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DO MULTI-FACTOR MODELS PRODUCE ROBUST RESULTS? ECONOMETRIC AND DIAGNOSTIC ISSUES IN EQUITY RISK PREMIA STUDY[1]

Summary: In recent decades numerous studies verified empirical validity of the CAPM model. Many of them showed that CAPM alone is not able to explain cross-sectional variation of stock returns. Researchers revealed various risk factors which explained outperformance of given groups of stocks or proposed modifications to existing multi-factor models. Surprisingly, we hardly find any discussion in financial literature about potential drawbacks of applying standard OLS method to estimate parameters of such models. Yet, the question of robustness of OLS results to invalid assumptions shouldn't be ignored. This article aims to address diagnostic and econometric issues which can influence results of a time-series multi-factor model. Based on the preliminary results of a five-factor model for 81 emerging and developed equity indices [Sakowski, Ślepaczuk and Wywiał, 2016a] obtained with OLS we check the robustness of these results to popular violations of OLS assumptions. We find autocorrelation of error term, heteroscedasticity and ARCH effects for most of 81 regressions and apply an AR-GARCH model using MLE to remove them. We also identify outliers and diagnose collinearity problems. Additionally, we apply GMM to avoid strong assumption of

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IID error term. Finally, we present comparison of parameters estimates and Rsquared values obtained by three different methods of estimation: OLS, MLE and GMM. We find that results do not differ substantially between these three methods and allow to draw the same conclusions from the investigated five-factor model.

Keywords: multi-factor models, asset pricing models, equity risk premia, OLS, MLE, GMM, autocorrelation, heteroscedasticity, outliers, collinearity, normality, econometric diagnostics.

JEL Classification: C15, G11, F30, G12, G13, G14, G15.

Introduction

Studies which focus on Capital Asset Pricing Model (CAPM) and try to explain equity risk premium for single stocks are quite numerous in financial literature. They start from seminal papers of Sharpe [1964], Lintner [1965] and Black, Jensen and Scholes [1972]. Consequently, next studies developed multi-factor models by introducing new risk factors, changing functional form of the model or adding new variables that define states of the market. Examples of such models, among others, are three-factor model of Fama and French [1992], four-factor model of Carhart [1997] or various multi-factor models of Rahim and Noor [2006], Chen, Novy-Marx and Zhang [2011], Fama and French [2012, 2015], Frazzini and Pedersen [2014].

Parameters in these models are typically estimated using standard OLS methods. This enforces the researcher to apply a set of strong assumptions which rarely are true. Many studies show that residuals from time-series models applied to excess returns are typically autocorrelated and heteroskedastic. For weekly and daily observations they are fat-tailed and often characterized by numerous outliers. Adding new factors to the model increases probability of negative multicollinearity effects. As the result, OLS estimators become typically inefficient which affects probability of type I and II errors and makes statistical inference incorrect.

Although properties of General Methods of Moments (GMM) estimators and their advantages over OLS methods are widely discussed (see for example Hansen [1982], Newey and McFadden [1994], Hansen, Heaton and Yaron [1996], Hansen [2002], Alastair [2005], Kirby [2006]), yet, surprisingly, the question of consequences of applying standard OLS method in multi-factor models for excess returns is rarely discussed in the financial literature.

The main objective of this study is to verify robustness of the OLS results presented in Sakowski, Ślepaczuk and Wywiał [2016a] with alternative MLE and GMM techniques which allow to avoid or ignore strong assumptions behind OLS method.

The structure of the paper is as follows. Section 2 describes the five-factor model and empirical data applied in the study. Section 3 presents methodological issues and reviews effects that show up when typical OLS assumptions are not valid. Section 4 presents model diagnostics and necessary steps to remove problems with problematic assumptions. In Section 5 we compare results obtained with OLS, MLE and GMM methods. The last section concludes.

2. The model and empirical data

2.1. The Model

Our analysis is based on five-factor time-series model, which is a modification of the four-factor model by Carhart [1997]. Parameters are estimated using OLS method. The functional form of our model is described by:

$$(R_i - Rf)_t = \alpha_i + \beta_{MKT,i}(Rm - Rf)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML:,i}WML_t + \beta_{VMC:,i}VMC_t + \ + \varepsilon_{i,t} \tag{1}$$

where:

$(R_i - R_f)$ is weekly return of equity index in excess to weekly risk free rate,
$(R_m - R_f)$ is equally weighted equity index less than risk free rate,
HML is the weekly premium on the book-to-market factor (high minus low),
SMB is the weekly premium on the size factor (small minus big),
WML is the weekly premium on winners-minus-losers factor (winners minus loosers),
VMC is the weekly premium on volatile minus calm (*VMC*) equity indices (volatile minus calm).

The first four factors are created following Carhart [1997] approach.

The $(R_m - R_f)$ factor represents weekly excess return of the market portfolio over the risk-free rate. The market portfolio consists of equally weighted all 81 equity indices.

The HML is a zero-investment portfolio that is long on the highest decile group of book-to-market (B/M) equity indices and short on the lowest decile group. The difference of returns of these extreme decile groups is calculated in each weekly interval, which finally constitutes HML factor.

The SMB is a zero-investment portfolio that is long on the highest decile group of small capitalization equity indices and short on the lowest decile group. The difference of returns of these extreme decile groups is calculated in weekly interval as well.

The WML is a zero-investment portfolio that is long on the highest decile group of previous 1-year return winner equity indices and short on its lowest decile group (loser equity indices). The difference of returns of these extreme decile groups is calculated again for each weekly interval.

Finally, the VMC factor, is the weekly premium on volatile minus calm equity indices and is obtained by subtracting the equal weighted average return of the lowest volatility equity indices from the equal weighted average return of the highest volatility equity indices (again, a zero-investment portfolio). The difference of returns of these extreme decile groups is calculated for each weekly interval. The definition of high or low volatility is based on 63 days realized volatility calculated separately for each equity index.

The discussion about possible extensions of this model, as well as analysis of dynamics of risk factors is presented in Sakowski, Ślepaczuk and Wywiał [2016a].

2.2. Empirical Data

Instead of using single stocks (eg. Fama and French [1992, 1993, 2012], Frazzini and Pedersen [2014], among others) we deploy our model to a wide broad set of equity indices covering the period 2000-2015. The analysis was performed on weekly data for 81 most representative and investable equity indices, from all continents[2]. We include data for 27 developed and 54 emerging markets indices. The data was obtained from Bloomberg. The detailed list of all equity indices and their descriptive statistics are presented in the Online Appendix[3].

The reason behind selection of weekly instead of monthly data was the intention to evaluate theoretical value of excess returns for the given equity index more frequently. Returns and risk factors were calculated after converting local prices to USD. Risk-free rate has been approximated by three-month LIBOR.

[2] For practical purposes we used only these indices which can be easily invested through options, futures or ETFs.

[3] The Online Appendix is available on: http://coin.wne.uw.edu.pl/qfrg/third-2016-appendix

3. Methodological issues. What can go wrong?

In our study, we estimate the five-factor model separately for 81 indices. Dealing with such a number of regressions encounters a question of practical nature:

1. Should we estimate all models with the same functional form and compare their results across all markets ignoring any diagnostic issues, as it is presented in financial literature for years?
2. Or rather should we perform all diagnostics for each model separately and puzzle out possible problems, which will most probably result in different model functional forms across investigated markets and hence make it difficult to compare results for them?

Taking into account that we do intend to compare parameter estimates and Rsquared coefficients across equity indices, the first approach seems to be more adequate and would allow us to analyze explaining power of models estimated for different markets. On the other hand, the process of polishing every regression to detect and resolve diagnostic problems is definitely a challenging, time-consuming and daunting task. This problem seems to be even more important in the process of performing rolling regressions, for example for investment strategy purposes. We address this issues in Sakowski, Ślepaczuk and Wywiał [2016b]. Hence, we are interested in finding an answer to the question of how risky is performing OLS estimations in time-series multi-factor model in equity risk premium studies and how robust are results from such regression if OLS assumptions are clearly violated.

In the process of estimation of multi-factor models using time-series data, we can potentially suffer from various econometric problems. The OLS assumes that error term is identically and independently distributed (IID). Regression results can be also affected if we observe strong collinearity among independent variables. For inference purposes we additionally have to assume that the error term is normally distributed. Also, estimates and theoretical values will be changed if sample contains outlying and influential observations.

The consequences of non-fulfilling these assumptions are widely known.

Autocorrelation and heteroskedasticity Autocorrelation or heteroskedasticity is very likely to be present in time-series multi-factor models, because of nature of stocks returns. If that is the case, the Gauss Markov theorem is not valid and OLS doesn't provide Best Linear Unbiased Estimators (BLUE). Although this doesn't introduce bias, the standard errors tend to be underestimated and hence the t-statistics overestimated.

One way to solve the problem of autocorrelation and heteroskedasticity is to deploy the AR-GARCH model which is estimated using Maximum Likelihood Estimation. The autoregressive part of the conditional mean equation will take into account autocorrelation, while time-varying variance can be addressed via conditional variance equation. An alternative solution to avoid assumptions about IID error term is to apply Generalized Methods of Moments (GMM) proposed by Hansen [1982]. GMM allows error term to be heteroskedastic and serially correlated and it does not require information about the exact distribution of the disturbances.

We will apply this two approaches in Section 4 and 5 and compare their results with those obtained using OLS method.

Multicollinearity Even in case of extreme (but still not perfect) multicollinearity OLS estimates are still unbiased, BLUE and consistent. Nevertheless, greater multicollinearity implies greater standard errors. When high multicollinearity is present, confidence intervals for coefficients tend to be wide and t-statistics tend to be very small. As a result, the null hypothesis will be harder to get rejected (higher probability of type II error), as the coefficients will have to be larger in order to be statistically significant.

Moreover, when two regressors are highly and positively correlated, their slope coefficient estimators tends to be highly and negatively correlated. As a result, if we overestimate the effect of one parameter, most likely we also underestimate the effect of the other. Because of increased variance, coefficient estimates tend to be very unstable across different samples and they are very sensitive to even small changes in the model. All that means that statistical power of the analysis is harmed and the process of specifying the correct model is difficult.

Hence, appropriate and careful identifying possible problems with multicollinearity is an important part of multi-factor model diagnostics. We address this question in Section 4.2.

Influential observations Regression results can be also affected by presence of outliers and influential observations. The decision whether such cases should be excluded from the sample or not is not obvious and depends on particular problem and particular data. Anyway, identifying such observations is important to take the right choice and determine the degree of influence. We focus on that issue in Section 4.3

Non-normality Distributions of equity returns are almost always characterized by fat tails. In multi-factor models, this will very likely result in leptokurtic distributions of residuals. Although normality is not necessary in OLS to maintain BLUE estimators, it still affects distributions of t-statistics and makes statistical inference very difficult. We assess non-normality of residuals in our models in Section 4.4.

Non-linear model functional form Another problem with estimating multi-factor models may arise when the data resembles clearly nonlinear patterns or relationships. Applying linear functional form of the model to the nonlinear data results in autocorrelation of residuals and may produce spurious results. We carefully address this question in Section 4.5.

4. Model diagnostics

To diagnose possible problems with our model we estimate its parameters using standard OLS approach and then investigate properties of residuals. We concentrate on detecting autocorrelation, heteroscedasticity and ARCH effects, possible collinearity among risk factors, indentifying influential observations, testing normality of residuals and finally verifying validity of linear relationship among indices excess returns and risk factors.

4.1. Autocorrelation of residuals, heteroscedasticity and ARCH effects

First, we apply Ljung-Box test to identify autocorrelation of standardized residuals and squared standardized residuals obtained using OLS. We report results in Table 1. We calculate LB statistics up to 1st, 5th and 9th autocorrelation coefficient. We can observe that autocorrelation (up to 9th lag) exists in almost 50% of models.

Additionally, we observe strong autocorrelation among squared standardized residuals. This indicates that we have strong ARCH effects (heteroscedasticity of residuals, volatility clustering effects) in residuals for all 81 regressions.

In order to remove autocorrelation and address the problem of heteroscedasticity we estimated five-factor AR(5)-GARCH(1,1) model. Orders of this model have been chosen arbitrarily. This model is estimated using Maximum Likelihood method. Ljung-Box test for autocorrelation of standardized residuals and squared standardized residuals are reported in Table 2. P-values from this table shows that autocorrelation and heteroscedasticity problem was addressed in almost all cases.

Table 1. Ljung-Box test for autocorrelation of excess returns and squared excess returns

	Returns p-value for LB up to lag			Squared returns p-value for LB up to lag		
Index	1	5	9	1	5	9
1	*2*	*3*	*4*	*5*	*6*	*7*
AEX	0.5525	0.7014	0.5710	**0.0000**	**0.0000**	**0.0000**
MERVAL	0.3615	**0.0006**	**0.0009**	**0.0004**	**0.0000**	**0.0000**
AS51	0.4996	0.3611	0.1680	**0.0000**	**0.0000**	**0.0000**
ATX	0.6346	0.4850	0.3694	**0.0000**	**0.0000**	**0.0000**
BHSEASI	**0.0343**	**0.0000**	**0.0000**	0.5990	**0.0000**	**0.0000**
BEL20	0.6010	0.9243	0.7755	**0.0000**	**0.0000**	**0.0000**
IBOV	**0.0035**	**0.0029**	**0.0010**	**0.0000**	**0.0000**	**0.0000**
SOFIX	**0.0012**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
SPTSX	**0.0455**	0.0708	**0.0364**	**0.0000**	**0.0000**	**0.0000**
IPSA	0.1230	0.0656	**0.0334**	**0.0000**	**0.0000**	**0.0000**
XIN9I	0.5080	**0.0429**	**0.0387**	**0.0053**	**0.0000**	**0.0000**
COLCAP	0.6961	**0.0002**	**0.0002**	**0.0084**	**0.0000**	**0.0000**
CYSMMAPA	0.2956	0.2005	0.0509	**0.0000**	**0.0000**	**0.0000**
CTXEUR	**0.0319**	**0.0005**	**0.0003**	**0.0000**	**0.0000**	**0.0000**
KFX	0.0617	**0.0288**	**0.0372**	**0.0000**	**0.0000**	**0.0000**
EGX30	0.1113	**0.0462**	**0.0190**	**0.0028**	**0.0000**	**0.0000**
TALSE	**0.0000**	**0.0000**	**0.0000**	**0.0059**	**0.0000**	**0.0000**
SX5E	0.4124	0.6826	0.5572	**0.0000**	**0.0000**	**0.0000**
HEX25	0.3747	0.6731	0.4197	**0.0000**	**0.0000**	**0.0000**
CAC	0.3672	0.7756	0.6453	**0.0000**	**0.0000**	**0.0000**
DAX	0.7891	0.3251	0.2752	**0.0000**	**0.0000**	**0.0000**
GGSECI	**0.0000**	**0.0000**	**0.0000**	0.1717	0.5871	0.6147
FTASE	0.1455	0.3365	0.4575	**0.0000**	**0.0000**	**0.0000**
HSI	0.4227	0.4824	0.5738	**0.0000**	**0.0000**	**0.0000**
M1HU	0.3894	0.2227	0.2786	**0.0000**	**0.0000**	**0.0000**
BUX	0.3092	0.2210	0.2465	**0.0000**	**0.0000**	**0.0000**
ICEXI	0.4759	**0.0000**	**0.0000**	0.5392	**0.0000**	**0.0000**
NIFTY	**0.0298**	**0.0054**	**0.0051**	**0.0000**	**0.0000**	**0.0000**
LQ45	0.9172	**0.0145**	**0.0033**	**0.0018**	**0.0000**	**0.0000**
ISXGI	**0.0006**	**0.0004**	**0.0001**	0.6917	0.5339	0.1658
ISEQ20P	0.3624	0.1874	0.0802	0.0971	**0.0000**	**0.0000**
TA.25	0.5416	**0.0146**	**0.0117**	**0.0000**	**0.0000**	**0.0000**
FTSEMIB	0.6984	0.8977	0.8009	**0.0000**	**0.0000**	**0.0000**
NKY	0.2464	0.2950	0.5002	**0.0038**	**0.0002**	**0.0000**
FNKEN2	0.1324	0.2733	**0.0321**	0.8021	**0.0109**	**0.0010**
KSX15	0.0564	0.1771	0.2683	**0.0000**	**0.0000**	**0.0000**
LSXC	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
RIGSE	**0.0358**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
VILSE	**0.0000**	**0.0000**	**0.0000**	**0.0053**	**0.0000**	**0.0000**
LUXXX	0.0924	**0.0033**	**0.0022**	**0.0000**	**0.0000**	**0.0000**
MBI	**0.0005**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
FBMKLCI	**0.0454**	**0.0198**	**0.0252**	**0.0281**	**0.0074**	**0.0058**
MALTEX	0.5539	**0.0292**	**0.0060**	**0.0000**	**0.0000**	**0.0000**
SEMDEX	0.5527	**0.0002**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Table 1 cont.

1	*2*	*3*	*4*	*5*	*6*	*7*
MEXBOL	0.0951	0.1407	0.0898	**0.0000**	**0.0000**	**0.0000**
MSETOP	**0.0000**	**0.0000**	**0.0000**	**0.0032**	**0.0000**	**0.0000**
MONEX20	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
MCSINDEX	0.0783	0.2878	0.5097	**0.0000**	**0.0000**	**0.0000**
FTN098	0.0768	0.1088	0.2102	**0.0000**	**0.0000**	**0.0000**
NZSE50FG	0.3520	0.6622	0.6207	**0.0001**	**0.0000**	**0.0000**
NGSEINDX	**0.0490**	**0.0042**	**0.0001**	**0.0000**	**0.0000**	**0.0000**
OBX	0.6942	0.5432	0.4553	**0.0000**	**0.0000**	**0.0000**
MSM30	0.0758	**0.0352**	**0.0004**	**0.0000**	**0.0000**	**0.0000**
KSE100	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
PCOMP	0.8228	0.0656	0.1111	0.0524	0.0756	**0.0153**
WIG20	0.7529	0.5561	0.6744	**0.0000**	**0.0000**	**0.0000**
PSI20	0.0802	0.0723	0.0961	**0.0000**	**0.0000**	**0.0000**
DSM	0.1172	0.0512	0.0716	**0.0000**	**0.0000**	**0.0000**
BET	**0.0300**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
RTSI	0.4130	0.6024	0.5886	**0.0000**	**0.0000**	**0.0000**
SASEIDX	0.6770	**0.0320**	**0.0242**	**0.0000**	**0.0000**	**0.0000**
BELEX15	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
MXSG	0.2355	0.2222	0.3223	**0.0000**	**0.0000**	**0.0000**
SKSM	0.5025	**0.0211**	**0.0266**	0.2111	**0.0112**	**0.0110**
SBITOP	0.6521	**0.0009**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
TOP40	**0.0106**	**0.0068**	**0.0128**	**0.0000**	**0.0000**	**0.0000**
KOSPI2	0.0913	0.3803	0.6704	**0.0000**	**0.0000**	**0.0000**
IBEX	0.3670	0.8491	0.7934	**0.0000**	**0.0000**	**0.0000**
CSEALL	**0.0000**	**0.0000**	**0.0000**	0.1878	**0.0132**	**0.0000**
OMX	0.2299	0.2872	0.0540	**0.0000**	**0.0000**	**0.0000**
SMI	**0.0005**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
TAMSCI	0.6236	0.3626	0.4314	0.1609	**0.0000**	**0.0000**
DARSTSI	**0.0216**	**0.0000**	**0.0000**	0.3907	**0.0020**	**0.0023**
SET50	0.8223	**0.0012**	**0.0007**	**0.0031**	**0.0000**	**0.0000**
TUSI20	0.0854	0.2206	0.0620	0.4386	**0.0214**	**0.0000**
XU030	0.0636	0.1079	0.0783	**0.0000**	**0.0000**	**0.0000**
UKX	**0.0241**	**0.0429**	**0.0226**	**0.0000**	**0.0000**	**0.0000**
PFTS	**0.0000**	**0.0000**	**0.0000**	**0.0006**	**0.0000**	**0.0000**
DUAE	0.2652	0.2269	0.0777	0.0891	**0.0000**	**0.0000**
SPX	**0.0360**	**0.0098**	**0.0062**	**0.0000**	**0.0000**	**0.0000**
VNINDEX	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Table presents Ljung-Box statistics for autocorrelation of standardized returns and squared standardized returns up to 1st, 5th and 9th lag. Null hypothesis denotes no autocorrelation. Weekly data for 81 emerging and developed indices. P-values lower than 0.05 were denoted with bold font.

Source: Own calculations.

Table 2. Ljung-Box test for autocorrelation of standardized residuals and squared standardized residuals for five-factor AR(5)-GARCH(1,1) model

	Std residuals p-value for LB up to lag			Squared std residuals p-value for LB up to lag		
Index	1	5	9	1	5	9
1	*2*	*3*	*4*	*5*	*6*	*7*
AEX	0.4354	0.9620	0.9438	0.4317	0.3253	0.3494
MERVAL	0.7723	0.7383	0.8840	0.7047	**0.0137**	**0.0396**
AS51	0.9499	0.9269	0.8281	0.7591	0.9996	0.9925
ATX	0.9403	0.8849	0.7691	0.4393	0.3076	0.4569
BHSEASI	0.9216	**0.0185**	**0.0315**	0.8656	0.8805	0.8875
BEL20	0.2961	0.6445	0.6523	0.6133	0.5275	0.6285
IBOV	0.8972	0.9367	0.9786	0.5752	0.2703	0.1708
SOFIX	0.9977	0.2929	0.2364	0.3616	0.9439	0.9443
SPTSX	0.5877	0.1481	0.2211	0.9965	0.9976	0.9994
IPSA	0.6219	0.9323	0.9672	0.6932	0.1169	0.0654
XIN9I	0.2035	**0.0020**	**0.0110**	0.4430	0.8058	0.8127
COLCAP	0.3109	**0.0059**	**0.0226**	0.7133	0.1928	0.2407
CYSMMAPA	0.9438	1.0000	0.8419	0.3295	0.4082	0.6220
CTXEUR	0.8969	0.8379	0.5950	0.8802	0.9421	0.7642
KFX	0.7611	0.2024	0.3552	0.4148	0.8719	0.8209
EGX30	0.9863	0.8213	0.2406	**0.0384**	**0.0118**	**0.0162**
TALSE	0.8163	0.6205	0.5037	0.3834	0.4170	0.4221
SX5E	0.7335	0.9473	0.8663	0.9157	0.9989	0.9992
HEX25	0.5481	0.8262	0.6829	0.5834	0.9030	0.6254
CAC	0.9288	0.9956	0.8562	0.6352	0.9837	0.9859
DAX	0.4991	0.8203	0.9109	0.6383	0.9489	0.8760
GGSECI	0.3561	**0.0041**	**0.0008**	0.0764	0.3443	0.6592
FTASE	0.8090	0.9386	0.8990	0.7225	0.9952	0.9988
HSI	0.8175	0.8166	0.7856	0.0865	0.2008	0.3803
M1HU	0.9545	0.4587	0.4391	0.4682	0.9454	0.9010
BUX	0.8256	**0.0110**	**0.0204**	0.7340	0.9734	0.9813
ICEXI	0.5702	**0.0108**	**0.0017**	0.4921	**0.0000**	**0.0000**
NIFTY	0.9807	0.4943	0.1501	0.3460	0.9317	0.9197
LQ45	0.8681	0.9232	0.8688	**0.0293**	0.0512	0.1381
ISXGI	0.1193	0.3880	0.4866	0.6659	0.9850	0.9928
ISEQ20P	0.9639	0.4046	0.4965	0.7220	0.2164	0.1864
TA.25	0.6666	0.9776	0.5426	0.0958	0.4769	0.7432
FTSEMIB	0.7839	0.9649	0.5807	0.9318	0.5737	0.7593
NKY	0.5518	0.5879	0.6151	0.7476	0.2550	0.4046
FNKEN2	0.9893	0.9822	0.2700	0.4898	0.7504	0.7698
KSX15	0.4340	0.8488	0.8061	0.3814	0.7258	0.7633
LSXC	0.0718	0.2492	0.4570	0.3636	0.9434	0.9910
RIGSE	0.6956	0.5326	0.2113	0.0519	0.2208	0.5228
VILSE	0.3248	**0.0000**	**0.0000**	0.6507	0.9758	0.9854
LUXXX	0.2207	0.2493	**0.0254**	0.7213	0.9742	0.9908
MBI	0.9769	**0.0425**	**0.0337**	**0.0402**	**0.0001**	**0.0004**
FBMKLCI	0.3434	0.6717	0.8523	0.9581	0.9963	0.9994
MALTEX	0.4191	0.0996	0.0800	0.9249	0.6221	0.8045

Table 2 cont.

1	*2*	*3*	*4*	*5*	*6*	*7*
SEMDEX	0.5964	0.1595	0.0808	0.3781	0.9621	0.8962
MEXBOL	0.9791	0.7811	0.5479	0.6350	0.9181	0.9070
MSETOP	0.1100	**0.0001**	**0.0000**	0.8040	0.9972	0.9969
MONEX20	0.5221	**0.0006**	**0.0004**	**0.0246**	**0.0257**	0.1117
MCSINDEX	0.4545	0.7381	0.7521	0.9183	0.9938	0.9953
FTN098	0.7818	0.5813	0.6721	0.5515	0.9487	0.8962
NZSE50FG	0.6773	0.9946	0.9950	0.7488	0.9701	0.9330
NGSEINDX	0.9187	0.2274	0.2043	0.2736	0.3955	0.2596
OBX	0.9961	0.7132	0.5348	0.6179	0.9159	0.9084
MSM30	0.2813	0.7150	0.4624	0.8498	0.9990	0.9859
KSE100	0.3872	0.1207	0.1893	0.7960	0.8933	0.9271
PCOMP	0.6326	0.7177	0.6918	0.4068	0.4676	0.3865
WIG20	0.6399	0.2192	0.1786	0.1446	0.5308	0.4051
PSI20	0.8498	0.9798	0.8967	**0.0369**	0.1197	0.3184
DSM	0.2082	0.1357	0.1575	0.7320	0.4714	0.4761
BET	0.3528	0.8453	0.7798	0.7876	0.6378	0.8210
RTSI	0.7874	0.8319	0.9266	0.2207	**0.0119**	**0.0158**
SASEIDX	0.2354	**0.0216**	**0.0438**	0.6707	0.9346	0.9683
BELEX15	0.2593	**0.0097**	**0.0128**	0.3555	0.8664	0.9271
MXSG	0.7991	0.0690	0.2270	0.2176	0.7579	0.6343
SKSM	0.5245	0.2066	0.2619	0.4677	0.8847	0.9406
SBITOP	0.9395	0.3572	0.3775	0.5697	0.9343	0.9746
TOP40	0.7271	0.8809	0.9318	0.8407	0.1141	0.1481
KOSPI2	0.6847	0.6578	0.6882	0.2857	0.6073	0.7721
IBEX	0.8188	0.9474	0.8358	0.5062	0.2622	0.4605
CSEALL	0.9747	0.9300	0.9346	0.9979	0.9774	**0.0014**
OMX	0.9694	0.5255	0.4758	0.8281	0.9044	0.9442
SMI	0.9059	0.9079	0.8906	0.9246	0.9977	0.9997
TAMSCI	0.9525	0.1757	0.2502	0.1434	0.1499	0.3298
DARSTSI	0.1376	**0.0000**	**0.0000**	0.8912	0.6311	0.8637
SET50	0.7929	**0.0455**	**0.0495**	0.4876	0.7595	0.8689
TUSI20	0.7019	0.3490	0.5152	0.7323	0.9971	0.9690
XU030	0.9136	0.4949	0.1462	0.5650	0.3508	0.5514
UKX	0.9895	0.1993	0.2069	0.5709	0.7543	0.8677
PFTS	0.8116	**0.0153**	**0.0051**	**0.0035**	**0.0022**	**0.0279**
DUAE	0.6013	0.9730	0.9770	0.9596	0.1243	0.2335
SPX	0.8115	0.9908	0.9899	0.3532	0.5658	0.3436
VNINDEX	0.4733	**0.0073**	**0.0069**	0.3302	0.8030	0.7458

Table presents Ljung-Box statistics for autocorrelation of standardized residuals and squared standardized residuals from five-factor AR(5)-GARCH(1,1) model up to 1st, 5th and 9th lag. Null hypothesis denotes no autocorrelation. P-values lower than 0.05 were denoted with bold font.

Source: Own calculations.

However, we have to note that in such approach applying only one particular order (5 lags) of the autoregressive model we end up either having to many lags for regressions where autocorrelation problem doesn't exist (e.g. the AEX index from Table 1) or having not enough lags for regressions where we observe autocorrelation up to level higher than 5 (e.g. the VILSE index from Table 2. The problem here is that to do this correctly, i.e. to add the correct number of lags to each regression in order to exclude the problem of autocorrelation, we would have to investigate autocorrelation functions for residuals for each of these regressions individually. This would even more challenging in case of estimating rolling regressions for investment purposes, when appropriate number of lags should be determined for every week for separate 81 regressions.

As discussed in Section 3, an alternative way to avoid unrealistic assumptions that error term is IID, is to apply Generalized Method of Moments (GMM). We follow this approach and compare GMM estimates in Section 5.

4.2. Collinearity

To identify possible collinearity we utilize two methods. First, we look at the scatter plots of five factors (Figure 1). Among ten pairs there, only in case of VMC versus R_m-R_f we observe a moderate correlation of 68%.

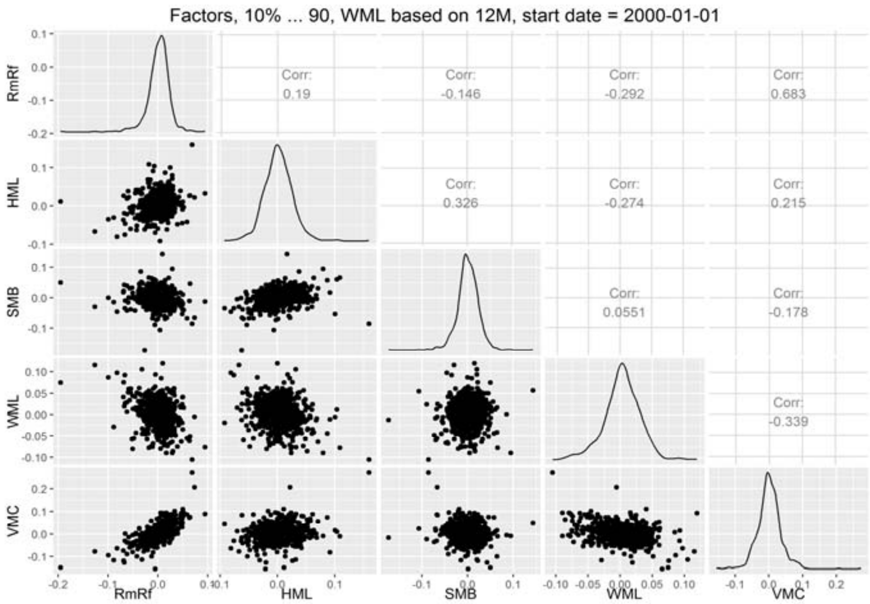

Fig. 1. Scatter plots for five factors used for all 81 regressions

This scatter plots present valus (returns) of five factors in the period from 2000 to 2015. Factors were build on top and bottom decile groups of appropriate criterions. WML returns are based on last 12 months. 794 observations used.

Source: Own calculations.

The second approach is to utilize the popular VIF (variance inflation factor) measure. In Table 3 we present VIF values for separate regressions. The rule of thumb says that multicollinearity is present if VIF is larger than 10. In our case for all regressions values are smaller than 5 which doesn't imply problems.

Table 3. VIF values for single regressions

Index	n	RmRf	HML	SMB	WML	VMC
KSX15	149	2.349	1.474	1.448	1.541	2.944
FNKEN2	204	3.188	1.614	1.594	1.965	4.126
LSXC	219	2.840	1.757	1.663	1.909	3.565
GGSECI	221	2.858	1.826	1.682	1.975	3.610
ISXGI	290	2.714	1.961	1.689	1.548	3.415
TUSI20	430	2.377	1.688	1.476	1.458	2.757
DARSTSI	434	2.381	1.677	1.473	1.456	2.750
DUAE	458	2.365	1.632	1.450	1.430	2.742
BELEX15	494	2.257	1.481	1.386	1.333	2.573
ISEQ20P	524	2.211	1.450	1.345	1.315	2.469
MBI	534	2.198	1.428	1.301	1.309	2.426
CYSMMAPA	551	2.148	1.415	1.276	1.311	2.396
BHSEASI	559	2.140	1.409	1.271	1.299	2.374
FTN098	588	2.087	1.365	1.240	1.263	2.284
NGSEINDX	597	2.074	1.357	1.240	1.252	2.267
XIN9I	609	2.060	1.347	1.235	1.245	2.264
SBITOP	625	2.040	1.336	1.220	1.247	2.230
MONEX20	629	2.039	1.337	1.224	1.245	2.232
COLCAP	662	2.023	1.336	1.255	1.238	2.223
M1HU	742	1.934	1.293	1.244	1.232	2.089
NZSE50FG	742	1.934	1.293	1.244	1.232	2.089
SOFIX	752	1.938	1.312	1.257	1.236	2.091
VNINDEX	765	1.912	1.322	1.257	1.236	2.068
all other	794	1.900	1.299	1.218	1.201	2.008

VIF coefficients for five risk factors calculated in USD in the period of 2000-2015. Group “all other” consists of: AEX, MERVAL, AS51, ATX, BEL20, IBOV, SPTSX, IPSA, CTXEUR, KFX, EGX30, TALSE, SX5E, EX25, CAC, DAX, FTASE, HSI, BUX, ICEXI, NIFTY, LQ45, TA.25, FTSEMIB, NKY, RIGSE, VILSE, LUXXX, FBMKLCI, MALTEX, SEMDEX, MEXBOL, MSETOP, MCSINDEX, OBX, MSM30, KSE100, PCOMP, WIG20, PSI20, DSM, BET, RTSI, SASEIDX, MXSG, SKSM, TOP40, KOSPI2, IBEX, CSEALL, OMX, SMI, TAMSCI, SET50, XU030, UKX, PFTS and SPX.

Source: Own calculations.

We also checked how exclusion of collinear variables affects estimates of parameters and their standard errors. It has occurred that the difference between two estimates were negligible.

4.3. Influential observations

The next part is focused on identifying influential observations. The first step involves analysis of scatter plots of excess returns of a given index (the dependent variable) versus all five factors. We present them in the Online Appendix[4]. A visual inspection of these scatter plots shows that there are only a few observations which might be considered as influential.

In the second step we utilize several popular measures which help identify influential observations. We examine leverage, studentized residuals, Cook's distance and DFBeta measures. Table 4 reports percentage of observations (for leverage and studentized residuals) or number of observations (for Cook's D and DFBetas) which exceed the assumed threshold.

Leverage measure points to those observations where we observe extreme or outlying values of the independent variables (i.e. risk-factors). Following Hoaglin and Welsch [1978] we recognize the observation as high-leverage if $h_{ii} > 2p/n$, where p is the number of parameters to estimate, n is the number of observations and h_{ii} is the diagonal element of the hat matrix, $H=X(X'X)^{-1}X'$. We observe that 3%-4% of observations can be regarded as leverage points, and hence potentially influential.

The threshold for studentized residuals is set to 0.5% and 99.5% quantile, so that we expect about 1% of observations to exceed it. Indeed, we observe that the percentage ranges from 1% to 2% which could be addressed to moderate leptokurtosis of the residuals.

Table 4. Influence ratios

	%	%	Count	Count	Mount	Count	Count	Count	Count
Index	Leverage	RStudent	Cook's D	DFBetas Intercept	DFBetas RmRf	DFBetas HML	DFBetas SMB	DFBetas WML	DFBetas VMC
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
AEX	4%	1%	0	0	**1**	0	0	0	0
MERVAL	4%	2%	0	0	0	0	0	0	0
AS51	4%	1%	0	0	0	0	0	0	0
ATX	4%	1%	0	0	0	**1**	0	0	0
BHSEASI	4%	1%	0	0	0	0	0	0	0
BEL20	4%	1%	0	0	0	0	0	0	0
IBOV	4%	1%	0	0	0	0	0	0	0
SOFIX	4%	1%	0	0	0	0	0	0	**1**
SPTSX	4%	1%	0	0	0	0	0	0	0
IPSA	4%	1%	0	0	**2**	0	0	0	**1**

[4] The Online Appendix is available on: http://coin.wne.uw.edu.pl/qfrg/third-2016-appendix.

Table 4 cont.

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
XIN9I	4%	1%	0	0	0	0	0	0	0
COLCAP	4%	1%	0	0	0	0	0	0	0
CYSMMAPA	4%	2%	0	0	0	0	0	0	0
CTXEUR	4%	1%	0	0	0	**1**	0	0	0
KFX	4%	1%	0	0	0	0	0	0	0
EGX30	4%	2%	0	0	0	0	0	0	0
TALSE	4%	1%	0	0	0	0	0	0	0
SX5E	4%	1%	0	0	0	0	0	0	0
HEX25	4%	2%	0	0	**1**	0	0	0	0
CAC	4%	1%	0	0	0	0	0	0	0
DAX	4%	1%	0	0	0	0	0	0	0
GGSECI	4%	1%	0	0	0	0	0	0	0
FTASE	4%	1%	0	0	0	0	0	0	0
HSI	4%	1%	0	0	0	0	0	0	0
M1HU	4%	1%	0	0	0	0	0	0	0
BUX	4%	1%	0	0	0	0	0	0	0
ICEXI	4%	1%	**1**	0	**2**	0	0	0	**1**
NIFTY	4%	1%	0	0	0	0	0	0	0
LQ45	4%	1%	0	0	0	0	0	0	0
ISXGI	4%	2%	**3**	**3**	**3**	**3**	0	**3**	**3**
ISEQ20P	5%	2%	0	0	0	0	0	0	0
TA.25	4%	1%	0	0	0	0	0	0	0
FTSEMIB	4%	1%	0	0	0	0	0	0	0
NKY	4%	1%	0	0	0	0	**1**	0	0
FNKEN2	4%	1%	0	0	0	0	0	0	0
KSX15	3%	1%	0	0	0	0	0	0	0
LSXC	5%	1%	0	0	0	**4**	**4**	0	0
RIGSE	4%	2%	0	0	0	0	0	**1**	0
VILSE	4%	1%	0	0	0	0	**1**	**1**	0
LUXXX	4%	1%	0	0	0	**1**	0	0	0
MBI	4%	2%	0	0	0	**1**	0	0	0
FBMKLCI	4%	1%	0	0	0	0	0	0	0
MALTEX	4%	1%	0	0	0	0	0	0	0
SEMDEX	4%	2%	0	0	**1**	0	0	0	0
MEXBOL	4%	1%	0	0	0	**1**	0	0	**1**
MSETOP	4%	2%	0	0	0	0	0	**1**	**1**
MONEX20	4%	2%	0	0	0	0	0	0	0
MCSINDEX	4%	1%	0	0	0	0	0	0	0
FTN098	4%	2%	0	0	**1**	**1**	**1**	0	**1**
NZSE50FG	4%	1%	0	0	0	0	0	0	0
NGSEINDX	4%	3%	0	0	0	0	0	0	0
OBX	4%	2%	0	0	0	0	0	0	0
MSM30	4%	2%	0	0	**1**	0	0	0	0
KSE100	4%	2%	0	0	0	0	0	0	0
PCOMP	4%	1%	0	0	0	0	0	0	0
WIG20	4%	1%	0	0	0	**1**	0	0	**1**
PSI20	4%	1%	0	0	0	0	0	0	0
DSM	4%	2%	0	0	0	0	0	0	0

Table 4 cont.

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
BET	4%	1%	0	0	0	0	0	0	0
RTSI	4%	1%	0	0	0	**1**	**2**	0	**1**
SASEIDX	4%	2%	0	0	0	0	0	0	0
BELEX15	5%	2%	0	0	0	0	0	0	0
MXSG	4%	2%	0	0	0	0	0	0	0
SKSM	4%	2%	0	0	0	0	0	0	0
SBITOP	4%	1%	0	0	0	0	0	0	0
TOP40	4%	1%	**1**	0	**2**	**1**	**1**	0	**1**
KOSPI2	4%	2%	0	0	**1**	0	0	0	0
IBEX	4%	2%	0	0	0	0	0	0	0
CSEALL	4%	2%	0	0	0	0	0	0	0
OMX	4%	1%	0	0	**1**	0	0	0	0
SMI	4%	2%	0	0	**1**	0	0	0	**1**
TAMSCI	4%	1%	0	0	0	0	0	0	0
DARSTSI	5%	3%	0	0	0	0	0	0	0
SET50	4%	1%	0	0	0	0	0	0	0
TUSI20	4%	2%	0	0	0	0	0	0	0
XU030	4%	1%	0	0	0	0	0	0	0
UKX	4%	1%	0	0	0	**1**	0	0	0
PFTS	4%	2%	0	0	0	0	0	0	0
DUAE	5%	1%	0	0	0	0	0	0	0
SPX	4%	2%	0	0	0	0	0	0	0
VNINDEX	4%	2%	0	0	0	0	0	0	0

Residuals are from five-factor model. Indices excess returns and risk factor returns are calculated in USD in the period of 2000-2015 on weekly data. For Leverage and studentized residuals we report percentage of observations which exceed the threshold. For Cook's D and DFBetas the number observations which are higher than selected threshold are reported.

Source: Own calculations.

The Cook's Distance measure helps identify observation, which, if excluded from regression, influence OLS estimates. We assume the threshold to be 1, after Cook and Weisberg [1982]. In Table 4 we report number of observations that exceeded this threshold. For vast majority of regressions there are no such observations, only for 3 cases we have only 1 up to 3 influential observations.

Finally, the DFBeta measure, proposed by Belsley, Kuh and Welsch [1980], indicates what influence an observation has on the particular regression coefficient. We took the suggested cut-off at $2^{-1/n}$, where n is the number of observations. Again, for most regressions there are no observations which could be regarded as influential. Only for several of them we observe up to 4 observations which, if excluded, would noticeably alter OLS estimates.

Having identified only a few observations which might be influential, we decided anyway not to remove them. The reason for such attitude is very practical. In reality we are not able to exclude returns which are just slightly higher in absolute values that these from the normal distributions because they are an imminent characteristic of financial markets.

4.4. Normality of OLS residuals

Normality of OLS residuals is verified by Jarque-Berra test. We report the results together with skewness and excess kurtosis coefficients on Figure 5. The null about normality of residuals is rejected in all cases but at the same time the comparison of skewness and excess kurtosis shows that deviation from normality is not excessive.

Table 5. Jarque-Berra p-values, skewness and kurtosis for residuals from five-factor model

Index	JB pvalue	skewness	excess kurtosis	index	JB pvalue	skewness	excess kurtosis	index	JB pvalue	skewness	excess kurtosis
AEX	0.0000	0.2606	2.5473	**NIFTY**	0.0000	-0.0648	1.8867	**PCOMP**	0.0000	0.8948	7.5579
MERVAL	0.0000	-0.3907	9.0459	**LQ45**	0.0000	0.0303	2.2066	**WIG20**	0.0000	0.2221	1.1837
AS51	0.0000	0.1635	0.6475	**ISXGI**	0.0000	3.3754	44.1027	**PSI20**	0.0000	0.0676	2.0035
ATX	0.0000	0.0371	2.1078	**ISEQ20P**	0.0000	-0.5498	8.6249	**DSM**	0.0000	0.1475	3.8629
BHSEASI	0.0000	-0.2551	2.6397	**TA.25**	0.0000	-0.0174	2.1041	**BET**	0.0000	0.0040	3.1406
BEL20	0.0000	0.3796	2.7249	**FTSEMIB**	0.0000	0.5367	5.2000	**RTSI**	0.0000	0.2702	3.6936
IBOV	0.0000	-0.1203	2.2265	**NKY**	0.0000	-0.0558	2.3650	**SASEIDX**	0.0000	-0.9476	6.5659
SOFIX	0.0000	1.1717	11.2222	**FNKEN2**	0.0000	-0.5050	0.7230	**BELEX15**	0.0000	0.2404	3.4216
SPTSX	0.0000	-0.2009	4.6272	**KSX15**	0.0000	0.5854	3.2367	**MXSG**	0.0000	0.2924	2.7672
IPSA	0.0000	0.5994	5.3865	**LSXC**	0.0000	3.2120	21.2272	**SKSM**	0.0000	0.4755	6.3278
XIN9I	0.0000	0.5646	2.5195	**RIGSE**	0.0000	0.8991	17.8460	**SBITOP**	0.0000	0.3014	1.1693
COLCAP	0.0000	-0.1717	2.3526	**VILSE**	0.0000	1.0657	9.8374	**TOP40**	0.0000	0.7365	6.3486
CYSMMAPA	0.0000	0.3352	3.0435	**LUXXX**	0.0000	-0.3995	6.5674	**KOSPI2**	0.0000	0.0840	1.9744
CTXEUR	0.0000	0.2072	1.2046	**MBI**	0.0000	1.2054	6.0953	**IBEX**	0.0000	0.3986	1.8961
KFX	0.0000	0.0116	1.1535	**FBMKLCI**	0.0000	0.0605	4.7510	**CSEALL**	0.0000	0.9261	5.9241
EGX30	0.0000	-0.0790	1.9626	**MALTEX**	0.0000	-0.1289	2.5031	**OMX**	0.0000	0.1942	1.0230
TALSE	0.0000	0.8766	3.8504	**SEMDEX**	0.0000	-0.0905	6.2359	**SMI**	0.0000	1.0188	7.5516
SX5E	0.0000	0.3938	1.9553	**MEXBOL**	0.0000	0.1524	2.2441	**TAMSCI**	0.0000	0.4147	3.9827
HEX25	0.0000	-0.0181	2.8725	**MSETOP**	0.0000	1.1295	8.7749	**DARSTSI**	0.0000	1.3844	7.3662
CAC	0.0000	0.3315	1.5005	**MONEX20**	0.0000	1.0007	3.8546	**SET50**	0.0000	0.0831	1.2762
DAX	0.0000	0.2112	1.5621	**MCSINDEX**	0.0000	-0.0212	2.1882	**TUSI20**	0.0000	-0.7044	10.7571
GGSECI	0.0000	0.7881	5.4357	**FTN098**	0.0000	0.1339	2.2204	**XU030**	0.0000	0.2597	8.3296
FTASE	0.0000	0.2577	2.7365	**NZSE50FG**	0.0000	-0.2048	0.8666	**UKX**	0.0000	0.1778	4.6265
HSI	0.0000	0.2395	1.4679	**NGSEINDX**	0.0000	-0.2053	2.8875	**PFTS**	0.0000	-0.1175	7.4515
M1HU	0.0000	-0.1534	0.7304	**OBX**	0.0000	-0.1223	2.5751	**DUAE**	0.0000	-0.0506	3.7732
BUX	0.0000	-0.1033	0.8815	**MSM30**	0.0000	-0.2315	6.8806	**SPX**	0.0000	-0.0943	2.1159
ICEXI	0.0000	-6.6535	97.9079	**KSE100**	0.0000	-0.6569	3.9280	**VNINDEX**	0.0000	-0.0280	3.1453

Jarque-Berra test p-values, skewness and kurtosis were calculated on weekly data.

Source: Own calculations.

This conclusion is more credible when we investigate histograms and QQ-plots for individual residuals series. Figure 2 presents them for residuals for AEX index regression. Histograms and QQ-plots for residuals from models for other indices are presented in the Online Appendix[5]. They resemble strikingly very similar pattern for other indices. They show that residuals have almost symmetric and follow a slightly leptokurtic distribution.

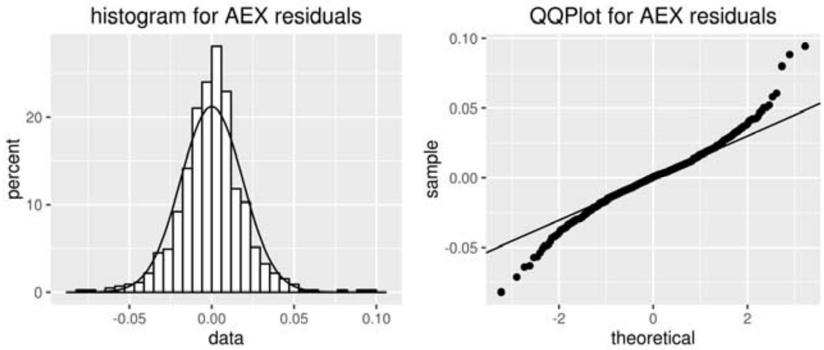

Fig. 2. Histogram and QQ-plot for residuals for AEX index

Residuals are from five-factor model estimated by OLS. Indices excess returns and risk factor returns are calculated in USD in the period of 2000-2015. Histograms and QQ-plots of residuals from regressions for other equity indices excess returns are presented in the Online Appendix available at: http://coin.wne.uw.edu.pl/qfrg/third-2016-appendix

Source: Own calculations.

Figure 3 presents dispersion of skewness and excess kurtosis coefficients of residuals obtained in 81 regression, separately for developed and emerging market indices. We can observe that skewness coefficients are in most cases pretty close to zero, while excess kurtosis in most cases is below value of 6, being somewhat more dispersed for emerging market indices.

[5] The Online Appendix is available on: http://coin.wne.uw.edu.pl/qfrg/third-2016-appendix.

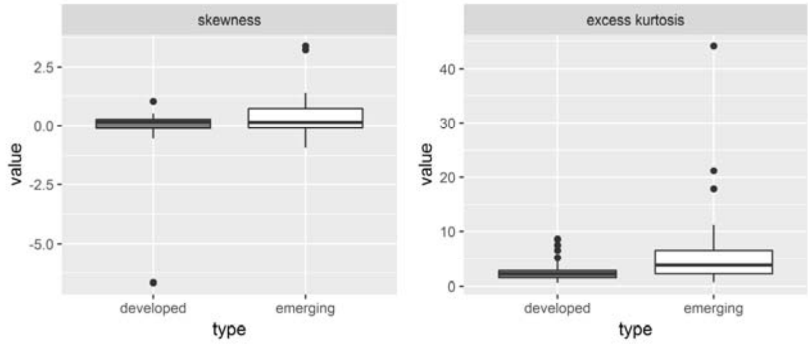

Fig. 3. Dispersion of skewness and excess kurtosis coefficients separately for developed and emerging market indices

Residuals are from five-factor model estimated by OLS. Indices excess returns and risk factor returns are calculated in USD in the period of 2000-2015.

Source: Own calculations.

Generally, we conclude that we observe a moderate non-normality in tested residuals, which is typical for regressions on weekly excess returns. Residuals don't seem to be asymmetric and distributions are moderately fat-tailed. In such conditions, the OLS estimate is still the best linear unbiased estimator (BLUE) of the regression coefficients.

4.5. Testing linear form of the model

Table 6 presents Ramsey RESET test for all regressions separately. We test the null hypothesis that linear functional form of regressions is correct. We include 2nd, 3rd or 4th powers of theoretical values as additional regressors. Their statistical significance would mean that they provide additional information to the model and hence its linear form is inappropriate.

Looking at p-values presented in Table 6 we can state that only in several cases linear functional form is not correct. Moreover, this happens only in case of emerging markets indices. Nevertheless, we do not consider this fact as a reason to change the linear functional form of the model. This rather suggests to include in the model additional explanatory variables which could better explain variability of excess returns for examined indices. The reason for such explanation can be to some extent illustrated by relatively low explanatory power of regressions for emerging markets.

Table 6. Ramsey Reset test

	p-value for powers				p-value for powers				p-value for powers		
Index	2	3	4	Index	2	3	4	Index	2	3	4
AEX	0.014	0.141	0.324	NIFTY	**0.008**	**0.009**	0.014	PCOMP	0.289	0.331	0.333
MERVAL	0.652	0.745	0.558	LQ45	0.134	0.159	0.231	WIG20	0.029	0.050	0.011
AS51	0.132	0.016	0.019	ISXGI	**0.000**	**0.000**	**0.000**	PSI20	0.334	0.535	0.462
ATX	0.620	0.108	0.340	ISEQ20P	0.184	0.037	0.028	DSM	0.024	**0.004**	**0.006**
BHSEASI	0.097	0.814	0.775	TA.25	**0.007**	**0.001**	**0.003**	BET	0.044	0.029	0.064
BEL20	0.430	0.697	0.850	FTSEMIB	0.015	**0.001**	**0.005**	RTSI	0.024	0.222	**0.003**
IBOV	0.237	0.738	0.239	NKY	0.814	0.142	0.073	SASEIDX	**0.000**	**0.003**	**0.001**
SOFIX	**0.004**	**0.004**	**0.003**	FNKEN2	0.819	0.456	0.510	BELEX15	0.257	**0.004**	0.017
SPTSX	0.166	0.819	0.427	KSX15	0.126	0.766	0.452	MXSG	**0.000**	0.175	0.016
IPSA	**0.002**	**0.000**	**0.000**	LSXC	**0.000**	**0.000**	**0.000**	SKSM	0.870	0.423	0.937
XIN9I	0.126	0.241	0.131	RIGSE	0.050	0.263	0.590	SBITOP	0.134	0.281	0.436
COLCAP	0.050	0.023	0.085	VILSE	0.627	0.037	0.065	TOP40	**0.000**	**0.000**	**0.000**
CYSMMAPA	0.399	**0.002**	0.081	LUXXX	0.167	0.419	0.583	KOSPI2	**0.009**	0.016	**0.003**
CTXEUR	0.620	0.041	0.420	MBI	0.719	0.192	0.281	IBEX	0.197	0.023	0.161
KFX	0.796	0.672	0.403	FBMKLCI	0.034	0.063	0.066	CSEALL	0.676	0.020	0.062
EGX30	0.012	0.049	0.059	MALTEX	0.176	0.412	0.525	OMX	**0.000**	**0.000**	**0.000**
TALSE	0.258	0.708	0.472	SEMDEX	**0.000**	**0.000**	**0.000**	SMI	0.760	0.230	0.158
SX5E	**0.005**	**0.001**	0.014	MEXBOL	0.074	0.313	0.492	TAMSCI	**0.006**	**0.003**	**0.005**
HEX25	**0.000**	**0.000**	**0.000**	MSETOP	0.034	**0.002**	0.617	DARSTSI	**0.001**	0.022	0.027
CAC	**0.004**	**0.004**	0.021	MONEX20	0.776	0.016	0.088	SET50	0.903	0.416	0.230
DAX	**0.001**	**0.002**	**0.005**	MCSINDEX	0.682	0.611	0.509	TUSI20	0.017	0.072	0.023
GGSECI	0.467	0.432	0.754	FTN098	**0.000**	**0.006**	**0.000**	XU030	0.426	0.338	0.094
FTASE	0.131	**0.005**	0.015	NZSE50FG	0.418	0.206	0.342	UKX	0.886	0.144	0.385
HSI	**0.000**	0.013	**0.008**	NGSEINDX	0.648	0.697	0.455	PFTS	0.216	0.029	0.180
M1HU	0.190	0.183	0.790	OBX	**0.007**	0.068	0.012	DUAE	0.494	0.251	0.123
BUX	**0.009**	0.933	0.356	MSM30	**0.000**	**0.000**	**0.000**	SPX	0.915	0.351	0.607
ICEXI	0.790	**0.002**	**0.001**	KSE100	0.051	**0.005**	0.023	VNINDEX	0.226	0.040	0.079

Residuals are from five-factor model. Indices excess returns and risk factor returns are calculated in USD in the period of 2000-2015. Null hypothesis for Ramsey Reset test state that linear functional form is correct. Table presents p-values for this hypothesis, respectively for adding 2nd, 3rd and 4th powers of theoretical values.

Source: Own calculations.

To confirm these conclusions, we present in the Online Appendix[6] scatter plots for the dependent variable (RiRf) versus five risk factors separately for all 81 regressions. We can see that for vast majority of models these relationships present a linear shape.

5. Results: OLS vs. MLE and GMM

In this section we compare parameter estimates obtained using three techniques: Ordinary Least Squares, Maximum Likelihood and Generalized Method of Moments.

[6] The Online Appendix is available on: http://coin.wne.uw.edu.pl/qfrg/third-2016-appendix

5.1. OLS

OLS estimates of a five-factor model are presented on Figure 4. Conclusions concentrate principally on differences between developed and emerging indices in terms of parameter dispersion and median values.

1. Developed countries have negative (but close to zero) alpha coefficients, which suggests that there are no excess returns which were not explained the five-factor model. On the other hand, alpha coefficients for emerging equity indices are more dispersed and on average positive although mainly insignificant.
2. Beta for (R_m-R_f) factor is on average higher for developed countries and additionally less diversified across countries when compared with to emerging markets.
3. Beta for HML factor is similar for developed and emerging markets, although we observe more dispersion for emerging equity indices.
4. The median values of SMB beta are negative for developed countries and lower compared to emerging markets. Their diversity is much higher for emerging markets as well.
5. Beta estimates for WML factor are very similar for both groups of indices.
6. Dispersion of beta estimates for VMC factor is much higher among emerging markets.
7. Finally, Rsquared coefficients show that models for developed markets have substantially higher explanatory power than those for emerging markets.

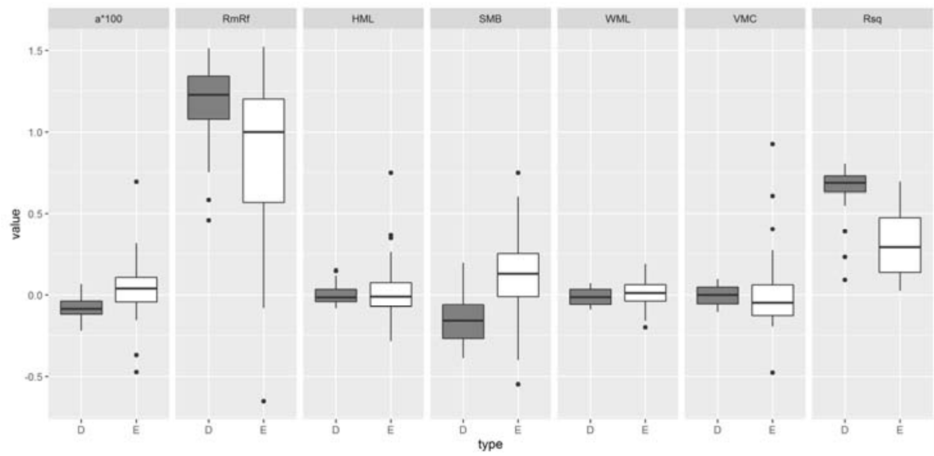

Fig. 4. Dispersion of OLS estimates and Rsquared values in five-factor model for emerging and developed indices separately

Model was estimated on weekly data in USD between 2000-2015. Grey box plots represent parameters estimates and Rsquared values for developed equity indices (D) while white box plots show the same for emerging markets (E).

Source: Own calculations.

5.2. MLE

Figure 5 shows dispersion of parameters estimates and Rsquared values for AR(5)-GARCH(1,1) five-factor model estimated with MLE.

The main observation is that, in general, parameters estimates are very similar to those obtained using OLS, at least in terms of median and dispersion. We detect only two minor differences. First, medians for alpha estimates are slightly higher, for both developed and emerging indices. Second, median estimates of (R_m-R_f) factor are lower, again for both developed and emerging indices.

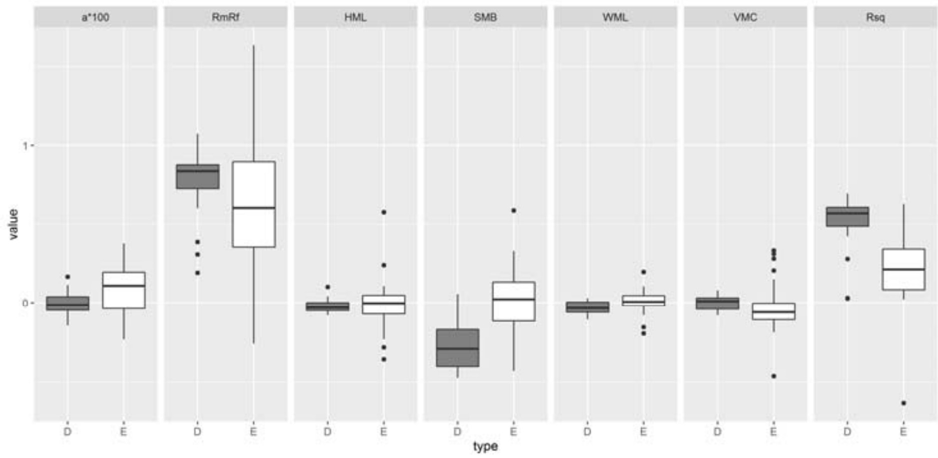

Fig. 5. Dispersion of MLE estimates and Rsquared values in five-factor AR(5)-GARCH(1,1) model for emerging and developed indices separately

AR(5)-GARCH(1,1) model was estimated on weekly data in USD between 2000-2015. Grey box plots represent parameters estimates and Rsquared values for developed equity indices while white box plots show the same for emerging markets.

Source: Own calculations.

5.3. GMM

Figure 6 shows parameter estimates and Rsquared values for five-factor model estimated with GMM.

Comparing this box-plots with those for OLS and MLE we cannot indicate any substantial differences. However, estimates of alpha and market risk factor (R_m – R_f) seem to be closer to MLE results from AR-GARCH model. This suggests that applying model or estimation method which takes into account non-IID error term produces similar results.

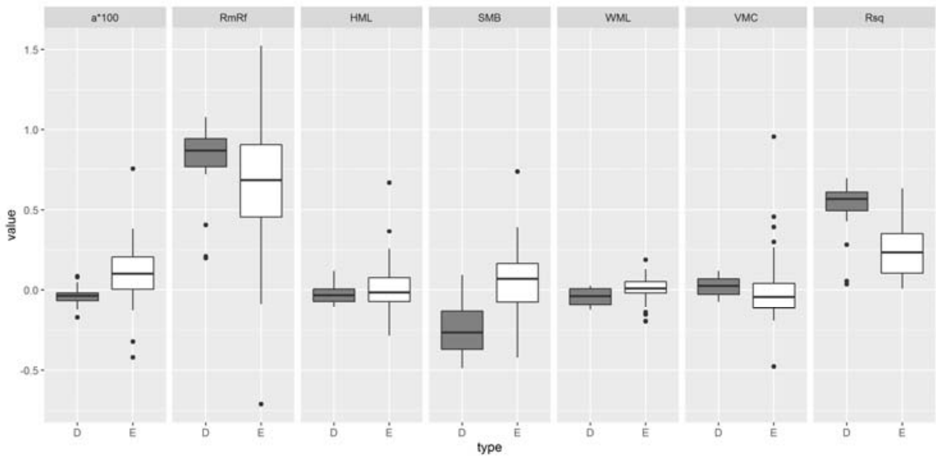

Fig. 6. Dispersion of GMM estimates and Rsquared values in five-factor model for emerging and developed indices separately

Model was estimated on weekly data in USD between 2000-2015. Grey box plots represent parameters estimates and Rsquared values for developed equity indices while white box plots show the same for emerging markets.

Source: Own calculations.

Conclusion

In this paper we investigated preliminary results of OLS estimation of a five-factor model for 81 emerging and developed equity indices presented in Sakowski, Ślepaczuk and Wywiał [2016a]. We verified robustness of these results to popular violations of OLS assumptions.

We found autocorrelation of error term and heteroscedasticity/ARCH effects for most of 81 regressions. To remove these, we applied AR-GARCH model using Maximum Likelihood Estimation.

We haven't identified any problems with collinearity among risk factors. We also haven't found any strong influential observations. On the other hand, we observe moderate leptokurtosis of OLS residuals and rejection of linear form of the model for some indices. To avoid strong assumptions of IID error term, we applied Generalized Methods of Moments to estimate model parameters.

In the last step we compare parameter estimates and models Rsquare coefficients obtained in OLS, MLE and GMM. We find that results do not differ substantially between these three methods of estimation and allow to draw the same conclusions from examined five-factor model.

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ANALIZA DIAGNOSTYCZNA WIELOCZYNNIKOWYCH MODELI OSZACOWAŃ PREMII ZA RYZYKO AKCYJNE

Streszczenie: W ostatnich latach liczne prace podejmowały temat empirycznej weryfikacji skuteczności modelu CAPM. Ich autorzy zaproponowali co najmniej kilka czynników ryzyka, które są w stanie wyjaśnić zróżnicowanie przekrojowe zwrotów rozmaitych aktywów finansowych. Zaproponowano także liczne modyfikacje istniejących modeli wieloczynnikowych. W bogatej literaturze rzadko jednak spotykamy dyskusję na temat konsekwencji stosowania standardowej Metody Najmniejszych Kwadratów do oszacowania parametrów tych modeli. Pytanie o odporność oszacowań wieloczynnikowych modeli wyceny aktywów finansowych uzyskanych za pomocą MNK na niespełnienie założeń nie powinno być jednak ignorowane. Celem niniejszego artykułu jest analiza diagnostyczna wyników oszacowań modelu pięcioczynnikowego dla 81 indeksów giełdowych [Sakowski, Ślepaczuk i Wywiał, 2016a]. Weryfikacja założeń modelu wskazuje na obecność autokorelacji i heteroskedastyczności czynnika losowego, a także występowanie efektów ARCH. Analiza obejmuje także identyfikację obserwacji wpływowych oraz weryfikację obecności współliniowości wśród czynników. W końcowej części prezentujemy porównanie oszacowań uzyskanych za pomocą Metody Najmniejszych Kwadratów, Metody Największej Wiarygodności oraz Uogólnionej Metody Momentów. Wszystkie trzy metody dają bardzo zbliżone oszacowania i pozwalają wyciągnąć ten sam zestaw wniosków dla analizowanego modelu pięcioczynnikowego.

Słowa kluczowe: modele wieloczynnikowe, modele wyceny aktywów, premia za ryzyko akcyjne, metoda najmniejszych kwadratów, metoda największej wiarygodności, uogólniona metoda momentów, autokorelacja, heteroskedastyczność, obserwacje odstające, współliniowość, diagnostyka modeli.