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MULTICRITERIA ANALYSIS OF CLASSIFICATION IN ATHLETIC DECATHLON

Abstract

Man's decathlon is an athletic contest which consists of 10 events, four of them measured in seconds and the remaining six in meters. Each athlete (alternative) is described by his 10 results (criteria). Current system of classification is based on aggregation of results using utility (scoring) functions which are defined exactly for each event. This system has been used since 1984 and the aim of the paper is to analyse it with respect to current conditions. The current method of classification is compared with several alternative methods which better reflect current top results of 10 events included in men decathlon. Proposed methods are applied to real results, specifically to the Olympic Games 2008.

Keywords

Multiple criteria decision making, decathlon, utility function.

Introduction

Decathlon is an athletic contest with a very long tradition which consists of 10 different events. Since ancient Greece it has been regarded as a measure of universality of athletes and is often called „king athletic event". For the first time a kind of athletic multicontest was scheduled in the 3rd Summer Olympic Games in Saint Louis in 1904. In 1914 the decathlon was established by the International Amateur Athletic Federation in the form in which it is known now. Since that year the decathlon consists of 10 events (3 sprints, 1 long distance run, 3 jump events and 3 throw events) and the order of all events within a contest is fixed. The contest is always scheduled in two consecutive days as follows:

1st day:

- 100 meters run,
- Long jump,
- Shot put,

– High jump,
– 400 meters run,

2nd day:

– 110 meters hurdles run,
– Discus throw,
– Pole vault,
– Javelin throw,
– 1500 meters run.

Principles of decathlon, its history and the current system of scoring are described in detail in an official IAAF paper [Diack 2004].

In general, ranking of contestants in decathlon is a multiple criteria decision problem (MCDM) with athletes attending the contest as alternatives and 10 criteria. The aim of the problem is to evaluate the alternatives (athletes) and rank them from the best to the worst. As it is clear, criterion values in this problem are not comparable – they are given either in seconds or in centimeters and moreover the values in seconds (centimeters) are incomparable directly. That is why the results of athletes in individual events are aggregated using so-called scoring tables which assign points to the performances in individual events. The athletes are ranked according to the total points achieved.

1. Current system of performance evaluation in decathlon

The scoring tables went over time through several evolution steps – [for details see: Diack 2004]. Their last change is dated 1984 and the tables constructed according to the following principles still stand today:

1. The scoring tables should differ from those used for individual event scoring.
2. The scores for different events should be comparable, so that equal skill levels in different events are rewarded with equal point levels.
3. The scoring tables should be one of the following:
   – linear in all events, or
   – slightly progressive in all events.
4. The tables should be applicable to all levels of performance, from youth to professional.
5. Men and women should have different tables.
6. Specialists' performances should be the basis for the scores in the tables.
7. The tables should be applicable in the future.

8. The total scores using the new tables for top world-class athletes should remain approximately the same.
9. As much as possible, the new tables should ensure that a specialist in one event cannot be better than top performances in the other events.

The current scoring tables were designed according to the above-mentioned principles by Czech mathematician dr. Trkal. They assign points based on performances of athletes using the following formulas:

$$u_{ik} = \mathrm{INT}[a_k(b_k - p_{ik})^{c_k}],\ k \in \mathrm{R},\ i \in \mathrm{A}, \tag{1}$$

$$u_{ik} = \mathrm{INT}[a_k(p_{ik} - b_k)^{c_k}],\ k \in \mathrm{T},\ i \in \mathrm{A}, \tag{2}$$

where

R is the index set of running events,
T is the index set of throw (jump) events,
A is the index set of athletes attending the contest,
u_{ik} is the number of points achieved by i-th athlete in the k-th event,
p_{ik} is the performance of the i-th athlete in the k-th event measured in seconds for running events and in centimeters for throw and jump events,
INT(x) is the integer part of x, and
a_k, b_k, c_k are specific parameters for the k-th event. The fixed values of parameters a, b and c are presented in Table 1 – [source: Diack 2004].

Table 1

Parameters of scoring functions

Event	a_k	b_k	c_k
100 meters	25.4347	18.00	1.81
Long jump	0.14354	220.00	1.40
Shot put	51.39	1.50	1.05
High jump	0.8465	75.00	1.42
400 meters	1.53775	82.00	1.81
110 m hurdles	5.74352	28.50	1.92
Discus throw	12.91	4.00	1.10
Pole vault	0.2797	100.00	1.35
Javelin throw	10.14	7.00	1.08
1500 meters	0.03768	480.00	1.85

It is not clear how the parameters for scoring functions are derived. Explanation of b_k parameters is not difficult – it is the performance rewarded by zero points. c_k parameters express the degree of progressiveness of the scoring function. The most progressive functions are those for all running events. On the contrary the functions for throw events are almost linear. Explanation of a_k parameters is not clear at all.

The total number of points for each athlete U_i is given as a simple sum of points rewarded in all events, i.e.

$$U_i = \sum_{k=1}^{10} u_{ik}, \qquad i = 1,2,...,n.$$

The presented approach has been applied since 1985 without any changes. Not every system is ideal. This one has several questionable features too. The most important one is that the number of points awarded in the events is not uniform – some of the events are rewarded by a higher number of points than the other ones. The differences are clearly shown in Table 2. This table presents current world records (WR) in all events and their appropriate numbers of points given by (1) and (2), as well as the average number of points in particular events of TOP100 historic performances in men decathlon – [source: Diack 2004; Westera 2006] and the author's own calculations.

Table 2

Differences in point rewards

Event	*WR*	u_{wr}	u_{100}
100 meters	9.58	1202	917
Long jump	895	1312	970
Shot put	23.12	1295	815
High jump	245	1244	859
400 meters	43.18	1156	899
110 m hurdles	12.87	1126	946
Discus throw	74.08	1383	808
Pole vault	614	1277	901
Javelin throw	98.48	1331	810
1500 meters	206	1218	711

2. Alternative definitions of scoring functions

Decathlon is a multiple criteria decision making problem with the aim to rank all alternatives (athletes). There are many methods based on different principles that can be used for multicriteria evaluation of alternatives. AHP/ANP, PROMETHEE class methods, ELECTRE class methods and aggregation using utility functions are the most often applied ones but for decathlon ranking only the last-mentioned approaches are applicable. That is why we suggest a modification of the current scoring functions in order to take into account differences in awarding points in decathlon events. Similar re-definitions of scoring functions were discussed by several researchers, e.g. [Cox and Dunn 2002; Cheng et al. 2003; Westera 2006; Taborski 2008]. The following four modifications of scoring functions (M1 – M4) are further discussed:

M1 – The scoring functions (1) and (2) remain unchanged but the parameter a_k is modified for all events to unify the number of points for world records on the level 1250 points (the sum of u_{WR} in Table 2 is approx. 1250). This model preserves the rate of progression of the utility function (parameter c_k) and the bounds (parameter b_k) that correspond to zero points performances.

M2 – Similar to the previous case – except that the parameter a_k is modified to approximate the point rewards of average TOP100 performances to the same level. The average value of the last column in Table 2 is approx. 864 and, that is why for the first event the parameter a_k is reduced by approx. 5.8% and for the last event it is increased by approx. 21.5%.

M3 – Linear utility functions with lower bound (zero points) on the same level as in formulas (1) and (2), i.e. parameter b_k, and upper bound (1250 points) for the current world record (it is not expected that an athlete can beat current world record in any event).

M4 – Linear utility function with the lower bound as in the previous case. The number of points is given by the following formula (for events measured in seconds):

$$u_{ik} = \frac{b_k - p_{ik}}{b_k - q_{ik}} 864 ,$$

where q_{ik} is the performance rewarded by TOP100 average points (Table 2).

Table 3 presents original value of a_k parameter and its modifications in models M1 and M2 given by own calculations.

Table 3

Parameters for alternative scoring functions

Event	a_k	M1 – a_k	M2 – a_k
100 meters	25.4347	26,4500	23,9646
Long jump	0.14354	0,1368	0,1279
Shot put	51.39	49,6100	54,4797
High jump	0.8465	0,8511	0,8514
400 meters	1.53775	1,6630	1,4779
110 m hurdles	5.74352	6,3800	5,2457
Discus throw	12.91	11,6700	13,8048
Pole vault	0.2797	0,2737	0,2682
Javelin throw	10.14	9,5250	10,8160
1500 meters	0.03768	0,0387	0,0458

3. Re-calculation of Olympic Games 2008 results

The alternative definitions of scoring functions presented in previous section are applied to the data set of decathlon results in Olympic Games 2008 in Beijing. The criterion matrix, i.e. the performances of first 15 athletes in descending ranking, is presented in Table 4 (source: official web pages of IAAF – www.iaaf.org). All running performances are given in seconds, long and high jump and pole vault in centimetres and remaining two events in meters (discus and javelin).

Table 4

Performances of first 15 athletes in Olympic Games 2008

	100 m	Long	Shot	High	400 m	110 m	Disc	Pole	Javel	1500 m
1	2	3	4	5	6	7	8	9	10	11
1	10.44	778	16.27	199	48.92	13.93	53.79	500	70.97	306.59
2	10.96	761	14.39	211	47.30	14.21	44.58	500	60.23	267.47
3	10.90	733	14.49	205	47.91	14.15	44.45	470	73.98	269.17
4	11.07	737	16.53	208	50.91	14.47	50.04	500	64.01	301.56
5	11.26	708	15.42	196	49.51	14.21	45.17	500	65.40	269.29
6	11.21	768	14.78	211	49.54	14.71	45.50	480	63.93	289.63

Table 4 contd.

1	2	3	4	5	6	7	8	9	10	11
7	10.53	756	15.15	196	47.70	14.37	48.39	430	51.59	268.94
8	11.12	729	13.23	205	49.65	14.37	45.39	520	60.21	272.90
9	10.85	704	15.09	199	47.96	14.08	50.91	460	51.52	271.62
10	10.80	770	13.67	199	48.47	14.71	40.41	480	60.27	266.77
11	11.15	704	14.36	211	50.90	14.51	49.35	480	67.07	287.03
12	11.02	723	16.26	202	51.56	15.51	47.43	510	62.57	281.34
13	10.89	729	14.79	196	48.98	14.06	39.83	480	67.16	289.60
14	11.19	719	13.78	199	49.99	14.73	44.09	470	71.44	277.96
15	10.64	707	15.82	196	49.66	13.90	36.73	470	65.60	300.49

Table 5 compares the total number of points and ranking of all athletes derived in standard way – formulas (1) and (2) – which is denoted as M0, with four alternative definitions M1 – M4 presented in the previous section.

Table 5

Comparison of original and alternative approaches

	M0	R	M1	R	M2	R	M3	R	M4	R	PII
1	8791	1	8787	1	8762	1	9674	1	8803	1	1
2	8551	2	8584	2	8556	2	9441	3	8520	3	2
3	8527	3	8551	3	8555	3	9450	2	8569	2	3
4	8328	4	8319	4	8324	4	9338	4	8497	4	4
5	8253	5	8273	5	8288	5	9248	5	8392	5	6
6	8241	6	8247	7	8235	7	9245	6	8376	7	11
7	8238	7	8272	6	8246	6	9176	10	8282	11	8
8	8220	8	8242	8	8227	8	9197	9	8306	9	5
9	8205	9	8241	9	8227	8	9171	11	8292	10	12
10	8194	10	8227	10	8197	11	9153	12	8247	13	9
11	8178	11	8180	11	8200	10	9206	8	8369	8	7
12	8154	12	8139	13	8184	12	9207	7	8377	6	10
13	8118	13	8151	12	8109	13	9124	13	8243	14	14
14	8055	14	8065	14	8082	14	9100	14	8258	12	13
15	7992	15	8035	15	7971	15	9017	15	8141	15	15

The results Presented are completed by ranking given by one of the most often used MCDM methods for evaluation of alternatives which is PROMETHEE II method even though it is clear that this kind of methods is not suitable for evaluation of decathlon athletes. The main reason is that this method compares each pair of alternatives with respect to all criteria.

That is why the final ranking depends on mutual relations of pairs of alternatives and rank reversal is not eliminated. This is unacceptable for decathlon purposes and the only method which can be used in this context is an application of utility functions. For all criteria (events) a linear preference function with a sufficiently high preference threshold was used in application of PROMETHEE method. Principles of PROMETHEE class methods are generally known. They were proposed by Brans and Vincke [1985]. Their basic description as well as information about original software support for MCDM problems including PROMETHEE class methods can be found in [Jablonsky 2007].

Table 6

Average point rewards given by models

Event	M0	M1	M2
100 meters	876	911	825
Long jump	899	856	800
Shot put	786	759	833
High jump	824	829	829
400 meters	850	919	817
110 m hurdles	925	1027	845
Discus throw	782	707	837
Pole vault	860	842	825
Javelin throw	794	746	847
1500 meters	674	691	819
MIN	674	691	800
MAX	925	1027	847

The comparison of results shows a close similarity of rankings given by original formulas and by models M1 and M2, i.e. by models based on utility functions with the same progression level as the original ones. Only a few rank reversals appear. On the other hand the linear utility functions (models M3 and M4) generate a quite different results as compared to the original ranking.

Table 6 shows differences in average point rewards for all events and for original model M0 and models M1 and M2. It is clear that the models M0 and M1 on the one hand and model M2 on the other hand are significantly different. Minimum and maximum average rewards differ by around 300 points in the first case and only by fewer than 50 points in the model M2. A similar relations holds for models M3 and M4 not presented in Table 6.

Conclusions

Analysis of decathlon results is a very interesting multiple criteria decision making problem of high importance. The paper presents current system of evaluation based on aggregation of performances in events into final point score and proposes several new definitions of scoring functions. The most promising definition has been introduced as model M2. It solves the problem of high differences in point rewards and preserves the current level of progression in individual events. Moreover, results of the model M2 keep the current final level of points and seem to be comparable to the current standard system.

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References

Brans J.P., Vincke, P. (1985), *A Preference Ranking Organisation Method (The PROMETHEE Method for Multiple Criteria Decision-Making)*, "Management Science", No. 31(6).

Cheng C.L, Tsai C.H., Chen L. (2003), *Applying Grey Relational Analysis to the Decathlon Evaluation Model*, "International Journal of The Computer, "The Internet and Management", No. 11(3).

Cox D.F, Dunn R.T. (2002), *An Analysis of Decathlon Data*, "The Statistician", No. 52(2).

Diack L. (2004), *IAAF Scoring Tables for Conbined Events* – accessible at http:// www.iaaf.org/mm/Document/Competitions/TechnicalArea/ ScoringTables_CE_ 744.pdf [08.03.2011].

Jablonsky J. (2007), *Optimisation and Modelling with Spreadsheets*, in: *Proceedings of the SOR'07*, BORI, Ljubljana.

Taborski J. (2008), *Quantitative Methods in Athletic Decathlon* (in Czech), VSE Praha.

Westera W. (2006), *Decathlon, Towards a Balanced and Sustainable Performance Assessment Method*, "New Studies in Athletics", March/April, pp. 37-48 – accessible at http://www.open.ou.nl/wim/publicationspdf/ [08.03.2011].