Olena GUMEN
National Technical University of Ukraine "Kyiv Polytechnic Institute"
Volodymyr DOVHALIUK, Viktor MILEIKOVSKYI
Kyiv National University of Construction and Architecture

GEOMETRIC MODELLING OF TURBULENT FLAT JETS IN ACCOMPANYING AND CONTRARY FLOWS

We offer an approach to describe turbulent jets in accompanying and contrary flows without using experimental coefficients and additional values (turbulent viscosity, mixing path length etc.). It is based on the theory developed by professor of Kyiv National University of Construction and Architecture A. Tkachuk. The analogy between an ideal liquid flow and developed turbulent flow allows describing surfaces of tangential velocity rupture by vortex sheets. We continue this research direction by analysis of jet flows with large-scale vortices (puffs) by geometric and kinematic analysis of this macrostructure. The results have good correlation with well-known experimental data. We analytically obtain conditions of the jet existence in contrary flow. This approach may help for efficient development of new air distribution devices of energy efficient air exchange organization.

Keywords: turbulent flow, turbulent jet, vortex, puff, method of singularities

INTRODUCTION

Various types of turbulent jet flows are used for ventilation and air conditioning [1]. The uniformity of air distribution in rooms is determined by jets development. Jet expansion rate usually regarded because of turbulent pulsations in the jet boundary layer and the ambient medium (in case of jet development in the flow). To describe turbulent pulsations usually we use values that cannot be obtained by measuring, such as turbulent viscosity, mixing path length, turbulent Prandtl number, turbulence kinetic energy, turbulence kinetic energy dissipation etc. The most widespread method of turbulent flow simulation is Computational Fluid Dynamics (CFD). Jets contain large-scale vortices (puffs). For flows with large vortices Large Eddy Simulation (LES) is used [2-8]. The main problem using CFD is impossibility of direct analysis of equations. The research process is similar to experiments.

Alternatively to these approaches Professor of Heat Gas Supply and Ventilation Department of Kyiv National University of Construction and Architecture A. Tkachuk [1] has developed a theory of turbulent boundary layers according to singularity method. The flow is regarded as a stream of ideal liquid containing small vortex flows as "singularities". This approach allows describing the effect of vortices directly without using additional values.

Our continuation of this research is an approach to the jet flow analytical description, which describes the jet boundary layer as an aggregate of large-scale vortices (puffs). This approach [9] gives the fundamental constant of the jet - tangent of expansion angle, which is 0.22. The main problem of this research is the presence of an imputed value that relies on complicated inflow to vortices near to the contact point. This value needs to be removed from consideration.

1. MAIN CONCEPTS OF THE JET MACROSTRUCTURE ANALYSIS

In free jets [9] there is a possibility of presenting the jet boundary layer (Fig. 1) as a vortex sheet with puffs rolling along free jet boundary.

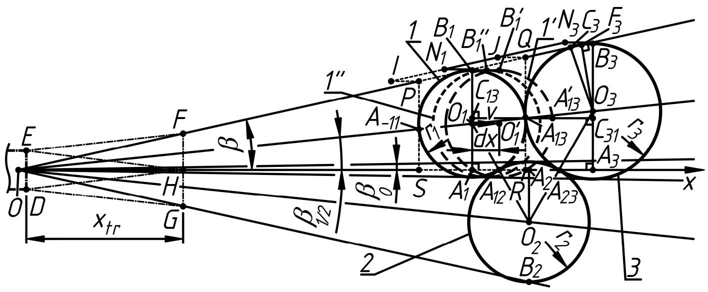

Fig. 1. Jet chart

A line connecting the centres of vortices actually coincides with the characteristics line, where velocity amounts to half of axial velocity. Let B is the jet half-width. Let x-axis be directed from the inlet slot centre in the jet direction.

In work [9] the free jet is presented as two sheets of puffs in chessboard order. The relation was obtained between the tangent Θ of jet expansion angle β, the tangent $\Theta_{1/2}$ of expansion angle $\beta_{1/2}$ of the line, where velocity v equals half of the axial velocity u_m, and the tangent Θ_0 of the angle β_0 of puff immersion in the adjacent layer. The error is up to 0.32% for $\Theta < 1$.

$$\Theta_{1/2} = 0.4655\Theta, \Theta_0 = 0.069\Theta \tag{1}$$

The correlation between the radius r_i and the abscissa x_i of any (i-th) vortex centre O_i is the following:

$$r_i = x_i\left(\Theta - \Theta_{1/2}\right) = x_i\left(\Theta_{1/2} + \Theta_o\right). \tag{2}$$

Let us consider Tolmien source, which flows from the infinitely thin slot at point O to the uniform accompanying or contrary flow with the velocity projected on x is u_∞. Let us mark vortices 1, 2 and 3 with radiuses r_1, r_2 and r$_3$ and the abscissas x_1, x_2, x_3 of vortex centres O_1, O_2, O_3. Let us designate as A_i the x-projection of the i-th vortex axis and designate as B_i the point of the puff which is most distant from x-axis. The points of contact between puffs i and j shall be designated as A_{ij}. The point of contact of puff 1 with the preceding adjacent vortex shall be designated as A_{-11}. The projection of point A_{13} on the line A_1B_1 shall be designated as C_{13}, while on the line $A_3B_3 - C_{31}$. Axis O_1 of vortex 1 moves with a forward velocity:

$$v = (u_m + u_\infty)/2 \tag{3}$$

in the direction of x-axis and goes away from it due to velocity gain. Contrary to [9] let us consider vortex movement during infinitely small time interval $d\tau$ step by step as gradual displacement along x-axis by an infinitely small value $dx_1 = v\,d\tau$ and corresponding infinitely small gain. After the time $d\tau$ vortex 1 will take position 1'. All its points will displace to corresponding points with a prime. If line 1 shell move with the ambient medium velocity u_∞ it will take position 1" at distance $dx_0 = u_\infty \mathrm{d}\tau$ from the initial position after the same time. All its points will displace to corresponding points with double prime. The distance between corresponding points of lines 1' and 1" is dx_Δ.

Let C_i be used to designate the point of intersection of vortex i and the jet boundary that is different from B_i. In the curvilinear triangle $B_1A_{13}C_3$ there is only an inflow of the medium to the jet that is normal to the x-axis [9]. And this is possible only if at least all the medium of the curvilinear figure $B''_1B'_1A'_{13}A''_{13}$ is consumed by the jet. If less media is consumed, vortex 1 will either produce air motion in the direction of x-axis or push it out from the jet, which contradicts the physical meaning of the problem. The area of this figure is:

$$dA_c = |B_1C_{13}|dx_\Delta = r_1\left(1 - \Theta_{1/2}\left(1 + \Theta_{1/2}^2\right)^{-1/2}\right)(u_m - u_\infty)\,d\tau/2. \tag{4}$$

By the work [9] the consumed area contributes to gain of the total area A_Σ only of the vortex 1 and the curvilinear triangle $A_{13}A_{23}A_{12}$. The outer part of inter-puff layer also increases. We will add a curvilinear triangle $B_1A_{13}C_3$ to A_Σ.

If the jet is formed in the contrary flow, it loses medium to the ambient flow. If the line B_1B_3 shell displaced with the velocity of the ambient flow u_∞ during the time $d\tau$, we will get a parallelogram $N_3N_1B_3B_1$. The area is:

$$dA_m = (|\mathrm{A_3B_3}| - |\mathrm{A_1B_1}|)|\mathrm{N_1B_1}| = -(\mathrm{r_1} + \mathrm{r_3})\Theta\, \mathrm{u_\infty d\tau}/(1 + \Theta_{1/2})^{1/2} \tag{5}$$

To get simple dependencies we will use a reference figure with area A_o instead of A_Σ. The total area A_Σ, which consumes dA_c, should be proportional to A_o with a factor:

$$a = A_o/A_\Sigma = \mathrm{const} \tag{6}$$

If jet boundaries are rectilinear, vortices and inter-puff layers are homothetic with homothety centre O. To ensure constant factor a we need homothety of the reference figures at every puff with the same homothety centre. We choose the reference figure to obtain simple equations. A jet with curvilinear boundaries can be regarded as near to rectilinear with sufficient accuracy for three adjacent puffs.

If the ambient flow is contrary, it is desirable to replace the figure $N_1N_3B_3B_1$ with an additional reference figure with area $dA_{m,0}$. The proportionality factor is

$$a_m = dA_{o,m}/dA_m = \text{const} \tag{7}$$

The balance equation (a term in braces is only for the contrary flow) will be:

$$dA_c = (dA_o/a) + \{dA_{m,0}/a_m\} \tag{8}$$

We will use a trapezoid PQRS as a reference figure for A_o. The lines PS and QR pass through the points A_{-11} and A_{13} normally to the x-axis. The points P and Q are on the jet boundary, and S and R - on the x-axis. If such trapezoids are built on every vortex, the entire jet will be filled except very small peripheral puff segments. The medial line is a width of the jet half, the high is $|A_{-11}A_{13}|\cos(\beta_{1/2})$. The area according to the equations (1, 2) is:

$$\mathrm{A_o} = 2\Theta \mathrm{x}_1^2[\mathrm{r}_1/\mathrm{x}_1]/\left(1+\Theta_{1/2}^2\right)^{1/2} = \mathrm{r}_1^2/\left(0.26725\left(1+(0.4655\,\Theta)^2\right)^{1/2}\right) \tag{9}$$

The change of the area A_o is calculated by the formulas (3, 9) for the time $d\tau$ at the puff 1 motion with the velocity v considering constant in the square brackets:

$$dA_o = 4\Theta x_1[r_1/x_1]dx_1/\left(1+\Theta_{1/2}^2\right)^{1/2} = 2\Theta r_1(u_m+u_\infty)d\tau/\left(1+\Theta_{1/2}^2\right)^{1/2} \tag{10}$$

The additional reference figure IJQP is obtained by displacing the line PQ together with the ambient flow during the time dτ. Analogically to the equation (5):

$$dA_{m,0} = -2r_1\,\Theta\,u_\infty\,d\tau/\left(1+\Theta_{1/2}\right)^{1/2} \tag{11}$$

Let us substitute equations, (1, 4, 10, 11) in the dependence (8). At $x \neq 0$

$$\Theta = (\tilde{a}\bar{u}/4)/\left(1+[\Theta_{1/2}/2\Theta]\tilde{a}\bar{u}\right)^{1/2} = (\tilde{a}\bar{u}/4)/\left(1+0.23275\,\tilde{a}\bar{u}\right)^{1/2} \tag{12}$$

where:
$\bar{u}$ - relative velocity of the ambient medium;

$$\bar{u} = \left(1-(u_\infty/u_m)\right)/\left(1+(u_\infty/u_m)\right) \tag{13}$$

$\tilde{a}$ - the reduced proportionality factor. If $u_\infty \geq 0$ it is equal to a. If $u_\infty < 0$

$$\tilde{a} = \left(\frac{1}{a} - \frac{(u_\infty/u_m)/\left(1+(u_\infty/u_m)\right)}{a_m}\right)^{-1} \tag{14}$$

2. THE JET EXPANSION TANGENT CALCULATION

Let us determine relative radiuses. By Pythagorean Theorem we solve $\Delta O_1A_1A_{12}$, $\Delta O_2A_2A_{12}$, $\Delta O_2A_2A_{23}$ and $\Delta O_3A_3A_{23}$. And using equations (1, 2) radiuses (and diameters $d_i = 2r_i$) form a geometrical progression with a denominator:

$$q = \frac{r_{i+1}}{r_i} = \frac{d_{i+1}}{d_i} = \frac{1+\Theta\left(1-2\left(\Theta_{1/2}/\Theta\right)\right)^{1/2}}{1-\Theta\left(1-2\left(\Theta_{1/2}/\Theta\right)\right)^{1/2}} = \frac{1+0.26268\Theta}{1-0.26268\Theta} \tag{15}$$

The result (15) is identical to [9] but without additional constructions. The equation (15) have no abscissa. So if the boundaries are curvilinear but we can linearise it between three adjacent puffs this equation has acceptable precision. The total area:

$$\begin{aligned} A_\Sigma = A_1 + A_{B_1A_{13}B_3} + A_{A_{13}A_{23}A_{12}} = A_1 + A_{O_1B_1B_3O_3} - A_{O_1B_1A_{13}} - \\ - A_{B_3O_3A_{13}} + A_{\Delta O_1O_2O_3} - A_{O_1A_{13}A_{12}} - A_{O_2A_{12}A_{23}} - A_{O_3A_{23}A_{13}}, \end{aligned} \tag{16}$$

where:
A_1 - area of the puff 1;
$A_{B_1A_{13}B_3}$, $A_{A_{13}A_{23}A_{12}}$ - areas of curvilinear triangles $B_1A_{13}B_3$ and $A_{13}A_{23}A_{12}$;
$A_{O_1B_1B_3O_3}$, $A_{\Delta O_1O_2O_3}$ - areas of the trapezoid $O_1B_1B_3O_3$ and the triangle $O_1O_2O_3$;
$A_{O_1B_1A_{13}}$, $A_{B_3O_3A_{13}}$, $A_{O_1A_{13}A_{12}}$, $A_{O_2A_{12}A_{23}}$, $A_{O_3A_{23}A_{13}}$ - areas of corresponding sectors.

After the areas calculation using formulas (1, 2, 6, 9, 16) we obtained intricate equations with solution shown on Figure 2 that can be calculated by the following:

$$\begin{aligned} a = 0.17857\Theta^7 - 1.02577\Theta^6 + 2.13547\Theta^5 - \\ - 1.68644\Theta^4 + 0.24324\Theta^2 + 0.1092\Theta + 0.9323 \pm 0.00018 \end{aligned} \tag{17}$$

The equation (7) for a_m using formulas (5, 7, 11, 15) is shown on the Figure 2. The numerical solution of the equations (12-14, 17) gives the result shown in Figure 3.

At $u_\infty = 0$ the solution is $\Theta_{calm} = 0.2179$ (by the formula (17) $\Theta_{calm} = 0.2180$). It repeats all decimals of the well known experimental value $\Theta_{calm} = 0.22$. Approximation formulas at the Figure 3 have deviation up to 0,171% at $u_\infty > 0$ and 0.16% at $u_\infty < 0$. The Figure 3 shows that $\Theta < 0{,}3483$. So we may use equation at the Figure 2.

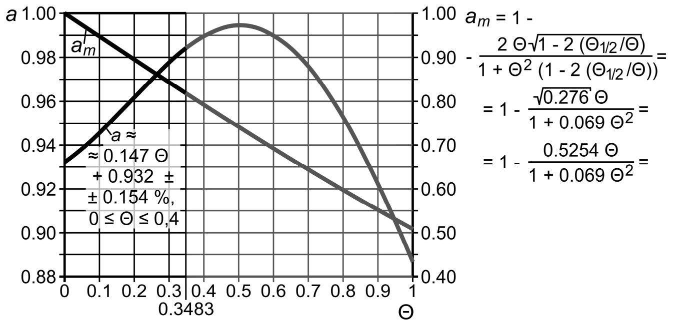

Fig. 2. The proportionality factors of the areas

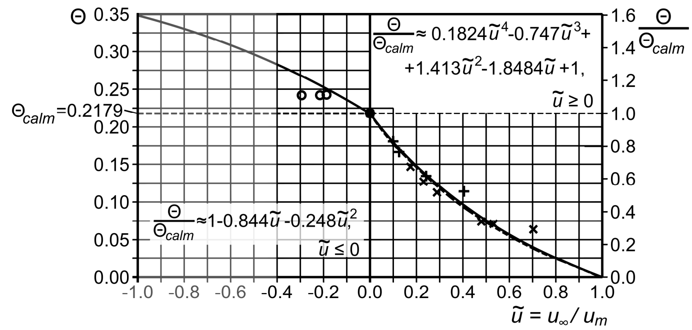

Fig. 3. Jet expansion tangent: – – our calculation data; - - - – theoretical data of H. Abramovych [10]; experimental data [10] of: + – O. Yakovlevskyi, × – B. Zhestkov and other, ○ – H. Abramovich and F. Vafin

3. JET EXISTENCE CONDITIONS

A jet that runs out of a finite width slot DE has an initial section DEFG in which the vortex structure is formed. If initial velocity profile is uniform, the velocity in jet core ΔEHD is equal to the initial one u_0 [1]. After the completion of formation in a transition section FG with abscissa x_{tr} and width 2 B_{tr} the jet corresponds to the Tolmien source (main jet section). The reference point O is the pole of the jet.

The velocity decay by momentum equation at the obtained expansion angle is given in work [10]. There is no experimental data for $u_m/u_\infty \le -0{,}3$. For this range there is a great deviation between calculation results and theoretical data [10].

We will use the momentum equation for transition and current sections with a similar velocity profile and macrostructure. After transformations:

$$B\left(\left(u_m/u_\infty\right)-1\right)^2 = B_{tr}\left(\left(u_0/u_\infty\right)-1\right)^2 \tag{1}$$

Maximum jet velocity u_m in the contrary flow tends to the flow velocity $u_\infty < 0$. If there are no other destroy reasons, in a section velocity u_m becomes zero and then change direction. It is a destroy. So, a jet in the contrary flow has limited length.

If we consider the boundary layer as a vortex sheet, the condition for destroy is a puff stop ($v = 0$). Using the dependence (3) we obtain a condition of the jet existence: the maximum jet velocity u_m may be greater than minus x-projection of the contrary flow velocity (always true for calm ambience and accompanying flow).

For existence of the main jet section it may contain some puffs. Stopped puff hinder previous puff movement. It deforms and destroys. The deformations allows short movement of a previous puff. And so on. So for main section development we need at least $n = 5$ puffs. The condition of the jet existence may be true along the way length equal to sum of the first n puff diameters. In the contrary flow the value of Θ changes slow (Fig. 3). We can accept an average value Θ_{av} between maximum and minimum value in the range of u_∞/u_{in}. By the dependency at the Figure 3, (15, 18) and formula of the sum of first members of the progression:

$$\left(\frac{u_\infty}{u_0}\right)_{cr} = \left(1 - 2\left(1 + 4.06959\frac{(1+0.26268\,\Theta_{av})^n - (1-0.26268\,\Theta_{av})^n}{(2-1.069\,\Theta_{av})(1-0.26268\,\Theta_{av})^{n-1}}\right)^{1/2}\right)^{-1} \quad (2)$$

Using the method of successive approximations $(u_\infty/u_{in})_{cr} = -0.4$ for the equation (19) after three iterations. It agrees with experimental data absence [10].

Further research will be focused on a more accurate analysis of the expansion of jets, which are laid on surfaces of different form. Then the macrostructure of jets in a flow situated at a certain angle to the jet will be analysed. This approach may be helpful for developers of the energy efficient air exchange organization.

CONCLUSIONS

1. An improved analytical description of free flat jets without the use of experimental values is suggested based on geometrical analysis of their structure. The results agree with commonly known experimental data.
2. Conditions of jet existence in the contrary flow are obtained. It is shown that if the ratio of ambient flow velocity to maximum velocity inside the jet is less than minus 0.4, the jet is destroyed by the ambient flow. It coincides with research data of H. Abramovich.

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MODELOWANIE GEOMETRYCNE BURZLIWYCH PLASKICH STRUMIENÓW W TOWARZYSZĄCYCH I PRZYPADKOWYCH PRZEPŁYWACH

Proponowane podejście do opisy burzliwych (turbulentnych) strumienów w towarzyszących i przypadkowych (зустрічних) strumieni bez użycia współczynników badawczych i stopniów dodatkowych (lepkość burzliwośti (turbulęntności), długość ścieżki mieszania itp.). Opiera się na teorii opracowanej przez profesora Kijóewskiego Narodowego Uniwersytetu Budownictwa i Architektury A.Y.Tkaczuka. Analogia między przepływami cieczy idealnej i rozwiniętym burzliwym pozwala do opisywania powierzchni pęknięcia części prędkośći tangencìalnej przez zasłoną wiru.

Kontynuacją tego kierunku badań jest opis strumienów przepływu o dużych wirach (kluby) za pomocą geometrycznej i kinematycznej analizy jej makrostruk-tury. Wyniki dobrze zgadzają się z powszechnie znanymi danymi eksperymental-nymi. Analitycznie uzyskane są warunki istnienia struminia w przepływie przeciw-nym. Takie podejście powinno pomóc w rozwoju nowych dystrybucyjnych urządzeń dla energooszczędej organizacji wymiany powietrza.

Słowa kluczowe: **przepływ burzliwy, strumień burzliwy, wir, kluby, metoda osobliwości**