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DESIGN OF OPTIMAL LINEAR SYSTEMS BY MULTIPLE OBJECTIVES

Abstract

Traditional concepts of optimality focus on valuation of already given systems. A new concept of designing optimal systems is proposed. Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. In MOLP problems it is usually impossible to optimize all objectives simultaneously in a given system. An optimal system should be tradeoff-free. As a methodology of optimal system design, De Novo programming for reshaping feasible sets in linear systems can be used. Basic concepts of the De Novo optimization are summarized. Possible extensions, methodological and actual applications are presented. The supply chain design problem is formulated and solved by De Novo approach.

Keywords

Optimization of given systems, design of optimal systems, multiple objectives, De Novo Programming, trade-offs free.

Introduction

Traditional concepts of optimality focus on valuation of already given systems. A new concept of designing optimal systems was proposed [Zeleny 1990 and others]. Mathematical programming under multiple objectives has emerged as a powerful tool to assist in the process of searching for decisions which satisfy best a multitude of conflicting objectives. Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. As a methodology of optimal system design, De Novo programming for reshaping feasible sets in linear systems can be used. The goal of this paper is to popularize the De Novo concept and present the literature review on it. The De Novo concept has been introduced by Milan Zeleny [see Zeleny 1990]. Basic concepts of the De Novo optimization are summarized. The paper presents approaches for solving the Multi-objective De Novo linear programming (MODNLP) problem, its possible extensions, methodological and actual applications, and an illustrative example. The approach is based on a reformulation of the MOLP problem by given prices of resources and a given budget. Searching for meta-optimum with a minimal budget is used. The instrument of optimum-path ratio is used for achieving the best performance for a given budget. Searching for a better portfolio of resources leads to a continuous reconfiguration and reshaping of system boundaries. Innovations bring improvements to the desired objectives and result in a better utilization of available resources. These changes can lead to beyond tradeoff-free solutions. Multi-objective optimization can be taken as a dynamic process. Possible extensions, methodological and real applications are presented. A supply chain design is formulated and solved by the De Novo approach.

1. Optimization of given systems

Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. In MOLP problems it is usually impossible to optimize all objectives simultaneously in a given system. Trade-off means that one cannot increase the level of satisfaction for an objective without decreasing it for another one. Trade-offs are properties of an inadequately designed system and thus can be eliminated through designing a better one. The purpose is not to measure and evaluate tradeoffs, but to minimize or even eliminate them. An optimal system should be tradeoff-free.

The multi-objective linear programming (MOLP) problem can be described as follows

$$\begin{aligned} \text{“Max”} \quad & z = Cx \\ \text{s.t.} \quad & Ax \le b \\ & x \ge 0 \end{aligned} \tag{1}$$

where C is a (k, n)-matrix of objective coefficients, A is a (m, n)-matrix of structural coefficients, b is an m-vector of known resource restrictions, x is an n-vector of decision variables. In MOLP problems it is usually impossible to optimize all objectives in a given system. For multi-objective programming problems the concept of non-dominated solutions is used [see for example Steuer 1986]. A compromise solution is selected from the set of non--dominated solutions.

Two subjects, the Decision Maker and the Analyst, have been introduced due to classification of methods for solving MOLP problems by information mode:

– Methods with a priori information.

The Decision Maker provides global preference information (weights, utility, goal values,…). The Analyst solves a single objective problem.

– Methods with progressive information – interactive methods.

The Decision Maker provides local preference information. The Analyst solves local problems and provides current solutions.

– Methods with a posteriori information.

The Analyst provides a non-dominated set. The Decision Maker provides global preference information on the non-dominated set. The Analyst solves a single objective problem.

Many methods from these categories have been proposed. Most of them are based on trade-offs. The next part is devoted to the trade-off free approach.

2. Designing optimal systems

Multi-objective De Novo linear programming (MODNLP) is a problem for designing an optimal system by reshaping the feasible set. By given prices of resources and a given budget, the MOLP problem (1) can be reformulated as a MODNLP problem (2).

$$\begin{aligned} \text{“Max”} \quad & z = Cx \\ \text{s.t.} \quad & Ax - b \le 0 \\ & pb \le B \\ & x \ge 0 \end{aligned} \tag{2}$$

where b is an m-vector of unknown resource restrictions, p is an m-vector of resource prices, and B is the given total available budget.

From (2) follows

$$pAx \le pb \le B$$

By defining an n-vector of unit costs $v = pA$ we can rewrite the problem (2) as

$$\begin{aligned} \text{“Max”} \quad & z = Cx \\ \text{s.t.} \quad & vx \le B \\ & x \ge 0 \end{aligned} \tag{3}$$

Solving single objective problems

$$\begin{aligned} \text{Max } & z^i = c^i x \quad i = 1, 2, \ldots, k \\ \text{s.t. } & vx \le B \\ & x \ge 0 \end{aligned} \tag{4}$$

z^* is a k-vector of objective values for the ideal system with respect to B.

The problems (4) are continuous "knapsack" problems, the solutions are

$$x_j^i = \begin{cases} 0, j \ne j_i \\ B/v_{j_i}, j = j_i \end{cases}$$

where

$$j_i \in \left\{ j \in (1,\ldots,n) \middle| \max_j (c_j^i / v_j) \right\}$$

The meta-optimum problem can be formulated as follows

$$\begin{aligned} \text{Min } & f = vx \\ \text{s.t. } & Cx \ge z^* \\ & x \ge 0 \end{aligned} \tag{5}$$

Solving the problem (5) provides the solution:

$$x^*$$

$$B^* = vx^*$$

$$b^* = Ax^*$$

The value B^* identifies the minimum budget to achieve z^* through solutions x^* and b^*.

The given budget level $B \le B^*$. The optimum-path ratio for achieving the best performance for a given budget B is defined as

$$r_1 = \frac{B}{B^*}$$

The optimum-path ratio provides an effective and fast tool for the efficient optimal redesign of large-scale linear systems. Optimal system design for the budget B:

$$x = r_1 x^*, \ b = r_1 b^*, \ z = r_1 z^*$$

If the number of criteria k is less than that of variables n, we can individually solve the problem individually and obtain synthetic solutions. Shi [1995] defined the synthetic optimal solution as follows: $x^{**} = (x_{j_1}^1, ..., x_{j_k}^k, 0, .., 0) \in R^n$, where $x_{j_q}^q$ is the optimal solution of [1995]. For the synthetic optimal solution a budget B^{**} is used. One can define six types of optimum-path ratios [Shi 1995]:

$$r_1 = \frac{B}{B^*}, \quad r_2 = \frac{B}{B^{**}}, \quad r_3 = \frac{B^*}{B^{**}},$$

$$r_4 = \frac{\sum_i \lambda_i B_i^j}{B}, \quad r_5 = \frac{\sum_i \lambda_i B_i^j}{B^*}, \quad r_6 = \frac{\sum_i \lambda_i B_i^j}{B^{**}}.$$

Optimum-path ratios are different. It is possible to establish different optimal system designs as options for the decision maker.

3. Extensions

The following extensions of De Novo programming (DNP) are possible:

– Fuzzy DNP.
– Interval DNP.
– Complex types of objective functions.
– Continuous innovations.

Fuzzy De Novo Programming (FDNP) uses instruments as fuzzy parameters, fuzzy goals, fuzzy relations, and fuzzy approaches [Li and Lee 1990].

Interval De Novo programming (IDNP) combines the interval programming and De Novo programming, allowing uncertainties represented as intervals within the optimization framework. The IDNP approach has the advantages in constructing an optimal system design via an ideal system by introducing the flexibility toward the available resources in the system constraints [Zhang et al. 2009].

Complex types of objective functions are defined. The generalization of the single objective Max ($cx - pb$) to the multi-objective form appears to be the right function to be maximized in a globally competitive economy [Zeleny 2010].

The search for a better portfolio of resources leads to continuous reconfiguration and "reshaping" of system boundaries. Innovations bring improvements to the desired objectives and the better utilization of available resources. The technological innovation matrix $T = (t_{ij})$ is introduced. The elements in the structural matrix A should be reduced by a technological progress. The matrix T should be continuously explored. The problem (2) is reformulated as an innovation MODNLP problem (6)

$$\begin{aligned} &\text{"Max"} \quad z = Cx \\ &\text{s.t.}\ \ TAx - b \le 0 \\ &\quad pb \le B \\ &\quad x \ge 0 \end{aligned} \tag{6}$$

The multi-objective optimization can be then seen as a dynamic process in three time horizons:

1. Short-term equilibrium:
   - trade-off,
   - operational thinking.
2. Mid-term equilibrium:
   - trade-off free,
   - tactical thinking.
3. Long-term equilibrium:
   - beyond trade-off free,
   - strategic thinking.

The process is illustrated by example 1.

Example 1

The MOLP problem is formulated as follows:

$$\begin{aligned} &\text{Max} \quad z_1 = x_1 + x_2 \\ &\text{Max} \quad z_2 = x_1 + 4x_2 \\ &3x_1 + 4x_2 \le 60, \\ &x_1 + 3x_2 \le 30, \\ &x_1 \ge 0, x_2 \ge 0. \end{aligned}$$

The MODNLP problem is formulated as follows:
Input: $p = (0.5, 0.4)$ $B = 42$,
unit costs $v = pA = (1.9, 3.2)$.

Max $z^i = c^i x \quad i = 1, 2, \ldots, k$ $\qquad z_1{}^* = 22.11,\ z_2{}^* = 52.50,$
s.t. $vx \le B$
$x \ge 0$

Min $f = vx$ $\qquad x_1{}^* = 11.98,\ x_2{}^* = 10.13$
s.t. $Cx \ge z^*$ $\qquad B^* = vx^* = 55.17$
$x \ge 0$ $\qquad b^* = Ax^* \quad b_1{}^* = 76.48,\ b_2{}^* = 42.39$

$$r_1 = \frac{B}{B^*} = 0.761$$

Optimal system design for B: $x = r_1 x^*$, $b = r_1 b^*$, $z = r_1 z^*$,
$x_1 = 9.12$, $x_2 = 7.71$, $b_1 = 58.23$, $b_2 = 32.25$, $z_1 = 16.83$, $z_2 = 39.96$.

The innovation MODNLP problem is formulated as follows:
Input: $p = (0.5, 0.4)$ $B = 42$,
the technological innovation matrix $T = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix}$,
unit costs $v = pTA = (1.48; 2.44)$,

$z_1{}^* = 28.38$, $z_2{}^* = 68.85$,

$x_1{}^* = 14.89$, $x_2{}^* = 13.49$,

$B^* = vx^* = 54.95$,

$r_1 = 0.764$,

$x_1 = 11.38$, $x_2 = 10.31$,

$z_1 = 21.69$, $z_2 = 52.62$.

The solutions in different time horizons are represented in Figure 1.

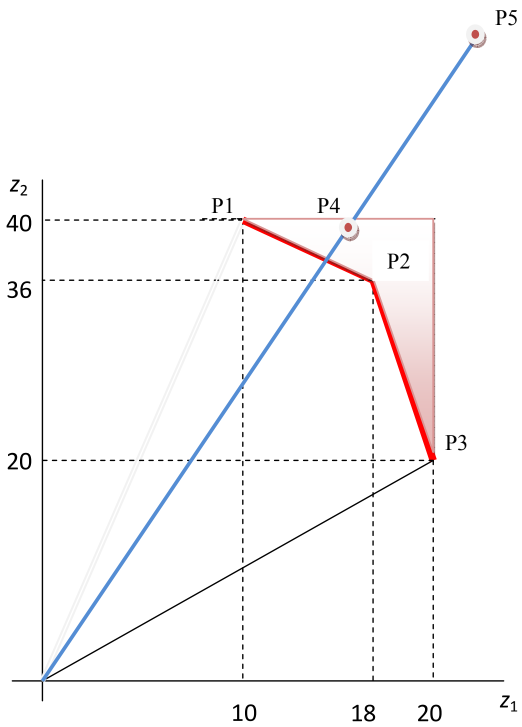

Figure 1. Solutions for the illustrative example

Figure 1 shows the non-dominated frontier (P1-P2-P3) for the MOLP problem, the solution (point P4) of the MODNLP problem and the solution (point P5) of the innovative MODNLP problem. The solution of the MODNLP problem is not fully trade-off free in this example. The solution of the innovative MODNLP problem shows the beyond trade-off free trajectory.

4. Applications

The tradeoff-free decision making has a significant number of method-logical applications. All such applications have the tradeoff-free alternative in common:

– Compromise programming – minimize distance from the ideal point.
– Risk management – portfolio selection – tradeoffs between investment returns and investment risk.
– Game theory – win-win solutions.
– Added value – value for the producer and value for the customer – both must benefit.

There are real applications of the De Novo approach. For example, the production plan for an actual production system is defined taking into account financial constraints and given objective functions [Babic and Pavic 1996]. The paper [Zhang et al. 2009] presents an Inexact DNP approach for the design of optimal water-resources-management systems under uncertainty. Optimal supplies of good-quality water are obtained with different revenue targets of municipal–industrial–agricultural competition under a given budget taken into account.

In the next part a supply chain design problem is formulated. Supply chain management has generated a substantial amount of interest from both managers and researchers. Supply chain management is now seen as a governing element in strategy and as an effective way of creating value for customers. A supply chain is defined as a system of suppliers, manufacturers, distributors, retailers and customers where material, financial and information flows connect participants in both directions [see for example Fiala 2005]. There are many concepts and strategies applied to the design and management of supply chains. The fundamental decisions to be made during the design phase are the location of facilities and the capacity allocated to these facilities. An approach to designing an economically optimal supply chain is to develop and solve a mathematical programming model. A mathematical program determines the ideal locations for each facility and allocates the activity to each facility so that the costs are minimized and the constraints of meeting the customer demand and the facility capacity are satisfied. A general form of the model for the supply chain design is given below.

Model

Our model of a supply chain consists of 4 layers with m suppliers, $S_1, S_2, \dots S_m$, n potential producers, $P_1, P_2, \dots P_n$, p potential distributors, $D_1, D_2, \dots D_p$, and r customers, $C_1, C_2, \dots C_n$.

The following notation is used:

a_i = annual supply capacity of supplier i,

b_j = annual potential capacity of producer j,

w_k = annual potential capacity of distributor k,

d_l = annual demand of customer l,

f_j^P = fixed cost of potential producer j,

f_k^D = fixed cost of potential distributor k,

c_{ij}^S = unit transportation cost from S_i to P_j,

c_{jk}^P = unit transportation cost from P_j to D_k,

c_{kl}^D = unit transportation cost from D_k to C_l,

t_{ij}^S = unit transportation time from S_i to P_j,

t_{jk}^P = unit transportation time from P_j to D_k,

t_{kl}^D = unit transportation time from D_k to C_l,

x_{ij}^S = number of units transported from S_i to P_j,

x_{jk}^P = number of units transported from P_j to D_k,

x_{kl}^D = number of units transported from D_k to C_l,

y_j^P = bivalent variable for build-up of fixed capacity of producer j,

y_k^D = bivalent variable for build-up of fixed capacity of producer k.

With this notation the problem can be formulated as follows:
The model has two objectives. The first one expresses the minimizing of total costs. The second one expresses the minimizing of total delivery time.

$$\text{Min } z_1 = \sum_{j=1}^{n} f_j^P y_j^P + \sum_{k=1}^{p} f_k^D y_k^D + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} c_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} c_{kl}^D x_{kl}^D$$

$$\text{Min } z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} t_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} t_{kl}^D x_{kl}^D$$

Subject to the following constraints:

– the amount sent from the supplier to the producers cannot exceed the supplier's capacity

$$\sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, ..., m,$$

– the amount produced by the producer cannot exceed the producer's capacity

$$\sum_{k=1}^{p} x_{jk} \le b_j y_j, \ \ j = 1, 2, ..., n,$$

– the amount shipped from the distributor should not exceed the distributor's capacity

$$\sum_{l=1}^{r} x_{kl} \le w_k y_k, \ \ k = 1, 2, ..., p,$$

– the amount shipped to the customer must equal the customer's demand

$$\sum_{k=1}^{p} x_{kl} = d_l, \ \ l = 1, 2, ..., r,$$

– the amount shipped out of producers cannot exceed units received from suppliers

$$\sum_{i=1}^{m} x_{ij} - \sum_{k=1}^{p} x_{jk} \ge 0, \ j = 1, 2, ..., n,$$

– the amount shipped out of the distributors cannot exceed quantity received from the producers

$$\sum_{j=1}^{n} x_{jk} - \sum_{l=1}^{r} x_{kl} \ge 0, \ \ k = 1, 2, ..., p,$$

– binary and non-negativity constraints

$$Y_{j,k} \in \{0,1\},$$
$$x_{ij}, x_{jk}, x_{kl} \ge 0, \ \ i = 1, 2, ..., m, \ j = 1, 2, ..., n, \ \ k = 1, 2, ..., p, \ \ l = 1, 2, ..., r.$$

The formulated model is a multi-objective linear programming problem. The problem can be solved by an MOLP method.

The De Novo approach can be useful in the design of the supply chain. Only a partial relaxation of constraints is adopted. Producer and distributor capacities are relaxed. Unit costs for capacity build-up are computed:

$$p_j^P = \frac{f_j^P}{b_j} = \text{cost of unit capacity of potential producer } j,$$

$$p_k^D = \frac{f_k^D}{w_k} = \text{cost of unit capacity of potential distributor } k.$$

Variables for build-up capacities are introduced:

u_j^P = variable for flexible capacity of producer j,

u_k^D = variable for flexible capacity of producer k.

The constraints for non-exceeding producer and distributor fixed capacities are replaced by the flexible capacity constraints and the budget constraint:

$$\sum_{k=1}^{p} x_{jk} - u_j^P \leq 0, \ \ j = 1, \ 2, \ ..., \ n,$$

$$\sum_{l=1}^{r} x_{kl} - u_k^D \leq 0, \ \ k \ = \ 1, \ 2, \ ..., p,$$

$$\sum_{j=1}^{n} p_j^P u_j^P + \sum_{k=1}^{p} p_k^D u_k^D \leq B.$$

Example 2

An example of the supply chain with 3 potential producers, 3 potential distributors, and 3 customers was tested. Data are presented in Tables 1, 2 and 3.

Table 1

Unit transportation costs

c_{ij}^P	D_1	D_2	D_3	c_{jk}^D	C_1	C_2	C_3
P_1	5	3	8	D_1	3	1	4
P_2	3	6	2	D_2	6	7	2
P_3	8	4	5	D_3	5	4	8

Table 2

Unit transportation time

c_{ij}^P	D_1	D_2	D_3	c_{jk}^D	C_1	C_2	C_3
P_1	4	2	3	D_1	6	1	2
P_2	3	2	2	D_2	3	2	5
P_3	1	5	3	D_3	1	4	2

Table 3

Capacity and costs for producers and distributors

	P_1	P_2	P_3	D_1	D_2	D_3
Capacity	250	300	200	300	200	300
Costs	150	200	180	50	60	90
unit cost	0.60	0.67	0.90	0.17	0.30	0.30

Customer demand: $d_1 = 100$, $d_2 = 150$, $d_3 = 200$.

We get the ideal objective values z^* by solving single objective problems. The interactive method STEM is used for finding a compromise non-dominated solution. The De Novo approach is used for the supply chain design. The results are compared in Table 4.

Table 4

Results for supply chain design

	Max z_1	Max z_2	Compromise	De Novo
x_{11}^P	0	0	0	0
x_{12}^P	200	0	50	0
x_{13}^P	0	0	0	0
x_{21}^P	250	0	200	250
x_{22}^P	0	0	0	0
x_{23}^P	0	250	100	100
x_{31}^P	0	200	100	100
x_{32}^P	0	0	0	0
x_{33}^P	0	0	0	0
x_{11}^D	100	0	0	0
x_{12}^D	150	150	150	150
x_{13}^D	0	50	150	200
x_{21}^D	0	0	0	0
x_{22}^D	0	0	0	0

Table 4 contd.

	Max z_1	Max z_2	Compromise	De Novo
x_{23}^D	200	0	50	0
x_{31}^D	0	100	100	100
x_{32}^D	0	0	0	0
x_{33}^D	0	150	0	0
u_1^P	250	0	250	0
u_2^P	300	300	300	350
u_3^P	0	200	200	100
u_1^D	300	300	300	350
u_2^D	200	0	200	0
u_3^D	0	300	300	100
z_1	2660	4670	3830	3644
z_2	2900	1350	1800	1700
B	460	520	730	444

The De Novo approach provides a better solution in both objectives and also with lower budget thanks to flexible capacity constraints. The capacity of supply chain members has been optimized with regard to flows in the supply chain and to the budget.

Conclusions

De Novo programming is used as a methodology of optimal system design for reshaping feasible sets in linear systems. The MOLP problem is reformulated by given prices of resources and a given budget. Searching for a better portfolio of resources leads to a continuous reconfiguration and reshaping of systems boundaries. Innovations bring improvements to the desired objectives and the better utilization of available resources. These changes can lead to beyond tradeoff-free solutions. Multi-objective optimization can be regarded as a dynamic process. The De Novo approach has been applied to the supply chain design problem; it provides a better solution than traditional approaches applied to fixed constraints. The De Novo programming approach is open for further extensions and applications.

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References

Babic Z., Pavic I. (1996), *Multicriterial Production Planning by De Novo Programming Approach*, "International Journal of Production Economics", No. 43.

Fiala P. (2005), *Information Sharing in Supply Chains*, "Omega", No. 33.

Li R.J., Lee E.S. (1990), *Fuzzy Approaches to Multicriteria De Novo Programs*, "Journal of Mathematical Analysis and Applications", No. 153.

Shi Y. (1995), *Studies on Optimum-Path Ratios in De Novo Programming Problems*, "Computers and Mathematics with Applications", No. 29.

Steuer R.E. (1986), *Multiple Criteria Optimization: Theory, Computation and Application*, Wiley, New York.

Zelený M. (1990), *De Novo Programming*, "Ekonomicko-matematický oborz", No. 26.

Zeleny M. (2010), Multiobjective *Optimization, Systems Design and De Novo Programming*, [in:] *Handbook of Multicriteria Analysis*, eds. C. Zopounidis, P.M. Pardalos, Springer, Berlin.

Zeleny M. (1990), *Optimal Given System vs. Designing Optimal System: The De Novo Programming Approach*, "International Journal of General System", No. 17.

Zhang Y.M., Huang G.H., Zhang X.D. (2009), *Inexact de Novo Programming for Water Resources Systems Planning*, "European Journal of Operational Research", No. 199.