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RISK-BASED APPROACH IN PORTFOLIO MANAGEMENT ON POLISH POWER EXCHANGE AND EUROPEAN ENERGY EXCHANGE

Introduction

Investors on the Polish Power Exchange POLPX may participate in the Day Ahead Market (DAM, spot market), the Commodity Derivatives Market (CDM, future market), the Electricity Auctions, the Property Right Market, the Emission Allowances Market (CO_2 spot) and the Intraday Market. All these markets differ with respect to an investment horizon length and the traded commodity.

The European Energy Exchange AG (EEX) in Leipzig is one of the European trading and clearing platforms for energy and energy-related products, such as natural gas, CO_2 emission allowances and coal. The EEX consists of three sub-markets (EEX Spot Markets, EEX Power Derivatives and EEX Derivatives Markets) and one Joint Venture (EPEX Spot Market). EEX is trying to become the leader among European Energy Exchanges assuming an active role in the development and integration process of the European market.

The choice of risk measure is an important step towards building a realistic picture of portfolio risk. In the literature, there has been a debate about the properties of various risk measures and which risk measure is best from a practical viewpoint. From a historical perspective, variance was suggested as a proxy for risk by Markowitz[1] as a part of a framework for portfolio selection that is still

[1] H.M. Markowitz: *Portfolio Selection*. „Journal of Finance" 1952, 7, 77-91.

widely used by practitioners. The disadvantages of variance as a measure of risk are well documented in the literature.

It has been pointed out in numerous empirical studies that the daily rates return of prices noted on energy markets exhibit autoregressive behavior, clustering of volatility, skewness and fat-tails. These phenomena should be accounted for by the probabilistic model, otherwise the risk measure may be unable to take into account appropriately the probability of extreme events.

In this paper we compare portfolio selection model based on use, as measure of risk, Conditional Value at Risk. We examine portfolios on electric energy spot markets based on linear daily rates of return of prices noted on POLPX and EEX from 1st January 2009 to 13th March 2013.

1. The traditional approach to portfolio optimization

The fundamental goal of portfolio theory is to optimally allocate investments between different assets. Mean-variance optimization (MVO) is a quantitative tool which allows to make this allocation by considering the trade-off between risk and return.

The classical Markowitz optimization problem, which constitutes the main theoretical background for the modern portfolio theory, is widely described and analyzed in literature, so we just briefly recall the mean-variance problem.

For given n risky assets the mean variance portfolio (MV) is the portfolio of assets that minimizes risk measured by the variance of portfolio return for a given covariance matrix $\sum$. It is a solution to the following problem[2]:

$$\begin{aligned} &\min\{x^T \sum x\} \\ &\max \mu^T x \\ &x \in S \end{aligned} \tag{1}$$

where:
x – vector of portfolio weights,
μ – vector of contracts means belong to portfolio,
S – set of acceptable results,
$\sum$ – covariance matrix.

[2] Ibid.

The simplest non-empty and bounded set X of feasible portfolios are usually considered as:

$$S = \{x \in \Re^n : \sum_{i=1}^{n} x_i = 1, \quad x \geq 0\}.$$

2. Downside risk portfolio selection

Since risk is an asymmetric phenomenon, a true risk measure should focus on the downside only. A risk measure which has been widely accepted since the 1990s is the value-at-risk (VaR). It was first popularized by JP Morgan and later by Risk Metrics Group in their risk management software. VaR became so popular that it was approved by bank regulators as a valid approach for calculating capital reserves needed to cover market risks.

VaR is defined as such loss of value, which is not exceeded with the given probability α at the given time period Δt, and given as follows[3]:

$$\mathrm{P}(W_{t+\Delta t} \leq W_t - VaR_\alpha(W)) = \alpha, \tag{2}$$

where:

W_t – a present value,

$W_{t+\Delta t}$ – a random variable, value at the end of duration of investment.

Equation (#2) describes VaR_α for short position. VaR_α answers the question: How much money can we lose over time period Δt with probability $1-\alpha$? The VaR quantity represents the maximum possible loss, which is not exceeded with the probability α.

For linear rates of return VaR_α we can write as a percentile of the order α of rates of return for short position:

$$\mathrm{P}(R_t \leq VaR_\alpha(R)) = \alpha \tag{3}$$

and for long position:

$$\mathrm{P}(R_t \leq VaR_{1-\alpha}(R)) = 1-\alpha, \tag{4}$$

where:

$\mathrm{R}_t = \frac{\mathrm{P}_t - \mathrm{P}_{t-1}}{\mathrm{P}_{t-1}}$ – a linear rate of return of contract,

$\mathrm{P}_t, \mathrm{P}_{t-1}$ – the prices of the instrument.

[3] P. Jorion: *Value at Risk: New Benchmark for Managing Financial Risk* (3rd ed.). McGraw-Hill, 2006.

It was found out, however, that VaR has an important disadvantage: it is not always subadditive. This means that VaR may be incapable of identifying diversification opportunities. There has been a good deal of criticism of VaR in the literature because of this shortcoming but it remains a widely used method for risk measurement by practitioners mainly because it has an intuitive interpretation and because it is required by regulation.

The fact that VaR may be unable to detect diversification opportunities raised an important debate as to whether it is possible to define a set of desirable properties that a risk measure should satisfy. This is, essentially, an axiomatic approach towards defining risk measures. A set of such properties was given by Artzner et al.[4] who defined axiomatically the family of coherent risk measures.

A representative of coherent risk measures which gained popularity is conditional value-at-risk (CVaR), also known as average value-at-risk or expected tail loss. CVaR is more informative than VaR about extreme losses and is always sub-additive, implying it can always identify diversification opportunities.

The CVaR quantity is the conditional expected loss given the loss strictly exceeds its VaR. In literature CVaR is also called Expected Shortfall (ES)[5]. For short position we can write:

$$CVaR_{\alpha}(R) = E\{R \mid R \geq VaR_{\alpha}(R)\}\,. \tag{5}$$

For long position we can write:

$$CVaR_{1-\alpha}(R) = E\{R \mid R \leq VaR_{1-\alpha}(R)\}\,. \tag{6}$$

CVaR is defined as the mean of the quantile of worst realizations. The definitions ensure that the VaR is never more than the CVaR, so portfolios with low CVaR mast have low VaR as well. Pflug[6] proved that CVaR is a coherent risk measure having the following properties: transition-equivariant, positively homogeneous, convex, monotonic, stochastic dominance of order 1, and monotonic dominance of order 2[7]. Moreover, various numerical experiments and studies

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4 P. Artzner, F. Delbaen, J.M. Eber, D. Heath: *Coherent Measures of Risk*. „Mathematical Finance" 1999, 9(3), 203-228.

5 W. Ogryczak, A. Ruszczyński: *Dual Stochastic Dominance and Quantile Risk Measures*. „International Transactions in Operational Research" 2002, 9(5), 661-680; S.A. Heilpern: *Aggregate Dependent Risks – Risk Measure Calculation*. „Mathematical Economics" 2011, 7(14), 108-122.

6 G.Ch. Pflug: *Some Remarks on the Value-at-risk and the Conditional Value-at Risk*. In: *Probabilistic Constrained Optimization: Methodology and Applications*. Ed. S. Uryasev. Kluwer, Dordrecht 2000.

7 G.Ch. Pflug: Op. cit.; R.T. Rockafellar, S. Uryasev: *Optimization of Conditional Value-at-Risk*. „The Journal of Risk" 2000, 2(3), 21-41.

considering portfolio optimization with CVaR point out that the minimization of CVaR leads to optimal solutions in terms of the VaR[8].

The portfolio selection model is based on two criteria mean-variance portfolio problem[9]:

– for short position is given as follows:

$$\begin{aligned} &\min CVaR_{\alpha} \\ &\max \mu^T x \\ &x \in S \end{aligned} \tag{7}$$

– for long position:

$$\begin{aligned} &\min CVaR_{1-\alpha} \\ &\max \mu^T x \\ &x \in S \end{aligned} \tag{8}$$

Using results of Steuer et al.[10] the problems (#7)-(#8) may be expressed in the form for short position:

$$\begin{aligned} &\min | CVaR_{\alpha} - \mu^T x | \\ &x_{\min} \le x_i \le x_{\max} \\ &\sum_{i=1}^{m} x_i = 1 \end{aligned} \tag{9}$$

and for long position:

$$\begin{aligned} &\min | CVaR_{1-\alpha} - \mu^T x | \\ &x_{\min} \le x_i \le x_{\max} \\ &\sum_{i=1}^{m} x_i = 1 \end{aligned} \tag{10}$$

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[8] R.T. Rockafellar, S. Uryasev: *Conditional Value-at-Risk for General Loss Distributions*. „Journal of Banking and Finance" 2002, 26(7), 1443-1471; S. Uryasev: *Conditional Value-at-Risk: Optimization Algorithms and Applications*. „Financial Engineering News" 2000, 14, 1-5.

[9] R.E. Steuer, Y. Qi, M. Hirscheberger: *Developments in Multi-attribute Portfolio Selection*. In: *Multiple Criterion Decision Making*. Ed. T. Trzaskalik. UE, Katowice 2006, 251-262.

[10] R.E. Steuer, Y. Qi, M. Hirscheberger: *Comparative Issues in Large-scale Mean-variance Efficient Frontier Computation*. „Decision Support Systems" 2011, 51(2), 250-255.

3. Empirical analysis

We build portfolios from POLPX and EEX on the basis of daily rates of return of prices from 1st January 2009 to 13th March 2013, because investors from spot energy markets make trading decision with one day horizon of investment. Because of negative energy prices on EEX linear rates of return were applied. In both analyzed markets investors can buy and sell electric energy in 24 independent contracts. We have observed that distribution of contracts is characterized by very high volatility, asymmetry and is leptokurtic.

We compare risk on portfolios built independently on these two energy markets and portfolios form POLPX and EEX together. We estimate *VaR* and *CVaR* using historical simulation method for $\alpha = 0.95$.

In table 1 we presented portfolios for investors who take up long position on POLPX. Based on problem (#10) we built three different portfolios.

Table 1

Portfolios on POLPX (long position)

Contracts	**Portfolio 1**			**Portfolio 2**			**Portfolio 3**		
	x	x_{min}	x_{max}	x	x_{min}	x_{max}	x	x_{min}	x_{max}
1	0,3451	0	1	0,0791	0	0,0853	0,0487	0	0,0583
2	0,012	0	1	0,0824	0	0,0876	0,0648	0	0,0477
3	0	0	1	0,086	0	0,0816	0,0412	0	0,0566
4	0	0	1	0,0917	0	0,0888	0,0501	0	0,0568
5	0	0	1	0,0914	0	0,0824	0,0489	0	0,0574
6	0	0	1	0	0	0,0735	0,0205	0	0,0585
7	0	0	1	0	0	0,0976	0	0	0,071
8	0	0	1	0	0	0,0895	0	0	0,0704
9	0	0	1	0	0	0,0114	0	0	0,0744
10	0	0	1	0	0	0,0939	0	0	0,0688
11	0	0	1	0,0124	0	0,0951	0	0	0,0901
12	0	0	1	0	0	0,0969	0,003	0	0,0719
13	0	0	1	0	0	0,0962	0,0721	0	0,0612
14	0	0	1	0	0	0,0958	0	0	0,0708
15	0	0	1	0	0	0,0922	0,0576	0	0,0672
16	0	0	1	0,0252	0	0,0892	0,0589	0	0,0642
17	0	0	1	0	0	0,09	0	0	0,065
18	0	0	1	0	0	0,0919	0,0677	0	0,0669
19	0	0	1	0,0745	0	0,095	0,07	0	0,07
20	0	0	1	0,0841	0	0,0989	0,0839	0	0,0872
21	0	0	1	0,0897	0	0,0985	0,0575	0	0,0735
22	0,2794	0	1	0,0901	0	0,099	0,0862	0	0,0652
23	0,3248	0	1	0,1035	0	0,1177	0,1087	0	0,0873
24	0,0387	0	1	0,0899	0	0,0931	0,0602	0	0,0683
Objective (#1)	0,0879			0,1185			0,1603		
Mean	0,0017			0,0031			0,0034		
VaR	0,0645			0,0768			0,1207		
CVaR	0,097			0,1388			0,1785		
SD	0,0401			0,0602			0,0618		

Portfolio number one consists only of night contracts. In the next two portfolios the real demand for electric energy in respective hours of the day was taken into consideration ($0 \le x_i \le x_{\max}$). In the second portfolio x_{max} was assumed to be equal to the real demand observed on POLPX for the contract in the studied period, augmented by 5%. In the third portfolio contracts are augmented by 2.5%. Based on these portfolios we can say that investors shouldn't buy electric energy in hour 7-10, 14 and 17.

In table 2 we presented portfolios for investor opening long positions on EEX. Based on problem (10) we built next three different portfolios. For every hour of the day we built two portfolios ($0 \le x_i \le x_{\max}$) under the same constraint as for POLPX. Based on these portfolios we can say that investors shouldn't buy electric energy in hour 1, 5 and 9.

In next step of the analysis the portfolios based on 48 contracts form POLPX and EEX were built. Table 3 presents results of the optimization problem (10). In general, when we compare risk measures by $CVaR_{0,95}$, risk on EEX is greater than risk on POLPX, so weights of contracts from POLPX are greater than weights of contracts from EEX, especially for night and early morning hours from 1 to 9. There is no significance difference between weights in hours during a day.

In the portfolio 7 the restriction for portfolio weights was used similar to portfolio 1 (for POLPX) and portfolio 4 (for EEX). For portfolios 8 and 9 x_{max} was assumed in the same way as for earlier constructed portfolios for POLPX and EEX.

Table 2

Portfolios on EEX (long position)

Contracts	Portfolio 4			Portfolio 5			Portfolio 6		
	x	x_{min}	x_{max}	x	x_{min}	x_{max}	x	x_{min}	x_{max}
1	2	3	4	5	6	7	8	9	10
1	0	0	1	0	0	0,0853	0	0	0,0583
2	0	0	1	0,002	0	0,0876	0,0242	0	0,0477
3	0,0003	0	1	0,0024	0	0,0816	0,012	0	0,0566
4	0,0012	0	1	0,0008	0	0,0888	0,0084	0	0,0568
5	0	0	1	0	0	0,0824	0	0	0,0574
6	0,0002	0	1	0	0	0,0735	0	0	0,0585
7	0,0004	0	1	0,0002	0	0,0976	0,0001	0	0,071
8	0	0	1	0,0011	0	0,0895	0,0006	0	0,0704
9	0	0	1	0	0	0,0114	0	0	0,0744
10	0,0146	0	1	0,0487	0	0,0939	0,0454	0	0,0688
11	0,0674	0	1	0,0641	0	0,0951	0,0648	0	0,0901
12	0,0561	0	1	0,0812	0	0,0969	0,0656	0	0,0719
13	0,0741	0	1	0,0701	0	0,0962	0,0674	0	0,0612
14	0,0678	0	1	0,0841	0	0,0958	0,0678	0	0,0708
15	0,0795	0	1	0,0689	0	0,0922	0,0599	0	0,0672
16	0,0011	0	1	0,0237	0	0,0892	0,0278	0	0,0642
17	0,0594	0	1	0,0548	0	0,09	0,0781	0	0,065
18	0,0794	0	1	0,0711	0	0,0919	0,0679	0	0,0669
19	0,0887	0	1	0,0785	0	0,095	0,0686	0	0,07
20	0,0675	0	1	0,0761	0	0,0989	0,0688	0	0,0872

Table 2 cont.

1	2	3	4	5	6	7	8	9	10
21	0,0724	0	1	0,0601	0	0,0985	0,0689	0	0,0735
22	0,0921	0	1	0,0699	0	0,099	0,0626	0	0,0652
23	0,0855	0	1	0,0711	0	0,1177	0,0729	0	0,0873
24	0,0923	0	1	0,0711	0	0,0931	0,0682	0	0,0683
Objective (#1)	0,079			1,1211			1,8401		
Mean	0,0145			-0,214			-0,374		
VaR	0,5584			0,3124			0,5147		
CVaR	1,0871			1,1399			1,2873		
SD	0,5521			1,9741			3,249		

Table 3

Portfolios on POLPX and EEX

Contracts	**Portfolio 7**			**Portfolio 8**			**Portfolio 9**		
	POLPX	EEX	x_{max}	POLPX	EEX	x_{max}	POLPX	EEX	x_{max}
1	0,2849	0,001	1	0,0833	0	0,0853	0,0583	0	0,0583
2	0,0245	0,0022	1	0,0817	0,0005	0,0876	0,0567	0	0,0477
3	0,0202	0,0011	1	0,0252	0,0001	0,0816	0,0566	0	0,0566
4	0,0206	0	1	0,0244	0,0004	0,0888	0,0297	0,0021	0,0568
5	0,0206	0	1	0,0243	0	0,0824	0,0248	0	0,0574
6	0,0204	0	1	0,024	0	0,0735	0,0245	0	0,0585
7	0,0216	0,0001	1	0,0232	0,0002	0,0976	0,0238	0,0001	0,071
8	0,0245	0	1	0,0234	0,0001	0,0895	0,0239	0,0002	0,0704
9	0,0235	0	1	0,0235	0	0,0114	0,024	0	0,0744
10	0,0206	0,0139	1	0,0224	0,0147	0,0939	0,0242	0,0144	0,0688
11	0,0201	0,0185	1	0,0239	0,0231	0,0951	0,0244	0,0217	0,0901
12	0,0234	0,019	1	0,023	0,0224	0,0969	0,0244	0,0224	0,0719
13	0,0254	0,0188	1	0,0239	0,0217	0,0962	0,0244	0,0222	0,0612
14	0,0203	0,0152	1	0,0207	0,0169	0,0958	0,0244	0,0169	0,0708
15	0,0204	0,0189	1	0,0239	0,0217	0,0922	0,0254	0,0223	0,0672
16	0,0201	0,0063	1	0,0244	0,0048	0,0892	0,0245	0,004	0,0642
17	0,0204	0,0139	1	0,0239	0,0148	0,09	0,0244	0,0145	0,065
18	0,0201	0,0189	1	0,024	0,022	0,0919	0,0278	0,0223	0,0669
19	0,0205	0,0196	1	0,0214	0,0229	0,095	0,0277	0,0233	0,07
20	0,0207	0,02	1	0,0242	0,0234	0,0989	0,0247	0,0239	0,0872
21	0,0209	0,0202	1	0,0245	0,0237	0,0985	0,0249	0,0242	0,0735
22	0,0208	0,0204	1	0,0245	0,0214	0,099	0,0251	0,0254	0,0652
23	0,0209	0,021	1	0,0247	0,025	0,1177	0,0294	0,0253	0,0873
24	0,0207	0,022	1	0,0243	0,0262	0,0931	0,0248	0,0262	0,0683
Objective (1)	0,3804			0,4874			0,4246		
Mean	-0,0222			-0,0171			-0,0162		
VaR	0,2267			0,2587			0,2239		
CVaR	0,33816			0,3721			0,3999		
Std. Deviation	0,84			0,7168			0,6241		

The negative value of portfolios return for POLPX and EEX together (see table 3) as well as for EEX (see table 2) can be the result of a negative electricity prices observed on EEX. The negative electricity prices ware first observed in 2009 on EEX as a result of demand and supply changes which come independently from price.

Conclusions

The risk of price changes on EEX is much greater than risk on POLPX. Contracts in night and early morning hour on POLPX are more attractive, but for odd hours contracts on two spot markets give very similar distance between risk and profit. Portfolios constructed for both electricity markets consist of contracts for all hours during the day in opposite to the portfolios built only for POLPX and EEX.

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ZARZĄDZANIE RYZYKIEM PORTFELA NA TOWAROWEJ GIEŁDZIE ENERGII (POLPX) I EUROPEJSKIEJ GIEŁDZIE ENERGII (EEX)

Streszczenie

W artykule dokonano analizy porównawczej modeli wyboru portfeli budowanych w oparciu o miarę ryzyka CVaR (warunkowaną wartość zagrożoną). Zbadano portfele na rynkach kontraktów krótkoterminowych energii elektrycznej z wykorzystaniem liniowych dziennych stóp zwrotu cen notowanych na Towarowej Giełdzie Energii (POLPX) i Europejskiej Giełdzie Energii EEX.