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COMPARISON OF MARKET RISK OF AN EQUITY ASSET CLASS MEASURED BY VALUE AT RISK AND MAXIMAL LOSS ACCORDING TO MONTE CARLO METHOD WITH FRACTIONAL BROWNIAN MOTION EVOLUTION OF THE PRICE AND HISTORICAL SIMULATION APPROACH

Summary: In this paper, author provides a comparison of market risk of the six equities from the Polish stock exchange. In order to calculate the risk, quantile-based risk measures have been used: Value at Risk and Maximal Loss. Two common approaches to calculate quantile-based measures have been used: Monte Carlo simulation and historical simulation. However, for the simulation of the future paths in the Monte Carlo approach, the fractional Brownian motion has been used instead of geometric Brownian motion.

Keywords: fractional Brownian motion, Monte Carlo, Value at Risk, Maximal Loss, Hurst exponent.

Introduction

Last events in the world economy show how important risk measurement and risk management are. There are several widely used risk measures, such as variance, standard deviation, entropic risk measure, superhedging price, Value at Risk and its different types, expected shortfall or conditional VaR, maximal loss, Greeks or Sharpe and Sortino ratios. None of them are perfect, but each of the above is used in order to evaluate different types of risks or evaluate the risk from a different perspective.

Author decided to use Value at Risk (VaR) and Maximal Loss (ML). VaR is commonly used market risk measure, but its disadvantage is that it is not a coherent risk measure (VaR fails to satisfy the second axiom – subadditivity [Jorion, 2007]), which has been introduced by Artzner et al. [1999]. On the other hand ML might be perceived as a weakly coherent risk measure [Studer, 1997]. They are both quantile-based measures, which might be a premise that the analysis based on them will be consistent and will not be contradictory with each other.

In contrast, other market risk measures, for example standard deviation or variance are not truly risk measures, but measures of dispersion around the mean. On the other hand ratios like Sharpe or Sortino ratios might be perceived as a good indicator or measure of risk, but only when we compare the results with other ratios from the stocks from the same sector of the economy, market capitalization or business model. Duration and convexity are measures used to evaluate risk involved in fixed income securities.

In order to calculate VaR and ML, the historical simulation and Monte Carlo simulation with a fractional Brownian motion evolution of the price have been performed. Fractional Brownian motion assumes that the underlying process is not random (*i.e.* is not a Markovian process) as opposed to the geometric Brownian motion. In order to show that the processes (equity prices) used in this research are not random, but have long range dependence, Hurst exponent has been calculated (see Table 1). Contrary to Monte Carlo simulation approach to calculate VaR and ML, historical simulation approach has been performed, as a common method to calculate market risk used in financial sector.

Main aim of this paper is to show the advantageous of using in market risk analysis Monte Carlo approach, in which the equity price evolution has the fractional Brownian motion features over the historical simulation approach. The analysis has been performed prior the 2007/2008 global financial crisis and after it. Author used price history starting since 2001 in simulation and parameter estimation and provided risk measure for the next 1 year ahead.

1. Value at Risk

"VaR is a summary measure of downside risk expressed in dollars, or in the reference currency" [Jorion, GARP, 2003, p. 250] (absolute VaR) or in the percentage (relative VaR). VaR is the maximum loss over a target horizon, such that there is a low probability that the actual loss will be larger (by loss in the equity asset class, the author means a decrease in the equity price). To evaluate the loss, which can occur and be bigger than calculated VaR, there is a different risk measure called expected shortfall, which is a mean of the α% of the worst case losses.

VaR ($VaR(\alpha,t)$) at fixed time t and probability α is defined as follows:

$$\mathrm{P}(S_0 - S_t \geq VaR(\alpha, t)) = \alpha, \tag{1}$$

where:

- S_0 and S_t are the initial value (price) and final value (price) of the financial instrument, respectively,
- α is a tolerance level.

VaR describes quantile of the projected distribution of losses over the target horizon. This measure shows how big losses can be at the given confidence level, but it is expressed as a positive number.

There are three core approaches for calculating VaR:

- historical simulation,
- Monte Carlo simulation,
- VaR calculated from assumed probability distribution (parametric VaR).

Only first and second methods are interesting from the point of view of this research.

To compute VaR for longer than 1 day period, it is a need to scale 1 day VaR by multiplying it by the factor of $\sqrt{t}$. This solution is derived from the $T^{1/2}$ rule, but at this point, it is explained that it could be used when data are normally distributed. For the purpose of this research we assume that logarithmic rates of return are normally distributed.

2. Maximal Loss

ML is strictly connected with the ruin theory and it is commonly used in insurance business, but ML can be easily applied in risk measurement of equity. ML has one main advantage comparing to VaR. Namely, value of ML is not strictly determined for a particular moment of time.

Moment of ultimate ruin is given by [Chrzan, 2006]:

$$T = \inf\{t : U(t) < 0\}, \tag{2}$$

where:

- $U(t)$ is a level of financial surplus at $t \geq 0$ moment,
- probability of ruin is a function given by:

$$\psi(u) = P\{U(t) < 0, \text{ for } 0 < t < \infty\}. \quad (3)$$

Ruin could occur in the situation presented below (Fig. 1).

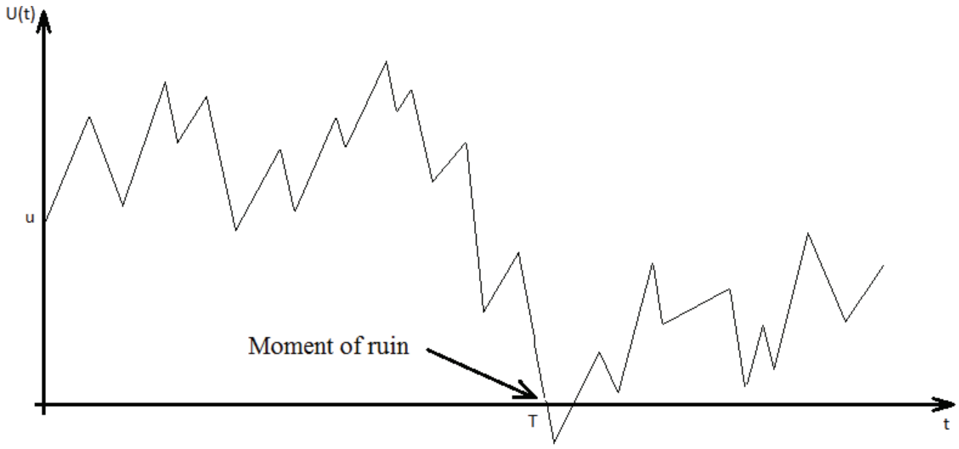

Fig. 1. Ruin

Source: Chrzan [2006, p. 326].

Ruin theory is not a core aspect of this research, that is why author avoided describing it in a more detail and focused only on ML. "ML is principally a methodology to determine the worst case scenario under normal market conditions, without ignoring the correlations among the risk factors" [Studer, 1997, p. 22].

ML, like VaR belongs to quantile-based measures of risk. ML, similar to VaR, is defined:

- over a given significant level (α),
- for some holding period t.

This definition looks similar to the VaR definition. However, there is one important difference: whereas for calculating VAR, the distribution of Profit and Loss (P&L) has to be known, ML is defined directly in the risk factor space Ω [Studer, 1995].

ML could be also defined as [Czernik, 2010]:

$$P\left(S_0 - \inf_{t' \in [0,t]} S(t') < ML(\alpha, t)\right) = 1 - \alpha . \quad (4)$$

Graphical illustration of ML is presented on the Fig. 2.

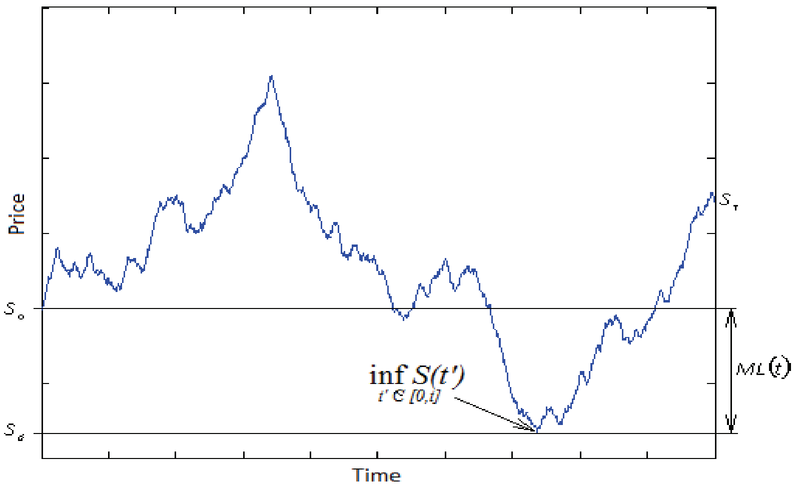

Fig. 2. Maximum loss for exemplary time series

Source: Czernik [2010, p. 38].

3. Fractional Brownian motion

It has been empirically proved that equity price moves are persistent processes [Mandelbrot, 1997; Peters, 1994; Barkoulas, Baum, Shiryaev, 1999; Cajueiro, Barbachan, 2003; Mastalerz-Kodzis, 2003], so author decided to use Fractional Brownian motion (fBm) to model future realizations of the price of an equity.

fBm is a generalization of the Brownian motion. A Gaussian process $B^H = \{B_t^H, t \geq 0\}$ is called fBm if it has zero mean and covariance function given by [Czernik, 2010]:

$$\mathrm{E}(B_t^H B_s^H) = R_H(t,s) = \frac{1}{2}(s^{2H} + t^{2H} - |t-s|^{2H}). \tag{5}$$

Covariance function can be as well described as follows [Czernik, 2010]:

$$\mathrm{E}(B_t^H B_s^H) = R_H(t,s) = \frac{V_H}{2}(s^{2H} + t^{2H} - |t-s|^{2H}), \tag{6}$$

where:

$$V_H = \frac{\Gamma(2-2H)\cos(\pi H)}{\pi H(1-2H)}, \tag{7}$$

$$\mathrm{E}B_t^H = 0. \tag{8}$$

This process was firstly research by A.N. Kolmogorov in *Wienersche Spiralen und einige andere interessante Kurven im Hilbertschen Raum* in the context of modelling turbulences and then studied by Mandelbrot and van Ness in *Fractional Brownian motions, fractional noises and applications*. Fractional Brownian motion is a process, which has the following properties:

- Self-similarity, for any constant $a > 0$, the processes $\{a^{-H} B_{at}^{H}, t \geq 0\}$ and $\{B_t^H, t \geq 0\}$ have the same probability distribution [Nualart, 2006].
- Stationary increments, i.e. $B_H(t+h) - B_H(h) \overset{d}{=} B_H(t) - B_H(0)$ for all $h > 0$ [Choi, 2008].
- Long range dependence [www 1]. Let $\{X(t), t \geq 0\}$ be a self-similarity process with stationary increments with $0 < H < 1$, with $E\left[X(1)^2\right] < \infty$ and define.
- $\xi(n) = X(n+1) - X(n)$.
- $r(n) = \mathrm{E}\left[\xi(0)\xi(n)\right] = \dfrac{1}{2}\left\{(n+1)^{2H} - 2n^{2H} + (n-1)^{2H}\right\}\mathrm{E}\left[X(1)^2\right]$.

  Then for $0.5 < H < 1$:
- $r(n) \underset{n\to\infty}{\sim} H(2H-1)n^{2H-2}\mathrm{E}\left[X(1)^2\right]$

and:

- $\sum_{n=0}^{\infty} |r(n)| = \infty$.

Fractional Brownian motion is not a Markovian process and is not a semimartingale, unless H = 0.5 [Czernik, 2010].

4. Fractional dynamics of the stock prices

Estimation of the VaR under Monte Carlo simulation is almost identical to estimation under historical simulation method with one core exception. In Monte Carlo simulation instead of calculating distribution from historical data, it is needed to conduct simulation of a distribution for available data.

Comparing to historical simulation approach, in Monte Carlo method there are no restriction concern sample size. This method is more flexible than historical simulation approach. It is available to generate complicated distribution and taking into account changes in data over time. As an example, it has been presented a simple construction of Monte Carlo simulation for a financial instrument described by geometric Brownian motion given by:

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{9}$$

where:

- S_t is a price of the instrument at time t,
- μ is the drift,
- σ is a volatility,
- W_t is a Wiener process.

In this paper, author considered a model based on fBm, which is given by [Iskra, 2010]:

$$dS = \mu S dt + \sigma S dB^H, \tag{10}$$

where: B^H is a fractional Brownian motion.

In the literature, there are plenty of the methods, how to simulate fBm, but author in this research used equation proposed by Czernik [2010]:

$$dB_t^H = B_{t+dt}^H - B_t^H = \kappa_H \left\{ \begin{matrix} \int_0^{t+dt} (t+dt-s)^{H-0,5} dB_s - \int_0^t (t-s)^{H-0,5} dB_s + \\ \int_{-\infty}^0 \left[(t+dt-s)^{H-0,5} - (t-s)^{H-0,5} \right] dB_s \end{matrix} \right\}, \tag{11}$$

where:

- increment of the Wiener process are approximated by: $dB \approx \varepsilon\sqrt{\Delta t}$,
- ε is a standard normal distribution random variable.

Author used pathwise integration model [Biagini et al., 2008], in which future realization of price with fBm evolution is given by:

$$S_t(t) = S_0 \cdot e^{\sigma B^{(H)}(t) + \mu(t)}. \tag{12}$$

5. Parameter estimation

Before going through the core part of this thesis, it is needed to estimate the parameters: H, μ and σ for a purpose of simulation future realization of the prices given by fBm. Author used Peters' methodology of estimation Hurst exponent, which looks as follows [1997]:

$$H = \frac{\log(R/S)}{\log(N/2)}, \tag{13}$$

where:
- R / S is rescaled range,
- N number of observations.

And maximum likelihood estimators proposed by Hu, Xia and Zhang [2009], which look as follows:

$$\hat{\mu} = \frac{t'\Gamma_H^{-1}Y}{t'\Gamma_H^{-1}t}, \tag{14}$$

$$\hat{\sigma}^2 = \frac{1}{N}\frac{(Y'\Gamma_H^{-1}Y)(t'\Gamma_H^{-1}t) - (t'\Gamma_H^{-1}Y)^2}{t'\Gamma_H^{-1}t}, \tag{15}$$

where:
- t is a vector of time from 1 to the length of Y,
- Y is a vector of annualized logarithmic rates of return,
- $\Gamma_H = \left[\left[Cov\left[B_{ih}^H, B_{jh}^H\right]\right]\right]_{i,j=1,2,\ldots,N}$.

In the table below there is a list of calculated parameters: μ, σ and H.

Table 1. Drift parameter, sigma parameter and Hurst exponent for equities used in research

Equity/Parameter	Drift	Sigma	Hurst exponent
Apator	0.083206%	50.32%	0.576452
Indykpol	-0.033696%	62.33%	0.602660
ING BŚ	0.013271%	22.05%	0.626872
PKN Orlen	-0.015897%	46.91%	0.562522
Wawel	0.091011%	24.34%	0.664060
Żywiec	0.038622%	34.65%	0.558025

Source: Own elaboration based on [www 2].

The estimated values of the Hurst exponent given in Table 1, show the persistency of the financial equity markets. Returns of the equity prices are perceived to be the random processes if the Hurst exponent is equal to 0.5. Estimated Hurst exponents for all the firms, which were the subject to this research are grater then 0.55. In estimating the parameters given in the Table 1, the time series' length of 7 years has been used in order to grasp the long range dependence. Based on that, it is concluded, that the returns of equity prices are persistent.

6. Risk analysis

In this section, the results of the loss estimation by means of a downturn in the equity prices has been presented. VaR and ML have been computed for the one year period ahead. The time series' length used in the calculation of VaR and

ML according to historical simulation approach is four years. Significant levels, which have been used are: 0.95, 0.975 and 0.99. Risk analysis has been performed in two different periods of time (before the global financial crisis − 2007 and after it − 2012). Results of those computations are presented in Table 2 for the year 2007 and in Table 3 for the year 2012.

Table 2. Absolute and relative Value at Risk and Maximal Loss under Monte Carlo approach, historical simulation approach and real occurred loses in 2007

Quantile	REAL LOSSES		HISTORICAL SIMULATION APPROACH		MONTE CARLO SIMULATION APPROACH	
	Apator					
	Quantile loss (%)[1]	Max Loss (%)	VaR (%)	ML (%)	VaR (%)	ML (%)
5%	14.6	23.7	37.3	99.5	21.7	22.4
2.5%	17.3	23.7	45.8	99.5	25.4	25.9
1%	19.2	23.7	59.6	99.5	29.4	29.7
	Indykpol					
	Quantile loss (%)	Max Loss (%)	VaR (%)	ML (%)	VaR (%)	ML (%)
5%	11.7	15.0	61.7	99.4	31.7	32.5
2.5%	12.2	15.0	74.1	99.4	36.7	37.2
1%	14.5	15.0	81.0	99.4	41.9	42.2
	ING BŚ					
	Quantile loss (%)	Max Loss (%)	VaR (%)	ML (%)	VaR (%)	ML (%)
5%	2.3	4.6	24.9	63.9	14.0	14.4
2.5%	3.3	4.6	33.2	63.9	16.6	16.9
1%	3.7	4.6	41.1	63.9	19.5	19.7
	PKN Orlen					
	Quantile loss (%)	Max Loss (%)	VaR (%)	ML (%)	VaR (%)	ML (%)
5%	3.9	17.4	36.4	67.9	17.6	18.2
2.5%	11.2	17.4	41.8	67.9	20.7	21.1
1%	14.3	17.4	48.8	67.9	24.0	24.3
	Wawel					
	Quantile loss (%)	Max Loss (%)	VaR (%)	ML (%)	VaR (%)	ML (%)
5%	9.5	15.2	37.2	87.3	17.3	17.8
2.5%	11.8	15.2	46.3	87.3	20.6	20.9
1%	13.2	15.2	54.2	87.3	24.1	24.3
	Żywiec					
	Quantile loss (%)	Max Loss (%)	VaR (%)	ML (%)	VaR (%)	ML (%)
5%	2.0	2.5	27.2	63.4	12.4	12.8
2.5%	2.4	2.5	34.5	63.4	14.7	15.0
1%	2.4	2.5	44.7	63.44	17.2	17.4

Source: Own elaboration based on [www 2].

[1] Loss by means of a downturn of an equity price at the respective quantile.

What is common for the risk estimation of the equities used in this paper (measured by VaR or ML) based on historical simulation approach is that this methodology in all cases overestimate the loss due to downturn in equity prices. The overestimation of the losses is mainly seen in the period after the financial crisis, as the data used as an input for the computation is very spiky.

On the other hand, risk measured by the Monte Carlo approach with the fBm features both overestimates and underestimates risk. For example in case of Apator, real losses at 97.5% percentile were 17%, while under the Monte Carlo simulation approach estimated losses at the same confidence interval were equal to 25%. Historical simulation approach estimates VaR at 46%. ML was equal to 24% in real world, while Monte Carlo simulation's ML were 26%. Historical simulation significantly overestimate risk: ML under this method was equal to 99.5% (more information has been presented on the Table 2).

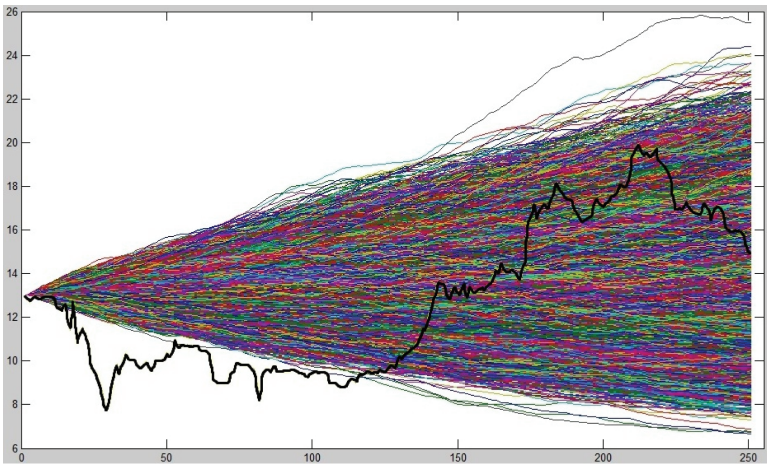

Fig. 3. Monte Carlo simulation and real equity price path (bold black line) for Apator

Source: Own elaboration based on [www 2].

Another example would be PKN Orlen equity. Differences between real losses and losses estimated from Monte Carlo approach are much smaller (14% and 1% for VaR and ML respectively) comparing to differences between real losses and historical simulation approach (34% (VaR) and 61% (ML)). Detailed information is in Table 2.

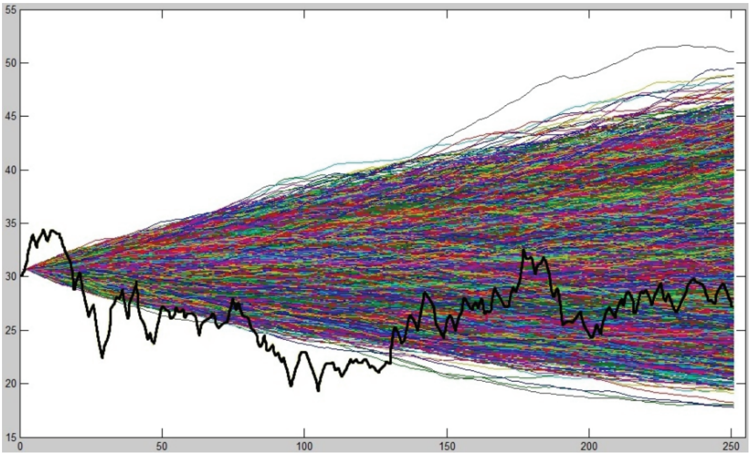

Fig. 4. Monte Carlo simulation and real equity price path (bold black line) for PKN Orlen

Source: Own elaboration based on [www 2].

In the Table 3, there is the same analysis for the same basket of equities, but a starting point of time was changed (after the world financial crisis). Time horizon for which risk have been estimated is year 2012.

Table 3. Absolute and relative Value at Risk and Maximal Loss under Monte Carlo approach, historical simulation approach and real occurred loses in 2012

Quantile	**REAL LOSSES**		**HISTORICAL SIMULATION APPROACH**		**MONTE CARLO SIMULATION APPROACH**	
	Apator					
1	2	3	4	5	6	7
	Quantile loss (%)	**Max Loss (%)**	**VaR (%)**	**ML (%)**	**VaR (%)**	**ML (%)**
5%	-5.3*	0.0	42.5	93.1	22.6	23.2
2.5%	-4.5	0.0	51.5	93.1	26.2	26.7
1%	-1.4	0.0	62.0	93.1	30.1	30.4
	Indykpol					
	Quantile loss (%)	**Max Loss (%)**	**VaR (%)**	**ML (%)**	**VaR (%)**	**ML (%)**
5%	9.6	16.8	56.4	90.6	32.9	33.7
2.5%	12.1	16.8	67.8	90.6	37.8	38.3
1%	15.1	16.8	76.4	90.6	43.2	45.0
	ING BŚ					
	Quantile loss (%)	**Max Loss (%)**	**VaR (%)**	**ML (%)**	**VaR (%)**	**ML (%)**
5%	21.6	25.5	34.5	84.2	14.6	15.0
2.5%	22.6	25.5	46.6	84.2	17.2	17.5
1%	23.7	25.5	55.4	84.2	20.1	20.3

Table 3 cont.

1	2	3	4	5	6	7
	PKN Orlen					
	Quantile loss (%)	**Max Loss (%)**	**VaR (%)**	**ML (%)**	**VaR (%)**	**ML (%)**
5%	27.7	31.7	47.1	85.4	19.6	20.2
2.5%	28.6	31.7	54.5	85.4	22.6	23.0
1%	29.3	31.7	63.3	85.4	25.9	26.1
	Wawel					
	Quantile loss (%)	**Max Loss (%)**	**VaR (%)**	**ML (%)**	**VaR (%)**	**ML (%)**
5%	-2.9	0.0	38.9	87.3	19.6	20.0
2.5%	-1.4	0.0	47.9	87.3	22.7	23.0
1%	-0.6	0.0	59.2	87.3	26.2	26.3
	Żywiec					
	Quantile loss (%)	**Max Loss (%)**	**VaR (%)**	**ML (%)**	**VaR (%)**	**ML (%)**
5%	15.1	18.9	32.7	71.4	13.8	14.2
2.5%	16.2	18.9	41.0	71.4	16.0	16.3
1%	17.0	18.9	50.3	71.4	18.5	18.7

* Negative values mean that during the period, no losses have been experienced.

Source: Own elaboration based on [www 2].

Real losses in 2012 calculated for the 99% percentile and maximal possible losses for ING BŚ equity were 24% and 25.5% respectively. Losses under Monte Carlo simulation estimated VaR and ML to be equal to 20% both (at the 99% significant level). For the same equity and the same significant level, losses under historical simulation approach were estimated at 78% (VaR) and at 84% (ML). This is significant overestimating the risk by the historical simulation approach.

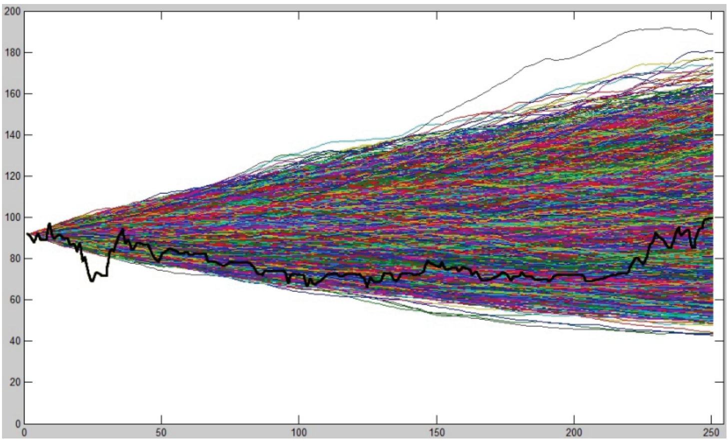

Fig. 5. Monte Carlo simulation and real equity price path (bold black line) for ING BŚ

Source: Own elaboration based on [www 2].

Losses for Żywiec for 99% quantile in 2012 were 17% (VaR) and 19% (ML). Monte Carlo simulation estimated both, VaR and ML at 19%, while historical simulation method highly overestimates risk − 50% and 71% for VaR and ML respectively.

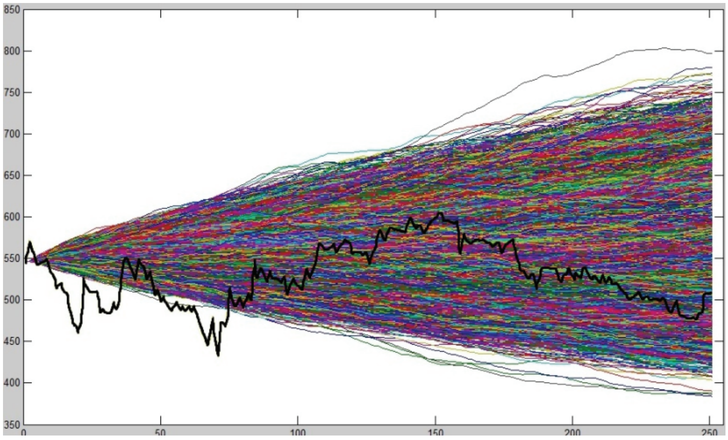

Fig. 6. Monte Carlo simulation and real equity price path (bold black line) for Żywiec

Source: Own elaboration based on [www 2].

Conclusion

This paper shows that the equity prices are persistent processes (*i.e.* all equities used in the paper have Hurst exponent bigger than 0.5). That makes Monte Carlo simulation with fBm evolution of the price a reliable approximation of the future price movements, due to the fact that this process is able to exhibit long range dependence. As well as, the analysis under the Monte Carlo approach is more precise tool in estimating the risk comparing with the historical simulation method − in all investigated equities, historical simulation method gives inaccurate results (*i.e.* risk has been overestimated). The conservatism of the estimates is especially seen in the estimates followed by the period of financial stress.

Monte Carlo approach with a fractional Brownian evolution of the price is time consuming and complex method, but it is worth including in the risk analysis of the equity markets, especially those, which exhibit long range dependence (*i.e.* those, which have been found not to satisfy the Markov property).

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PORÓWNANIE RYZYKA INWESTYCJI W UDZIAŁOWY INSTRUMENT MIERZONEGO ZA POMOCĄ MIARY VALUE AT RISK ORAZ MAKSYMALNA STRATA ZGODNIE Z METODĄ MONTE CARLO, GDZIE EWOLUCJA CENY JEST DANA UŁAMKOWYM RUCHEM BROWNA ORAZ SYMULACJĄ HISTORYCZNĄ

Streszczenie: W niniejszym artykule autor dokonuje analizy ryzyka rynkowego akcji giełdowych sześciu spółek z Warszawskiej Giełdy Papierów Wartościowych. Dla celów analizy zostały wybrane dwie kwantylowe miary ryzyka: wartość zagrożona ryzykiem (ang. Value at Risk, VaR) oraz maksymalna strata (ang. Maximal Loss). Analizę przeprowadzono na podstawie metody Monte Carlo oraz symulacji historycznej. Jednakże w metodzie Monte Carlo przyszłe wartości cen są dane ułamkowym ruchem Browna, a nie – jak podpowiada praktyka rynkowa – geometrycznym ruchem Browna.

Słowa kluczowe: ułamkowy ruch Browna, symulacja Monte Carlo, wartość zagrożona ryzykiem (ang. Value at Risk, VaR), maksymalna strata (ang. Maximal Loss), eksponent Hursta.