

Superficiem atq ſoliditatem Pyramidis invenire.

Reſolutio et Demonſtratio

I Pro ſuperficie

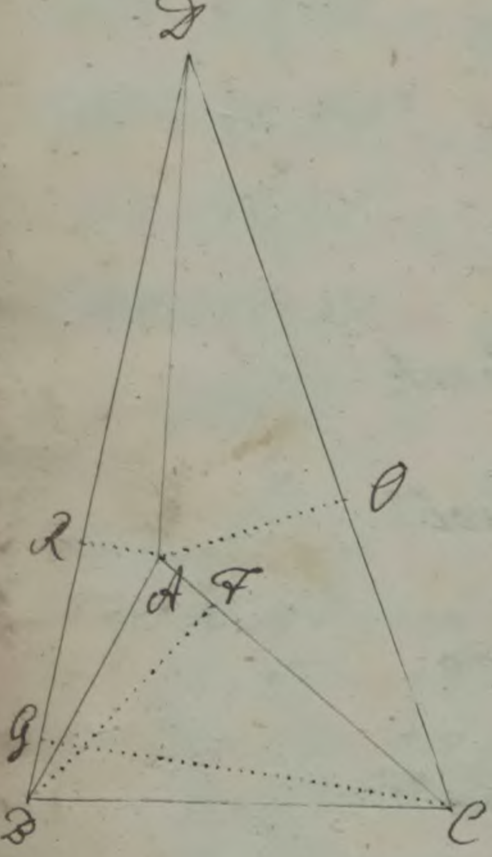
1) Inventam per § 182 et § 50. 53. Baſin.

2) Inventas Triangulorum lateralium Areas § 182.

3) Adde D. F. § 418.

II Pro ſoliditate.

Baſin inventam duc in tertiam Altitudinis Partem aut Factum ex Altitudine in Baſin divide per 3 D. F. per § 496.



Schema Calculi.

Esto DA = 250‴. AR = 75‴. AO = 50‴

AC = 103. BF = 170‴. CG = 185‴

DB = 405‴ DC = 397.

Ergo

$\triangle DAB = \frac{DB \times AR}{2} = \frac{405''' \times 75'''}{2} = 15187\tfrac{1}{2}'''q$

$\triangle DAC = \frac{DC \times AO}{2} = 397''' \times 25''' = 9925$

$\triangle ABC = \frac{AC \times BF}{2} = 163''' \times 85''' = 13855$

$\triangle DBC = \frac{DB \times CG}{2} = \frac{405 \times 185}{2} = 37462\tfrac{1}{2}$

Superficies Pyramidis = 76430.‴q

Pro soliditate
Basis ABC = 13855‴q
Altit. DA = 750

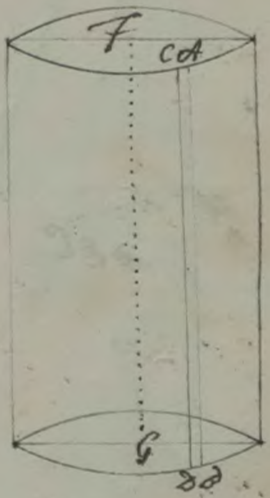
692750
27710

3)3463750

$1154583\frac{1}{3}$‴ = solid. Pyramidis ABCD.

§90. Theorema 11.

Superficies Cylindri Recti demtis Basibus aequalis est Rectangulo sub Peripheria et Altitudine Cylindri. Et superficies Coni Recti, demta Basi aequalis est Triangulo cujus Basis est Pphia, Altitudo autem Latus Coni.



Demonstratio

Membrum I.

Quoniam Cylindrus pro Prismate infinitangulo haberi potest §500. D. atq AB itemq CD lles et = les §. 424. D et §400. A. ABCD est Rectglum §70.138.139. D et superficies Cylindri demtis Basibus tot ei aequiter Rglis, quorum Altitudo communis est Altitudo Cylindri FG, vel AD, quot ♄

♄ Bases in Pphiam Circuli Ergo desinunt. §497. D. hoc est Basis omnium Rectangulorum, quibus superficies Cylindri demtis illis Basibus aequatur est Peripheria, Altitudo autem ipsa Cylindri Altitudo AD §471. A. Q.E.1

Membrum II

Similiter Conus pro Pyramide infinitangula haberi potest §800.θ. adeoq; subtensa Δli in Periphiam desinit §491.θ. Ductis ergo Coni Lateribus AD, DC aequalibus inter se §423.θ et 800.θ. Demitte ex A normalem AE ad DC §119.θ. quae erit Altitudo Δli ADC. §126.θ. jungeq; FE. §81.θ.

Quare cum Axis Coni AF normalis ad FC et DF. §422.θ. Ergo

∠AFE = ∠AFC. §26.θ.
sed FE = FC. §26.θ
et AF = AF. §40.A.

AE = AC. §99.θ.

h. e. Altitudo Trianguli ADC = Lateri Coni.

Quare cum superficies Coni demta Basi tot cingatur Triangulis, quorum communis Altitudo est Latus Coni, quot Bases in ipsam Periphiam desinunt; idem enim Discursu simili de reliquis Triangulis infinite parvarum Basium demonstrabitur; Ergo superficies Coni demta Basi aequalis est

Triangulo, cujus Basis est Pphia Circuli Baseos, Altitudo autem Latus Coni. §417.A. Q. E ll. D.

§91. Corollarium.

Ergo superficies Coni demta Basi invenitur, multiplicando Pphiam in Lateris semissem et contra §. 182. Θ et §185.

§92. Problema XLIV

Data Diametro et Altitudine Cylindri invenire illius Superficiem atq soliditatem.

Resolutio et Demonstratio

I Pro superficie

1) Ex Diametro quaere Pphiam Circuli Basis §64. atq Aream §66

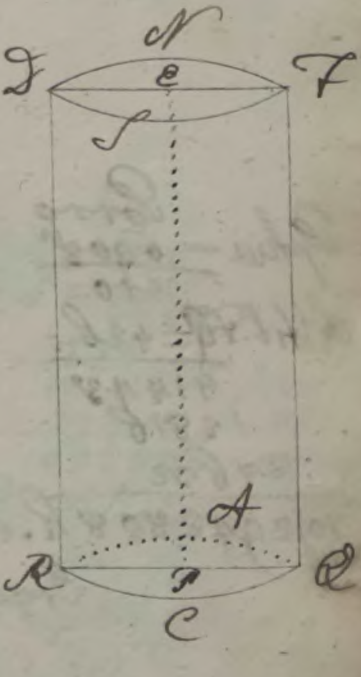
2) Inventam duc in 2 et prodibunt Bases AC et DNFS. §427.

3) Pphiam Nbr. I. inventam duc in Altitudinem datam, quod factum exhibet superficiem demtis Basibus §90

4) Adde facta Nbr. II. et III. D. F. §417.A.

II Pro soliditate.

Duc Basin AC in Altitudinem FQ D. F. p. §500. Θ.



Schema Operationis

Sit $RQ = 220''$, $EP \perp FQ = 426'''$

Ergo

$10 : 314 = 220'' :$ Pphiam

220

6280

628

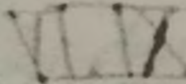
)69080 Ergo Pphia = $6908''$

$\frac{1}{4} R \cdot Q = 55'''$

34540

34540

10)379940



Ergo Ar. Circuli Al = $37994'''9$

2 = 2

Ar. Circ. Al + DoMFS = 75988

Porro

Pphia = $6908''$ 10

Altit. FQ = 426

41448

13816

27632

10)2942808 h. e. Superf. Cyl. demti Basibus = $294280\frac{8}{10}''''$

Ergo tota superficies = $370268\frac{8}{10}''''$

Pro soliditate

Basis Al = $37994'''9$

Altit FQ = 426

227964

75988

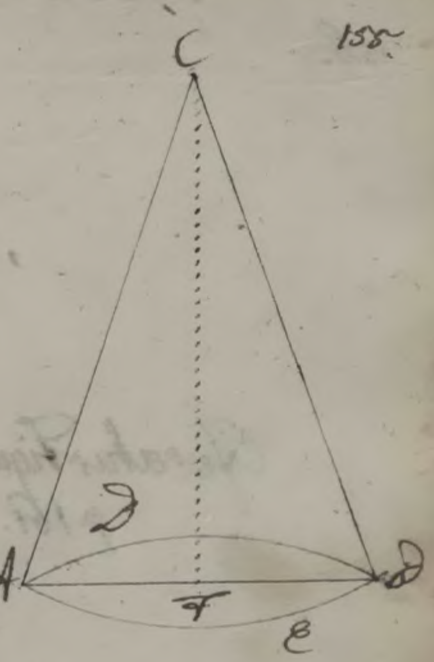
151976

Soliditas Cyl = 16185444 m c.

§93. Problema XLV

Data Diametro, Latere et Axe Coni recti invenire superficiem atque soliditatem.

Resolutio et Demonstratio.



I Pro superficie

1) Ex data Diametro quaere Peripheriam §64. atque Aream §66 ut innotescat Basis.

2) Eandem Peripheriam duc in dimidium Latus aut contra, factumque erit superficies Coni demta Basi. §91.

3) Adde Facta Membri 1. et 2. D. F. per §478.

II Pro soliditate

Eandem Coni Basin duc in $\frac{1}{3}$ Axis §. 9. i. e. Altitudinis D. F. p. §500. B.

aut Basin duc in Axem, factumque triseca. D. F. p. §185. A.

Schema Calculi

Sit $AB = 2'$, $CF = 4'$, $CB = 4\,2''$

Ergo per §64.

$100 : 314 = 2' :$ Peripheriam.

$628''' =$ Peripheria

$50''' = \frac{1}{4}$ Diam. AB

$31400'''' =$ Ar. Bas. ADFBE ♄

♄

Peripheria $= 628''$

$\frac{1}{2}\ CB = 210$

6280

1256

$131880'' =$ superf. Coni

$31400 =$ Basis

$163280 =$ Superf. totius

Pro Soliditate Coni

Area Basis $= 31400''''$

CF h. e. Altit. Coni $= 4\ 00''$

$3\times$ Solid. Coni $= 12560000'''$

Soliditas Coni $= 4186666\frac{2}{3}$

§45 Ar.

158.

894. Scholion
Quodſi latus datum non ſit inno
teſcet per §195 B ex Quadratorum
Axis et ſemiſſis Diametri ſumma
extrahendo radicem quadraticam
eritq; $CD = \sqrt{CF^2 + FD^2}$

895. Problema XLVI

Afferatur Figura p. 161.

Metiri ſuperficiem atq; ſoliditatem
Coni recti truncati h. e. Plano
quopiam CKDL ipſi Baſi AHB pa-
ralle ſecti, datis illius Altitudine
GH atq; Baſium Diametris AB et HL

Reſolutio

I Pro ſuperficie.
1) Aufer Diametro majore aeq
minorem, ut innoteſcat AH.
2) Quadratum Differentiae ad
Quadrato Altitudinis Coni trun-
cati
3) Extrahe Radicem quadrati-
cam §256. A ut innoteſcat AL.
4) Infer: ut Differentia AH ad ſem
Diametrum minorem LG ita La-
tus Coni truncati AL ad reliquam
totius Partem LC. §314 A.
5) Adde AL et LC ut innoteſcat Latus
totius Coni.

6) Infer: ut differentia semidia-
metrorum ad altitudinem EH co-
ni truncati, ita semidiameter AF
ad altitudinem coni totius CF. §314. A.

7) Ex datis Latere CA, et Diametro
AB quaere superficiem majoris
Coni CAB dempta Basi §98.

8) Similiter ex datis Latere CE et
Diametro ED, quaere Superficiem
Coni minoris CED dempta Basi §c.

9) Aufer Mbro 8. inventam a Mbro
7) inventa superficie.

10) Differentiae adde Bases Coni
truncati AB, GK, DQ §66.
inventas. Q. F. §47. A

II Pro soliditate
1) Ex datis Diametro AB et Axe
CF produc soliditatem Coni
majoris CAB. §93
2) Ex Axe CF aufer FG et remane-
bit Axis CG Coni minoris CED
3) Ex hoc et Diametro minore
ED produc soliditatem Coni
minoris §93 np. ECD
4) Aufer hanc ex illa Mbr. 1 inventa
Q. F.

Demonſtratio

Demiſsa ex C ⊥li CH in Baſis

Diametrum AB. §119. θ

cum et GF ⊥lis ad AB p. H. et 422 θ

Ergo CH ∾ GF. §138. θ.

cumq GC & DA ∾ AE/BZ p. H

Ergo GC ∾ HF. §448. θ.

CH = GF } §167. θ.
CG = HF }

Productis ergo AC et GF. §81. θ

quia ∠GFA = ∠R. p. H.

∠CAF ∠R. p. H. §422.1

∠GFA + CAF ∠res 2R: §42 A

AC et GF coëunt. §141. θ.

adeoq ob CH ∾ GF. p. d.

AH : HF = AC : CE. §349. θ.

ſed GC = HF. p. d.

AH : GC = AC : CE §10 A.
Q. E. I.

Ergo EC + CA = EA. §40. A.
Q. E. II.

CH ⊥ ad AB p. f.

AC = $\sqrt{AH^2 + CH^2}$. §195 θ

ſed CH = GF. p. d.

AC = $\sqrt{AH^2 + GF^2}$. §10 A
Q. E. III.

Tandem

$\angle CAH = \angle EAF$. §40 A.

$\angle CHA = \angle EFA$. §92 θ.

$\triangle CAH$ aq∠gl. $\triangle EAF$. §155. θ.

Ergo

$AH : HC = AF : FE$. §352. θ.

Q. E. N. D.

Reliqua per ſe patent.

Schema Calculi

Sit $AB = 2'$ $CD = 1'$. $GF = 2'$

Ergo $AF = 1'$. $CG = 5'$

atque $AH = 5''$ per Mbr. 1.

Quare per Mbr. 2.

$AH^2 = 25''q$

$HC^2 = 400''q$

$AH^2 + HC^2 = 425''q \mid 206'''+ = AC$ p. Mbr. 3.

```
 4|25
 4
 ·|25
 (4)
  |25|00
  4|00
```

Porro

$5'' : 5'' = 206''' : CE$

$CE = 206'''$ p. M. 4.

$AC = 206$. p. d.

$AE = 412'''$. p. M. 5.

5'' : 20'' = 10'' : EF

∴) 200f40'' p. M. 6

Ulterius

100 : 314 = 2' : Pph. AZDMA.

628''' = Pph. AZDMA

206''' = ½ AE

3768

1256

129368'''' = Supf. Coni. Mai. AED p. N. 7.

Nota vero Area Circuli majoris minoris expeditius inveniri poterat per §499. inferendo:

$AD^2 : CD^2$ = Area Circuli AZ : Ar. Circ. CQ

h. e.

4' : 1' = 129368'''' : 32342''''

100 : 314 = 1' : Pph. CQ

314''' = Pph. CQD

103''' = ½ CE.

942

314

32342'''' = Supf. Coni min: CED p.

97026'''' = Supf. Coni truncati dem-tis Basibus N. 9.

628''' = Pph. AZDN

50''' = ¼ Diam. AD

31400'''' = Basi majori AZFDN

314''' = Pph. CQDR

25 = ¼ Diam CD

1570

628

7850'''' = Basi minori CQGDR

136276'''' = Superficiei Coni truncati. p. Nbr. X.

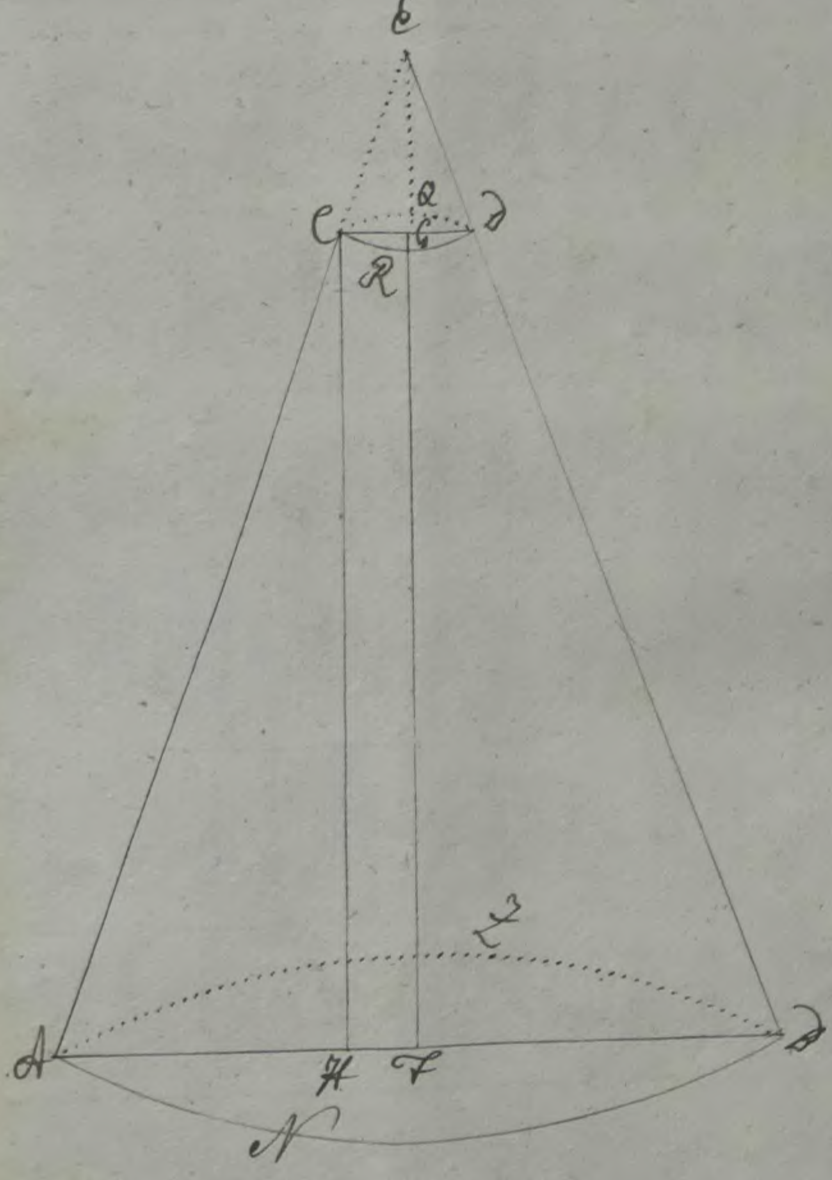
Pro soliditate
Basis major = $31400''^{q}$
EF = 4 00
3) 12560000
Soliditas Coni Majoris = $418666\tfrac{2}{3}'''^{c}$
EF = 40''
GF = 20''
EG = 20''
Basis minor = $7850'''^{q}$
EG = 20'''
3) 157000
Soliditas Coni minoris = $52333\tfrac{1}{3}'''^{c}$
Soliditas Coni truncati = $4134333\tfrac{1}{3}'''^{c}$

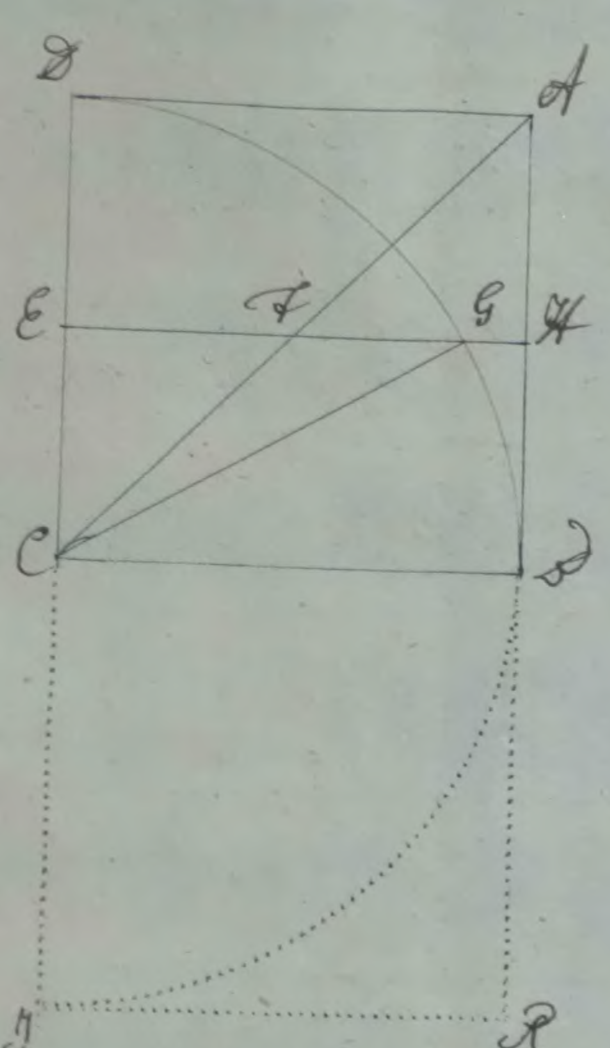
Similiter in aliis

§96. Theorema 12. cf. Fig 162.
Sphæra est ad Cylindrum super æquali Basi et ejusdem Altitudinis uti est 2 : 3. h. e
Sphæra : Cylindr = 2 : 3

Demonstratio

Concipe, descripto super DC qua=





drato §130 Θ. ductisq. Diagonali C
et Quadrante DB circa axem com-
munem DC moveri Quadratum
DABC, Quadrantem DCB et Δlu
R∠glum DCA; atq. facile liquet, ha
ipso motu a prima Cylindrum §42
Θ. a secunda Hemisphaerium §420
Θ. a tertia figura Conum rectum
§422. Θ. describi. Est autem soli
his omnibus Altitudo eadem)
§126. Θ. np. DC. Quare si illa in
Discos infini~~te~~te parvae Crassi
dissectos, Basiq. =los concipias,
Numerus hic sectionum infi
te parvarum in omnibus ide
est.
Assumto ergo quolibet pcto E
EH = CB aut DA. §135. Θ.

erit

HE Cylindri } Sectionis cu
EG Hemisphaerii } Basi =la
EF autem Coni } Semidiamet

idquod ex Genesi patet.

Duc CG. §81.

Quia $CG = CD$. §26 } θ.
$EH = CD$. §139)

$EG = EH$. §41. A.

et $CG^2 = EH^2$. §44 A. et §175. θ

= Quadrato semidiametri sectionis Cylindri cum basi ∥la.

Porro:

$DA \approx EF$

$\angle DAC = \angle FEC$. §132. θ.

$\angle DCF = \angle ECF$. §40. A.

Δ DAC aeqiang. Δ EFC. §155. θ.

$DA : EF = DC : EC$. §353. θ.

sed $DA = DC$. §68. θ.

Ergo $EF = EC$. §152. A.

Ergo $EF^2 = EC^2$. §44. A. et 175 θ

= Quadr. semidiam. sectionis Coni cum basi ∥la.

Est autem ob

$\angle ADC = \angle FEC$. p.d.

Δlum EGC. R∠lum §92. θ.

Ergo $CG^2 = EG^2 + EC^2$. §189. θ.

et $CG^2 = EC^2 = EG^2$. §43. A.

cumque $CG^2 = EH^2$. p.d.

Ergo

$EH^2 - EC^2 = EG^2$. §10 Ar.

h. e.

Quadratum ½ Diam. sectionis
Cylindri demto Quadrato ½ Diame-
tri sectionis Coni = Quadrato ½ Diametri sectionis Hemi-
sphaerii

Ergo et
Quadratum Diametri sectionis Cylindri demto Quadrat
Diametri sectionis Coni = Quadrato Diametri section
Hemisphaerii §. 42 A.

Sunt autem Circuli ut Quadrata Diametrorum. §. 49 A.

Ergo
Circulus sect. basis Cylindri demto Circulo sectionis
basis Coni = Circulo sectionis basis Hemisphaerii &c.
Quare ob altitudinem solidorum horum eandem
p. 41 2 C. adeoque et summam omnium sectionum ista-
rum infinite parvarum eandem erit: Solidum Cylindri
solida Coni = Solido Hemisphaerii §. 47. A.

Sed soliditas Coni est = ⅓ sol. Cylindri § sat.

Ergo
$\frac{3}{3}$ solid. Cylindr. − ⅓ sol. Cylindr. = sol. Hemisph. §. 10, 20 A.
$\frac{2}{3}$ solidi Cylindri DACB = h. e. solido Hemisphaerii §. 21 A. Idem
simili discursu, probabitur de altero Cylindro et Hemisphae-
rio, productis motu Quadrati IDA atque Quadrati CIDB esse

$\frac{2}{3}$ solidi Cylindri CIDR = Solid. Hemisphaerii

$\frac{2}{3}$ sol. Cylind. DACB + $\frac{2}{3}$ sol. Cyl. CIDR = solid. Sphaerae §. 42. 41
$\frac{2}{3}$ sol. Cyl. DACB + sol. Cyl. i.e. CIDR = sol. Sphaerae § 31 A.
$\frac{2}{3}$ Solid. Cyl. IA = Solido Sphaerae
Ergo 2 × solid. Cyl. IA = 3 × solid. Sphaerae §. 44. A.
Ergo solid. Sphaerae : solid. Cylindri = 2 : 3. § 311 Ar.
Q. E. D.

897. Corollarium

Hinc facillime soliditas Sphaerae invenitur, multiplicando np. soliditatem Cylindri ejusdem cum Sphaera Basis et Altitudinis in $\frac{2}{3}$

Schema Operationis

Esto Altitudo = 100''. adeoq; et
Diameter = 100''
Ergo Pphia = 314'' 360
et $\frac{1}{4}$ Diam = 25

1570
628

Ar. Bas. Cyl. = 7850''q
Alt = 100''

Solid Cyl. = 785000''c
$\frac{2}{3}$ = $\frac{2}{3}$

$\frac{2}{3}$ Solid. Cyl = $\frac{1570000''c}{3}$
= 523333$\frac{1}{3}$''c
= Solid. Sphaerae 896

Esto Telluris Diameter = 1720. Germ. Mill.
Ergo Area Circuli = 2322344. Mill. Germ. quadrat.
= 1720. M. G.

46446880
16256408
2322344

3994431680 Mill. Germ. cubic.
$\frac{2}{3}$) 7988863360

Soliditas Telluris = 2662954453$\frac{1}{3}$ Mill. Germ. cubic.

§98. Theorema 13

Cubus Diametri est ad Sphaeram quam proxime uti 300:157.

Demonstratio

Esto Diameter = 100 = D. Ergo

$D^3 = 1000000$

Sed Sphaera = $523333\frac{1}{3}$ §97.

$= \frac{1570000}{3}$ Ergo

$D^3 : \text{Sphaer.} = 1000000 : \frac{1570000}{3}$ §146.

$= 3000000 : 1570000$ §149 A.

$= 300 : 157$. Q.E.D.

§99. Corollarium

Ergo Sphaerae soliditatem datae ejus Diametro invenies ad 300, 157 et Cubum datae Diametri quaerendo quartum ppalem §314. A. h. m.

Esto D = 1720. Mill. Germ.

Esto $D^3 = 5088448000$. M. G. cub.

Ergo 300 : 157 = 5088448000 : Solid. Sphaer. Tell.

$$\begin{array}{r} 5088448000 \\ 157 \\ \hline 35619136000 \\ 25442240 \\ 5088448 \\ \hline 3\,)\,798886336000 \\ \hline 2662954453\frac{1}{3} \end{array}$$

Mill. Germ. cub. = solid. Tell. uti §97.

§100. Theorema 14.

Sphæra æquatur Pyramidi cujus Basis superficiei Altitudo autem Radio ejusdem Sphæræ æqualis est

Demonstratio.

Concipe Sphæræ Superficiem in Quadrata infinite parva eo quidem usque subdivisam esse ut a Planis rectilineis jam non amplius differant atq ex Sphæræ Centro ad singulos Quadratorum horum Angulos rectos ductas esse Lineas concipe. Sphæra ergo in infinitas Numero Pyramides resolvitur. §418. & quarum Altitudo communis est Radius, Vertice in Centro coeunt Bases autem superficiei Sphæræ æquales sunt. Quibus ergo in summam collectis Sphæra omnino æqualis est Pyramidi cujus Basis est superficies Altitudo autem Radius Sphæræ §404.

Q. E. D.

§101. Definitio 4.

Circulus Sphæræ maximus est qui per Centrum ejus transit.

§102. Theorema 15

Superficies Sphaerae est ad circulum ejus maximum = 4:1.

Demonstratio.

Esto superficies Sphaerae = S.
Circulus ejus maximus = M.
Diameter = D
Peripheria = P

Soliditas Sphaerae = $\frac{2}{3}$ Solid. Cyl. ejusd. Basi et Altit. §97.
Solid. Cyl. = Basi × Altit. §92
$= \frac{1}{4} DP \times D$
$= \frac{D^2P}{4}$ Ergo
Soliditas Sphaerae $= \frac{2D^2P}{12}$ §10. A.
$= \frac{1}{6} D^2P$. §204. Ar.
Est autem
Solid. Sphaerae = Pyramidi ejusdem Basis §100.
Solid. Pyram. = Basi × $\frac{1}{3}$ Alt. 99.
Solid. Sphaerae = Superf. Sphaer. × $\frac{1}{3}$ Alt. §10 A.
Sed Altitudo = Radio = $\frac{1}{2}$ D.
Solid. Sphaerae = S × $\frac{1}{3}$ × $\frac{1}{2}$ §10 A. h. e.
Soliditas Sphaerae = $\frac{1}{6}$ D × S.
$\frac{1}{6} D^2P = \frac{1}{6} D \times S$. §41 A.
D × P = S. §45 A. Atqui M = $\frac{1}{4}$ D × P. §62
Ergo
S : M = D × P : $\frac{1}{4}$ D × P. §145 A.
S : M = 1 : $\frac{1}{4}$ §160. 144 Ar.
S : M = 4 : 1. §159. 144

Q. E. D.

§103. Corollarium 1.

Quia $D \times P = S$ p.d. ad §99. Ergo Superficies Sphaerae aequalis est Rectangulo ex Diametro in Pphiam Circuli maximi.

§104. Corollarium 2.

Cumq; $S : M = 4 : 1$ §102.
Ergo $4 \times M = S$. §311.

§105. Corollarium 3.

Inde quidem Superficies Sphaerae invenitur.
Vel ducendo Pphiam Circuli maximi in Diametrum §103.
Vel circulum maximum ducendo in 4 §104

Schema Operationis

I Esto $D = 100$. Ergo
$P = 314$
Ergo $D \times P$ 31400 §103.
$= S$ Vel
quia $M = \frac{1}{4} D \times P$. §62
h. e $M = \frac{31400}{4}$
Ergo $4M = 31400 = S$.
§104.

II sit $D = 1720$. Mill. Germ

Ergo $P = 5400\frac{4}{5}$ Mill. G. §64.

$= \frac{27004}{5}$

cum q. $D = 1720$

540080

189028

27004

$5)\,46446880$

Ergo $D \times P = 9289376$. M. G = Superficiei Tell.

Vel

Quia $M = 2322344$. §66.

et $4 = 4$

$4 \times M = 9289376$. M. G.

§106. Corollarium 4.

Ex hactenus inventis nova Ratio inveniendæ soliditatis Sphæræ innotescit, ducendo np. Superficiem Sphæræ §105. cognitam in $\frac{1}{3}$ Radii s.q. i.e. in $\frac{1}{6}$ Diametri ejusdem Sphæræ

Schema Operationis.

I sit $D = 100$ ergo $S = 31400$

$\frac{1}{3}$ Radii $= \frac{50}{3}$

1570000

Solid. Sphær $= 3$

$= 523333\frac{1}{3}$

Vel

$S = D \times P = 31400$

$\frac{1}{6} D = \frac{100}{6}$

Solid. Sphæra $= \frac{3140000}{6}$

$= 523333\frac{2}{6}$

II Sit $D = 1720$ M. G. Ergo $\frac{1}{2} D$ h. e. Radius $= 860$

et $D \times P = S = 9289376$ M. G. quadrata.

$\frac{1}{3}$ Radii $= \frac{860}{3}$

55736256

74315008

3) 7988863360

Solid. Globi terr. $= 2662954453\frac{1}{3}$ M. G. cubic

Vel

$D \times P = S = 9289376$. M. G. Q.

$\frac{1720}{6}$

185787520

65025632

9289376

6) 15977726720

Solid. Globi terr $= 2662954453\frac{1}{3}$ M. G. C.

§1077. Problema XLVII

Invenire ſuperficiem atq ſoliditatem quinq Corporum regularium

Reſolutio et Demonſtratio

1) Soliditatem atq ſuperficiem ſub invenire. §86.

2) Eſt autem Tetraëdrum Pyramis §427. θ. Octaëdrum gemina Pyramis §428. θ. ſimiliter Dodecaëdrum atq Jcoſaëdrum ex Pyramidibus in puncto comuni intermedio coëuntibus compoſita ſu §418. θ. quorum illud ſub Baſibus Pentagonis duodecim, hoc autem ſub trigonis viginti contine §429. 430. θ. Hinc corporum horum ſoliditas invenietur per §89.

3) Superficies autem Corporum horum innoteſcet, ſi Baſis una Pyramidum iſtarum in quos reſolvitur ſolidum ducatur in Numerum, a quo corpus denominatur, ope

ƀ ƀ

pro Tetraëdro in 4
Octaëdro in 8
Dodecaëdro in 12
Jcoſaëdro in 20
} §427. 428 ug §430 θ.

Q. E. I. et D.

ƀ ƀ

§108. Definitio

Retia figuras planas dico, quibus solida terminantur.

§109. Problema XL.IIX

Rete pro Cubo describere

Resolutio.

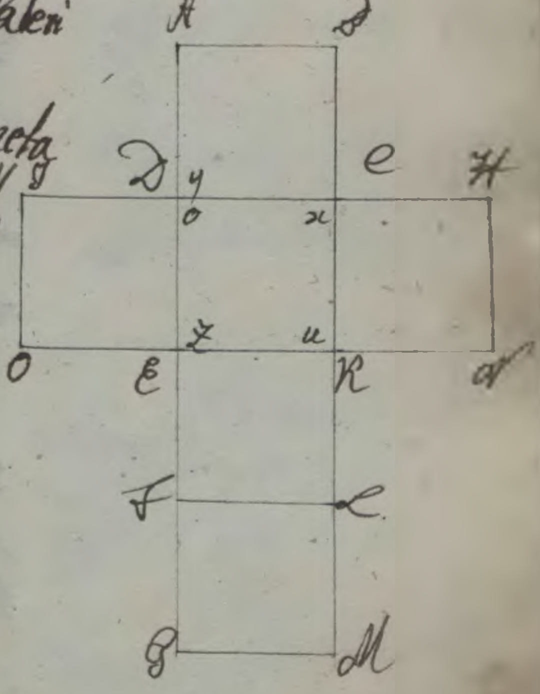
1) Latere Cubi vel dato vel arbitrarie assumto fac Quadratum AC. §110 θ.

2) Productis lateribus AD, BC, DC

3) Fac DE = EF = FG = ID = CH = lateri Cubi DC.

4) Per singula Divisionum puncta age ≈las IO, HN, CN, FL, GM, cum AD, et DC. §135 θ.

D. F.



Demonstratio.

DE ≈ CK p. C.

DC ≈ EK p. C.

DC = EK } §139 θ.

ED = CK }

ED = DC p. C.

DC = CK = KE = ED §40 A.

Cumq; AE ſit Recta p. ſ. erit
$\angle y + o = 2R$. §93. Θ.
~~$\angle y$ = R. p. ſ.~~
$\angle o = R$. §43. A.
$\angle z = R$. §169. Θ. & 43 A.
ſimiliter etiam.
$\angle u = R$ } &c.
$\angle x = R$ }
Ergo DK eſt Quadratum Lateris DC §68. Θ.

Simili Diſcurſu evincitur DO, KH, KF, FE eſſe Quadrata ejusdem Lateris DC. Ergo Figura deſcripta eſt Rete Cubi §426. Θ. §108. Q. E. D.

§110. Problema XLIX

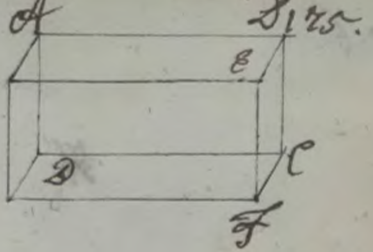
Rete pro Parallelepipedo deſcribere.

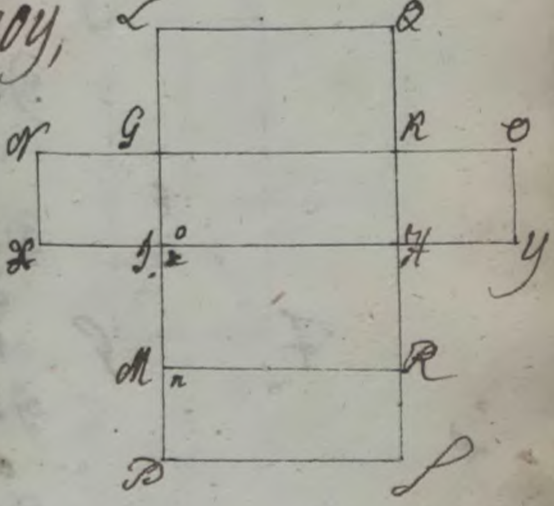
Reſolutio

1) Fac Pllgmum rect∠glum GH ∾ et = DF. §171. 344 Θ
2) Productis utrinq; Lateribus GF, KH, GK, FH. §8e. Θ.

3) Fac GL = AB
IM = AB itemq.
GN = AE
KO = AE et
MP = GI = FC.

4) Per singula Divisionum pcta age Plas LQ, IX, MR, PS, NX, OY, cum GK et GI. §135. θ. D. F.





Demonstratio

GP ≅ KS } p. C.
GI ≅ MP }

GH = MS §176. θ
GH = DF. p. C.

DF = MS. §41 A
IH ≅ MR.
∠x = ∠n §137. θ.
sed GM est Recta p. C.
∠o+x = 2R. §93. θ.
sed ∠o = R. p. C.

∠x = R. §43 A.
hinc ∠n = R. §92. θ
adeoq. MS est Rectglum §70 θ.
et MS ∠ GH [illegible] 79. θ.
sed DF ∼ GH p. C.

MS ∼ DF. §381 θ.

5

5

Simili Discursu patet
LK ≅ IR ≅ Alatq.
IN ≅ KY ≅ EC

Ergo
Figura descripta est Rete Parallelepipedi §43 θ. et 108.

Q. E. D.

§111. Problema I.

Rete pro Prismate describere.

Resolutio

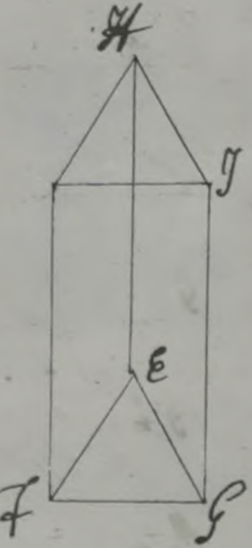
1) Construe Basin Prismatis v. c. triangularis Δlum ABC ≅ Δlo EFG

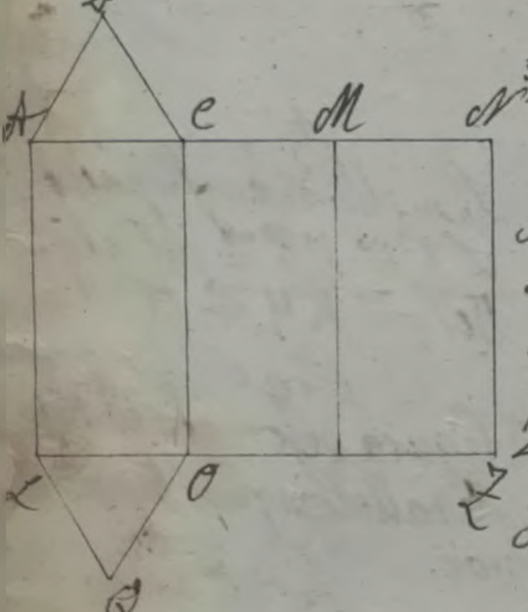
2) Producto Latere AC, fac CM = CB et MN = BA

3) Ad pctm A vel C excita AL ⊥ lem ad AC. §120. Θ. aequalem Altitudini Prismatis EH vel IG pf.

4) Per singula pcta C, M, N duc ∥ las AL et AN. §135. Θ. aut 16.

5) Super LO fac Δlum LOQ ≅ et = Δlo ABC vel EFG. §95. Θ. D. F.





Demonstratio

Sunt enim AO, OM, MZ R̄gula sub Lateribus Δli Basis et altitudine Prismatis contenta. p. C. et §70. 72. 139. Θ. Simq. ΔABC ≅ ΔLQO ≅ EFG p. C.

Ergo Figura descripta est Rete Prismatis Trigoni §419. Θ. 108.

Q. E. D.

§112. Scholion.

Simili modo Retia pro Prismatibus Polygonarum Basium fiunt.

§113. Problema LI.

Rete pro Cylindro describere.

Resolutio et Demonstratio.

1) Duc Rectam AC altitudini datæ vel pro arbitrio assumtæ Cylindri æqualem.

2) Produc utrinque eandem §82 ut AB = CD = Radio Basis Cylindri.

3) Radiis istis centris B et D describe Circulos §83. θ qui erunt Cylindri Bases. §424. θ

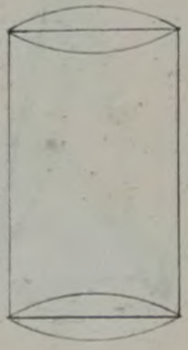
4) Ex Basis semidiametro quære Circuli Pphiam §64.

5) Hanc secundum scalam modicam ad ∠R transfer ex A in AE. §5 et 8. hujus et 120 θ ut fit AE = Pphiæ.

6) Per C et E duc ∥las EF et CF, cum AE et EC. §135. θ. §16.

Ergo CE est Rectangulum sub Pphia Basis Cylindri et Altitudine comprehensum. Ergo Figura descripta est Rete Cylindri §424. θ et §90

Q. E. R. et D.



§114. Problema LII.

Rete pro Pyramide deſcribere.

Reſolutio et Demonſtratio.

Eſto Pyramis Baſis triangular

1) Radio DC = fa vel fb, fc deſcri Arcum. §83. θ.

2) In hoc coapta Chordas aequal Lateribus Trianguli Baſis datæ Pyramidis §307. θ. h. e.

$CE = ab.$
$EG = ac$
$GH = bc$

3) Super Chordarum una fac △. $GHS \cong \triangle$lo abc. §98. θ. D. 7. §418. θ.

§115. Scholion

Simili ratione retia Pyramid Baſium polygonarum conſtruant

§116. Theorema 6.

Circulorum Pphiæ ſunt inter ſe ut Diametri vel Radii.

Demonſtratio.

Eſto Pphia unius Circuli = P
ejus Diameter = D,
et Area = A.
Eſto Pphia alterius Circuli = p,
ejus Diameter = d
et Area = a.

$$\left.\begin{array}{l}\text{Quia } A = \frac{1}{4} D \times P \\ \text{et } a = \frac{1}{4} d \times p\end{array}\right\} \S 62.$$

Ergo

$A : a = \frac{1}{4} D \times P : \frac{1}{4} d \times p$. §145. A.

$A : a = D^2 : d^2$. §499. G.

$$\left.\begin{array}{l}\frac{1}{4} D \times P : \frac{1}{4} d \times p = D^2 : d^2. \ \S 144 \\ D \times P : d \times p = D^2 : d^2. \ \S 159 \\ P : p = D : d. \ \S 163\end{array}\right\} A.$$

Q.E.I.

Ergo et

$P : p = \frac{1}{2} D : \frac{1}{2} d$ §160. et 144 A.

h. e. ut Radii §25. G.

Q.E.II.D.

Aliter

Quia Circuli Polygona infinitorum Laterum referunt §498 G.
Omnes autem sint inter se similes
§23. 78. 79. G.

Ergo

$P : p = D : d$. §483. G.

adeoque etiam

$P : p = \frac{1}{2} D \overset{P}{:} \frac{1}{2} d$. §160. 144 A.

Q.E.D.

§117. Theorema 17.

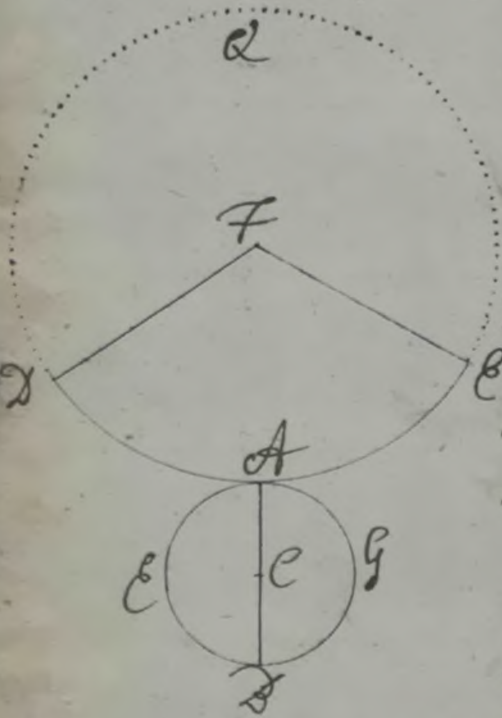
Peripheria Basis Coni recti DAE
est ad Pphiam DEQ Latere Con
EF tanquam Radio describenda
uti est semidiameter Basis A
ad Latus Coni

Demonstratio.

Quia superficies Coni recti =
Δlo cujus Basis est Pphia altitu
Latus Coni. §90. Ergo.
Si Pphia Basis AEBG fiat = Arcu
DAE et
Latus Coni FE = Radio
Superficies Coni recti aequatur
Sectori Circuli. Latere Coni tan
quam Radio descripti cujus
Arcus DAE aequalis est Pphiae
Basis AEBG. §69 sed
AEBG : DEQ = AC : FE. §116.
AEBG = DE. per hyp.
DE : DEQ = AC : FE. §10 A.
Q. E. D.

§118. Corollarium.

Hinc datis duobus Radiis inae
qualibus in Elementis determina
tis invenies Arcum DE Pphiae ma
joris DEQ, qui sit aequalis toti Pphiae minori



ad FE, AC et 360° quaerendo quartum numerum ppalem per §314 Arith.

§119 Problema L.III

Rete pro Cono delineare.

Resolutio et Demonstratio.

1) Diametro Basis AB describe Circulum §23. G.

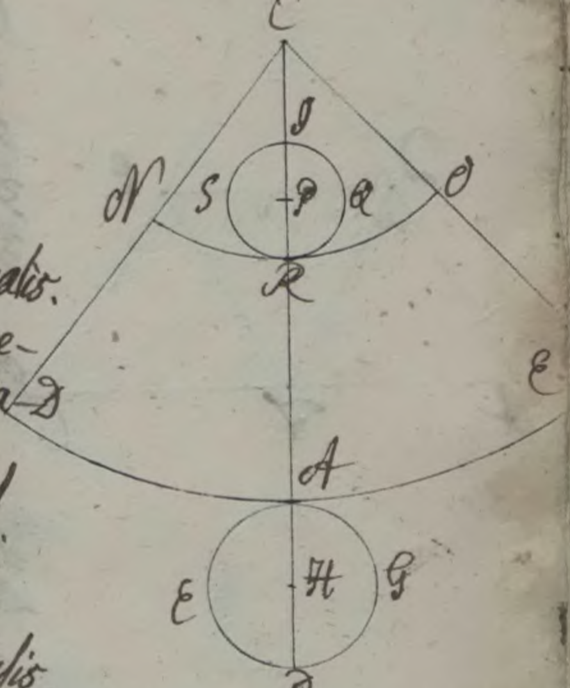
2) Eandemque Diametrum produc, donec Coni Lateri CA fiat aequalis.

3) Quaere ad latus Coni semidiametrum Basis et 360° quartum ppalem §314. G.

4) Radio CA describe Arcum et

5) Ad C fac angulum DCE = Arcui Numero invento §23.

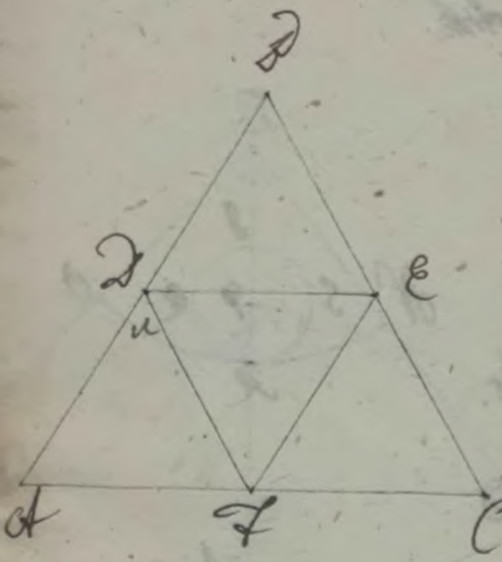
Eritque sector DCE una cum Basis Circulo EDGA Rete Coni §117. et 118. hujus atque §423. G. Q. E. R. et D.



§120 Corollarium

Quodsi ex A in R transferas Latus Coni truncati atque ad 360° Arcum NO h. e. angulum NCO atque CR quaeras Numerum quartum ppalem §314 et invenies Radium RP Circuli cujus Peripheria = Arcui NO §118. et §196 adeoque Rete Coni truncati. Nam cum CDE sit Rete integri Coni majoris §119 CNO Rete Coni minoris §.

Ergo NODE + RSIQ + EDGA Rete Coni truncati §95.



§121 Problema LIV

Rete Tetraëdri deſcribere

Reſolutio

1) Super Recta AC data vel aſſumta deſcribe Triangulum aeq∠t §96 θ.

2) Diſeca Latera in D, E, F, §112 θ.

3) Junge Rectas DE, EF, FD. §87. θ
D. F.

Demonſtratio.

AD : DB = AF : FC. p. C.

DF ≈ BC. §349. θ

∠B = ∠u §132. θ.

Sed ∠A = ∠A. §40 Ax.

△ ABC aeq∠gl. △ ADF. §155. θ.

AB : AD = BC : DF. §353. θ.

Sed AB = BC. p. ſ.

Ergo AD = DF. §152. A.

Porro

∠u = ∠B p. d.

∠A = ∠B p. C. §55. θ.

∠u = ∠A. §41 A.

AF = DF. §160 θ.

△ ADF eſt aequilaterum §55.

Simili diſcurſu erit

△ DBE et △ EFC aequilaterum.

$BC \cong EF$ p.d.
$DE \cong EC$ p.l.

$\triangle DBE = \triangle ECF$. §177. θ. similiter
$\triangle DBE = \triangle DFA$

$\triangle DBE = \triangle ECF = \triangle DFA$. §41 A.

Tandem

Quia $BE \cong FA$ } p.d.
$DB \cong FE$ }

DF est Pllgmm. §72 θ.

$\triangle DBE = DEF$. §167. θ.

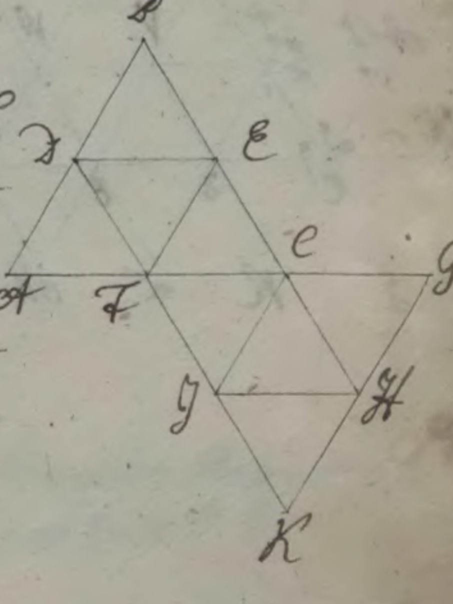
Quare cum Triangulum $DBE = DEF = ECF = DFA$. §41. A.

Ergo

descripta Figura est Recte Tetraëdri §427. θ et 108.

§122. Corollarium.

Quod si AC continuetur ut $FG = AC$ et reliqua fiant uti § antec. Figura descripta erit Rete Octaëdri, uti ex demonstratione § anteced. additis addendis satis liquet. Cf. §428. θ. et §108.

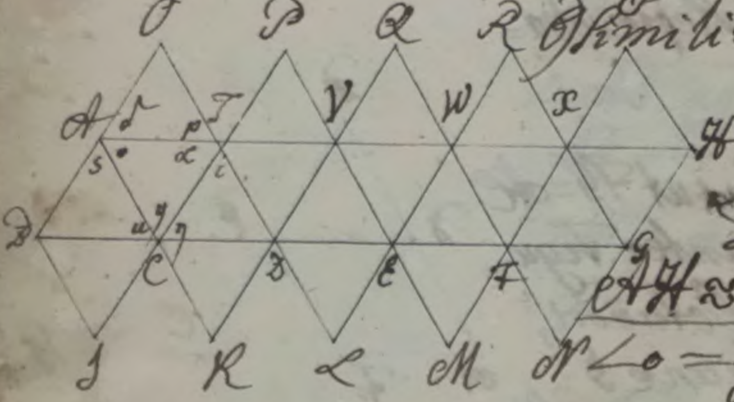


§123. Problema LV.

Rete pro Icosaëdro describere

Resolutio.

1) Construe Triangulum æqlate
§96 θ.
2) Producto Latere DC fac CB=
BE = EF = FG = DC.
3) Per A duc ≈lam AH. §125. θ.
4) Fac AT = TV = VW = WX = D
= DC
5) Duc Rectas AK per A et C
OL per T et B
PM per V et E etc.
6) Similiter OD per A et D
OI per T et C
QK per V et B.
Q.F.



Demonstratio

AH ≈ DG p. f.
d ∠o = ∠u §132 θ.
cumque AT = et ≈ DC p. f.
et AD ≈ TC §139 θ.
∠y = ∠s §132 θ.
△ADC ∼ △lo ACT §155. 352. 341 θ.
sed ∠o = ∠u } p d
∠y = ∠s } p d
AC = AC §40 A.
△ADC = △lo ACT. §114. θ.

Porro $AH \cong CB$.

$\angle\alpha = \angle\eta$. §132 Th.

cumq $AT \cong$ et $= CD$ p. h.

β $AC \cong TD$. §139. Th.

$\angle y = \angle i$ §132. Th.

$\Delta ACT \sim \Delta CTD$. §153. 352. 341. Th.

cumq $CT = CT$. §40 Ax.

$\Delta ACT = \Delta CTD$. §144. Th.

Idem simili discursu de Triangulis TDY, WVE, WCF, FWX, XFG, GHA ostendendum.

Tandem $AC \cong TD$ p. h. Ergo et

$AH \cong OL$. §82. Th.

Ergo $\angle y = \angle o$. §132 Th.

et ob $DO \cong BS$ p. d.

$\angle o = \angle a$

Δlum $OAT \sim \Delta$lo ACT. §153. 341, 352 Th.

atq ob $AT = AT$.

$\Delta OAT = ACT$. §114. Th.

Idquod cum simili prorsus ratiocinio de reliquis Triangulis omnibus pateat, inde quidem viginti e Numero Triangula ista sunt aequilatera similia §376. Th. et aequalia. §41 Ax. Ergo Icosaëdri Rete descripta erit §430 Th. §168.

Q. E. D.

186.

§127 Problema LVI

Rete Dodecaëdri constituere.

Resolutio.

1) Describe regulare Pentagonum §326. aut 405. & vel §38.

2) Applicata ad A et B Regula d Rectas AG et BF ipsi AB aequal

3) Eodem modo duc rectas AJ, HC, DL, KD &c.

4) Intervallo Lateris Pentagon fac Intersectiones in Q ex G et L in R ex N et

5) Similiter construe Pentagona α, β, γ, δ, η Q. F.

Demonstratio

Ostendendum est omnia Pentagona ista esse aequalia inter se et ordinata Id quod h. m. evincetur: BAEC est Quadrilaterum Circulo inscriptum p. C.

$\angle u = \angle C$. §2 16 b

$\angle C = \angle$lo Pent. regul. p. C.

$\angle u = \angle$lo Pent. regul. §41

Simili Diſcurſu erit
$\angle a = \angle o$ Pentag. Regul.
Sed $AG = GQ = AD$. p. C.

AGQLD eſt Pentagonum ordinatum §299 & §39

Cumque AD ſit Latus utrique Pentagono commune p. C. Ergo
Pentagonum ABCDE ≅ Pentagono ADLQG. §86. &.
Idem ſimili Diſcurſu de reliquis Pentagonis demonſtrabitur.

Ergo

Figura deſcripta eſt Rete Dodecaëdri §429. &. 108.

Q. E. D.

§125. Theorema 18.

Tetraëdrum, Octaëdrum, Icosaedrum, Cubus et Dodecaëdrum ſunt Corpora regularia nec praeter hoc quinque aliud eſſe poſſibile.

Demonſtratio

Quia Corpora regularia ſunt ſolida Figuris ordinatis atque aequalibus terminata. §430. 429. 428. 427. 426. &. hoc ſane unum eſt oſtendendum, Figuras ordinatas iſtas et aequales, in Verticibus ſuis

Angulorum planorum ita qu
dem coire ut ∠lum ſolidum
conſtituant, h.e. ut ſimul ſum
ſint minores 4 ∠lis Rectis p
§453 θ.
Quare
Aſsumta Figurarum regularium
prima, np. Triangulo æquilater
patet ∠lum quemvis $= \frac{2}{3}R$. §14
θ.
Ergo
tres ∠li, qui ad minimum requi
runtur ad ſolidum ∠lum §416.
in communi Puncto concurren-
tes efficient $3 \times \frac{2}{3}R = 2R$.
Id quod fit in Tetraëdro §427 f.
Q.E.I.

Aſsumtis quatuor ∠lis Triangu-
li æquilateri liquet Summam
∠lorum Planorum illorum æqua
lem eſse $4 \times \frac{2}{3}R = \frac{8}{3}R$.
$= 2 + \frac{2}{3}R$.
Id quod fit in Octaëdro §428. θ.
Q.E.II.

Aſsumtis quinque ∠lis ejusdem æ-
quilateri Trianguli Summa erit.
$= 5 \times \frac{2}{3}R = \frac{10}{3}R$.
$= 3 \times \frac{1}{3}R$.
Id quod fit in Icosaëdro §430. θ.
Q.E.III

Assumtis autem sex ∠lis Trianguli æquilateri, quia illorum summa
$= 6 \times \frac{2}{3} R.$
$= 4 R.$
manifestum est nullum solidum Corpus esse possibile, quod terminetur Triangulis ordinatis, quorum sex verticibus suis coëuntia ∠lum efficiant solidum §451. θ.
Q. E. N.

Porro
Quadrati ∠lus $= R.$ §68. θ.
Trium ergo Quadratorum ∠li in uno puncto a diversis plagis concurrentes efficient 3 Rectos.
Id quod fit in Cubo. §426. θ.
Q. E. V.

Liquet autem ex §433 θ quatuor Quadratorum ∠los h. e. 4 R. ∠los solido constituendo majores esse. Inde quidem unum solummodo solidum est meris Quadratis æqualibus terminatum np. Cubus. Tandem Angulus Pentagoni regularis
$= \frac{6}{5} R.$ §327. θ. Ergo.

compoſitorum tri
Summa $= 3 \times \frac{6}{5} R = \frac{18}{5} R$
$= 3 + \frac{3}{5} R.$
adeoq. ∠res < 4 Rectis §453 b.
Idquod fit in Dodecaëdro
Q. E. VI.
Plurium autem r. e quatuor ∠lori
Pentagoni regularis ad unum
Punctum conſtitutionem impoſ-
ſibilem eſſe patet ex §453 b.
Nam $4 \times \frac{6}{5} R = \frac{24}{5} R = 4 + \frac{4}{5} R.$
Poſtremo quia
∠lus Hexagoni ordinati = 120 §3
adeoq. tres = 360
∠lus Heptagoni ordinati = $128\frac{4}{7}$ &c.
adeoq. tres = $385\frac{5}{7}$
Figuræ iſtæ cum ſequentibus
ordinatis omnibus, quorum
∠li pro Laterum Numero eti
creſcant, neq. ∠los ſolidos §453. b.
neq. Corpora ſolida regularia
terminabunt. Q. E. D.

§126. Problema LVII

Virgulam cylindrimetricam
conſtruere h. e. cujus ope haud
difficulter invenitur Numerus

Mensurarum Fluidi alicuius in Vase Cylindrico contenti

Resolutio.

1) Diametrum Vasis Cylindrici v.c. Canthari unius PQ transfer in Rectam infinitam DA.

2) Adjunge huic ad ∠ R. aliam infinitam AF. §157 θ.

3) Fac A1 = AD

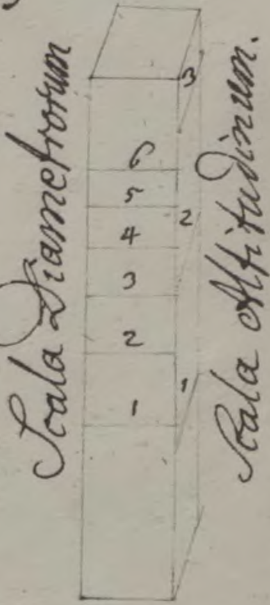
4) Duc D1, quae erit Diameter vasis duorum Cylindrorum PQG sub ejusdem Cylindri Altitudine.

5) Hanc ex A transfer in 2, et duc D2, quae erit Diameter Vasis trium Cylindrorum, qualium PQG est unus, sub eadem cum PQG altitudine.

6) Similiter hanc ex A transfer in 3, ducta D3 quae erit Diameter 4 Cantharorum qualium PQG.

7) Simili modo invenies reliquas plurium Cantharorum Diametros qualium PQG est unus np. A4, A5 etc.

8) In alterum Virgulae latus transfer has Diametros inventas A1, A2 etc. in alterum autem Cylindri tui PQG Altitudinem PR, quoties fieri potest. D.F.



Aliter.

Possunt autem Diametri istorum Cylindrorum unum, duos, tres &c. Cylindros PQD sub Altitudine eadem ipsius PQD capientes, per Calculum inveniri atq; in Diametri PQ particulis decimalibus centesimis atque millesimis, per modum Scalae Geometricae subdivisae. cf. §. et §. in Virgulam transferri.

Schema Calculi.

Esto Diameter PQ = 1,000

Ergo $PQ^2 = AD^2 = 1000000$

adeoq; $2 \times AD^2 = 2000000$ f. h. e. $DA^2 + 1A^2$

$1D^2 = §189$

```
2|00|00|00
1|00
 (2)
 96
  4.|0.0
  2 |81
  1 |19.|0.0
     (28 |2)
  1 |12  |96
        6|04 negliguntur.
```

Similiter.

$DA^2 + (DA)^2 = (D2)^2$: §189.

sed $A_2 = 1D$ p. C.

$1D = 1,414.$

$A^2 = 1,414.$

Proinde

$(Ae)^2 = 1999396$

$Ad^2 = 1000000$

$(Ae^2) + Ad^2 = 2999396 = f(de)^2$ § 41 Ar.

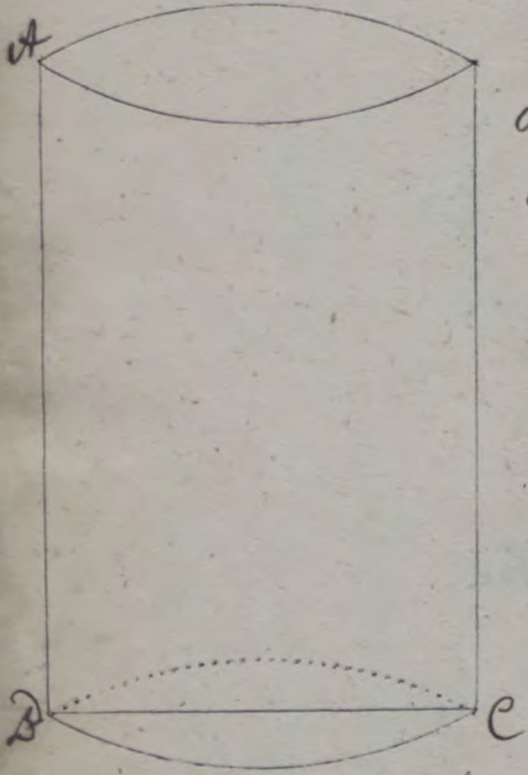
$1,732 = de$

```
   1
  1|99
  (2)
 1|89
  |10|93
  |(34)
  |10|19
     |64|96
     |34|6
```

Inde quidem Tabula

Mensura	Diametri.
1	1,000
2	1,414
3	1,732
4	2,000
5	2,236
6	2,449
7	2,645
8	2,828
9	3,000

cf. Wolf. Geometr lat. §582.



Demonstratio.

Sunt enim Cylindri sub eadem Altitudine inter se ut Quadrata Diametrorum Basium §503. et Hinc sub eadem Altitudine Quadratum Basis Diametri Vasis alicuius duas, tres, quatuor etc. Mensuras capientis est duplum, triplum, quadruplum etc. Quadrati Basis Diametri Vasis alterius unam solummodo Mensuram capientis. Extractis ergo Radicibus §256 Ar. quadraticis per Resolutionem aequationum vel etiam Geometrice per Resol. quam cum $DI^2 = DA^2 + AI^2$ §198 et $= 2\times DA^2$. Ergo $DI = \sqrt{2\times DA^2}$. Invenies Diametros ipsas Vasorum, duas, tres, quatuor etc. Mensuras capientium sub eadem cum Vasis unius Mensurae Altitudine. Quare adplicata Virgula Cylindrimetrica a Parte Diametrorum ad diametrum Vasis Cylindrici

Schema Calculi.

Sit per Constructionem hoc § pho Virgulam cylindricam.

$AB = 4$. $DC = b$. hinc

Capacitas Cylindri $AB \times DC$ $= 4\times b$ = eq. tali-um Vasorum, qualium [illegible] est unum innoteſcit, quot Menſurarum ſit Vas aliquod Cylindricum habens eandem Altitudinem cum Cylindro unius Menſuræ.

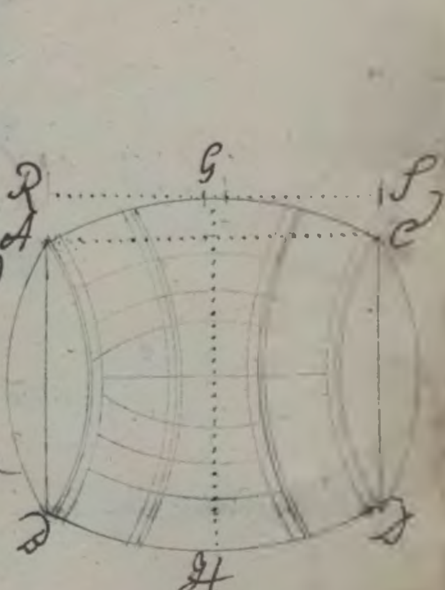
Porro adplicata Virgula eadem a Parte Altitudinum intelligitur, quoties Altitudo unius Menſuræ contineatur in Altitudine Vaſis Cylindri integri. Quod ſi ergo hanc in Baſis Diametrum ducas, prodibit Menſurarum Numerus quæſitus Q. E. D.

§127 Problema L.IIX

Invenire Capacitatem Dolii. h. e. Numerum Menſurarum in ipſo contentarum.

Reſolutio

1) Legitime adplicata Virgula §126 ad utramq. Dolii Diametrum AB, CD, obſerva eandem Quantitatem.



2) Cum Experientia conſtet, Rigore licet Geometrico haud in diem nondum Demonſtratum Dolium pro Cylindro haberi poſſe, cujus Baſis inter Fundum et Ventrem ſit media æquidifferens; Ergo

inter AD et GH quore mediam aqui differentiam. §351. A.

3) Numerum inventum duc in Longitudinem Dolii AC, per §126 demonstrata inventam, factumq; erit.

Schema Calculi.

Esto $AD = CD = 3$ } §126.
$RS = AC = 4$ }
$GH = 7$

Ergo quia
$AD = 3$
$GH = 7$
$AD + GH = 10$ Ergo
$\frac{1}{2} \times AD + GH = 5$ §351 A.
$RS = 4.$
Capacitas Dolii $= 20$.

Similiter in aliis.

§128. Scholion I.

Quodsi Basium Diametri fuerint inaequales, addendae sunt, earumque semisumma pro Diametro fundi s. basis assumenda.

Schema Calculi.

Esto $AD = 3$. $CD = 4$ $GH = 7$.
$AC = RS = 4$.

Hinc $AB + CD = 7$

et $\frac{1}{2}\ AB + CD = \frac{7}{2}$

$GH = 7$

$\frac{1}{2} \times (AB + CD) + GH = \frac{21}{2}$

Ergo per §351. Arith. erit media æquidifferens $= \frac{21}{4}$

ſed Altitudo $= 4$

Capacitas Dolii $= 21$.

§129. Scholion 2.

Plura dabunt Wolffius Geom. lat. §589 atq. Auctores ab eodem excitati. Inprimis autem Joh. Matthias Haſius in eximio Tractatu quem de Pithometriæ Theoria et Praxi conſcripſit. Witt. 1728 4^{to}

Finis Geometriæ Practicæ


KSIĄŻNICA
CIESZYŃSKA