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ANALYSIS OF INCENTIVE COMPATIBLE MULTICRITERIA DECISIONS FOR A PRODUCER AND BUYERS PROBLEM

Abstract

The paper deals with analysis of incentive compatible multicriteria decisions within a computer-based multiagent framework. This general question is discussed on an example of a market decision problem, where a producer is introducing a product and some buyers are considering the purchase of the product. Decisions of the producer and buyers are multicriterial. Each of the buyers is seeking a product variant according to his own preferences. The producer decides which variant of the product is introduced to the market. In order to incentivize the decisions, one of his criteria takes into account an aggregated satisfaction of the buyers. A multiagent computer-based system has been constructed for supporting mulicriteria analysis made by buyers and by the producer. Selected results of an interactive session made with use of the system are presented and analyzed.

Keywords

Incentive compatible decision mechanisms, multiagent systems, multicriteria optimization.

Introduction

The paper relates to a wider direction of research dealing with analysis of incentive compatible multicriteria decision mechanisms with use of multi-agent computer-based systems. In the research the situations are analyzed in which there are agents with their own interests and trying to realize their own egoistic goals. The effects of their decisions depend also on decisions of other agents. Each agent has his own private information and, in general, he does not like to reveal the information to others. The subject of the research includes decision mechanisms leading to motivation compatibility. The mechanisms can be constructed by harmonization of agents' activity to secure the effect-tiveness of the whole system. The incentive compatibility in market mechanisms was analyzed previously by Toczyłowski [2003, 2009]. The ideas developed in the papers have inspired the research presented here.

In this paper an analysis of incentive compatible decisions is performed on an example of a two stage mechanism in which a producer and his potential buyers participate. In the first stage, each buyer makes independent multicriteria analysis of a possible variant of a product and selects the variant preferred according to his individual criteria. In the example presented, each buyer minimizes a cost criterion and maximizes a criterion defined by the usefulness of the product. In the second stage, the producer also performs multicriteria analysis in the set of possible variants of the product but with respect to his criteria, including a profit criterion. The reputation of the product on the market has been assumed as one of important criteria of the producer. The reputation is expressed by an aggregated measure of satisfaction of buyers with the variant offered by the producer.

A special multiagent computer-based system has been designed. It enables problem formulation and supports multicriteria analysis made by buyers and by a producer. The system has been implemented using Optimization Software for Operations Research Applications [AIMMS]. Information about the AIMMS environment can be found on www.aimms.com and [Bisschop and Roelofs 2009]. Details referring to the functionality of the system, its implementation and user instructions can be found in the Eng. diploma thesis [Skorupiński 2010]. The system secures the confidentiality of information of users playing roles of buyers and a producer. The producer has no access to the information introduced by the buyers, nor to results of their analysis made with the use of the system.

We could imagine that the system is at the disposal of an institution, whom both the producer and the buyers trust. The institution secures the confidentiality of the individual information and performs market analysis of the new product among potential buyers. It also supports the producer in the selection of the product variant which would be favorable with respect to his criteria but would also have a good reputation on the market.

This paper includes a mathematical formulation of multicriteria optima-zation tasks for buyers and for the producer. The tasks are solved by the system during multicriteria analysis with use of the reference point method [Wierzbicki 1986; Wierzbicki et al. 1993, 2000]. A question arises how to define and derive buyers satisfaction levels with respect to the variant of the product offered on the market. Then, how to calculate a cumulative reputation of the product variant on the market. Some proposals are presented.

Series of interactive sessions have been made with use of the system. Different results have been obtained showing possible behaviors of buyers and of the producer, as well as relations among solution variants chosen by them. The results of one of the sessions are presented in the paper. In the final remarks, directions of further research are discussed.

1. Mathematical description

A producer is going to offer a new variant of his product to a set L of buyers. The variants of the product that can be produced are described by a vector of decision variables $x \in D \subset R^n$, where D is a set of admissible vectors of the variables. The set D is not given explicitly. We assume that it is given by a set of linear constraints of the form: $A\, x^{\mathrm{T}} \leq b$, where A and b are matrix and vector of coefficients respectively.

The analysis of the product variants is performed in two stages.In the first stage each buyer can generate, review and analyze nondominated product variants in his space of criteria, using the reference point method [Wierzbicki 1986; Wierzbicki et al. 1993, 2000]. The following optimization tasks are formulated:

$$\max_x \{[\phi(r,a,y)] : x \in D \subset R^n\}, r \in R^n, a \in R^n,$$

where Φ denotes a scalarizing achievement function, r and a are vectors of controlling parameters. The vectors r and a play the roles of the reservation and aspiration points, respectively. The criteria y are selected variables of the vector x. They include buyers' criteria, such as:

- e – economic attributes of the product, including purchasing and operating cost covered by the buyer,
- u – usefulness of the product due to quality, technological advantage, reliability.

A nondominated solution is derived for reservation and aspiration points given by a buyer, solving the optimization problem:

$$\max z + \varepsilon \sum_{k \in \bar{X}} z_k \ ,$$

subject to constraints of the reference point method:

$$
\begin{aligned}
& z \le z_k, \forall k \in \overline{X}, \\
& z_k \le \gamma (x_k - r_k)/(a_k - r_k), \forall k \in \overline{X}, \\
& z_k \le (x_k - r_k)/(a_k - r_k), \forall k \in \overline{X}, \\
& z_k \le \beta (x_k - a_k)/(a_k - r_k) + 1, \forall k \in \overline{X},
\end{aligned}
$$

and constraints of admissible values of the variables x:

$$A\,x \le b.$$

In the formulation z, z_k, x denote variables, $\overline{X}$ is a set of criteria indexes, $\gamma > 1$ and $0 < \beta < 1$ are parameters of the achievement function applied in the above formulation.

Analysis is performed by each buyer in a number of iterations. In each iteration a given buyer assumes the reservation and aspiration points according to the reference point method. The computer-based system solves the above problem and calculates the respective variant, nondominated in the set D.

We have assumed that the reservation point of each buyer is not selected arbitrarily but is defined on the basis of the BATNA concept, similarly as in the assumptions of the procedures supporting cooperative decisions [Kruś 2002, 2004, 2008]. The BATNA concept (Best Alternative to Negotiated Agreement) is widely applied in negotiations [Fisher and Ury 1981; Raiffa 1982]. It means the best alternative a negotiating party can obtain if negotiations will not succeed. In our case, it relates to a product, which is already accessible on the market and can be compared to the variants of the product offered by the producer. We assume that each buyer is interested in a variant proposed by the producer if the variant is better than that defined by his BATNA. The BATNA concept is important for the calculation of the buyer satisfaction, proposed further in the paper.

For a reservation point given in this way each buyer assumes some number of different aspiration points. The system derives the respective nondominated variants, so the buyer can compare the variants according to his preferences. Finally, he is asked to indicate his preferred variant. The above actions are made independently by all the buyers with use of the system. The system stores information about the variants indicated by all the buyers, what completes the first stage.

The criteria of the producer include a profit obtained from the product variant offered on the market and a reputation among buyers accepting the variant offered. The profit equals sales revenues minus total expenses corresponding to the product variant. The profit criterion y_{profit} is calculated by:

$$y_{profit} = (p_e x_e - p_u x_u)\sum_{l \in L} v_l \ ,$$

where v_l is a binary variable indicating which buyer accepts the product variant offered. A simplifying assumption has been made that the revenues are in proportion to the variable x_e denoting costs covered by the buyer, and the producer's expenses are in proportion to the usefulness x_u, with coefficients p_e and p_u, respectively.

The reputation is defined as an aggregated measure of satisfaction levels of buyers accepting the variant proposed by the producer. The satisfaction level of a given buyer is calculated for the product variant offered by the producer when the buyer has already performed the multicriteria analysis, and has indicated the preferable nondominated variant. A buyer can not accept a given variant if the variant is dominated by the buyer's reservation point.

An interval scale has been assumed to measure the satisfaction level of each buyer. The scale has to be normalized with respect to different buyers and should be manipulation-free. The interval scales are constructed based on two uniquely defined points. The Celsius temperature scale defined by the temperature of ice thawing and the temperature of water boiling serves as an example. We have assumed that the satisfaction level of a buyer is measured, based on his reservation point (with a lower level $s_{lo} = 0$) and on the accessible variant preferred by him (with the upper level $s_{up} = 100$). Of course, an arbitrary variant may have assigned a satisfaction level lower than 0, or greater than 100. Discussions of different types of scales and their applicability to measuring can be found in [Torgerson 1958; Coombs, Dawes and Tversky 1970].

In our research, we also discussed other definitions of the scale and of different ways of calculating the satisfaction level. It seems natural to take the aspiration point chosen by a given buyer as a variant with the maximum satisfaction level equal to 100. In this case, the buyer can manipulate the distance of the aspiration point to the reference point. Increasing the distance, he could influence the producer's decisions, increasing his importance in comparison to other buyers.

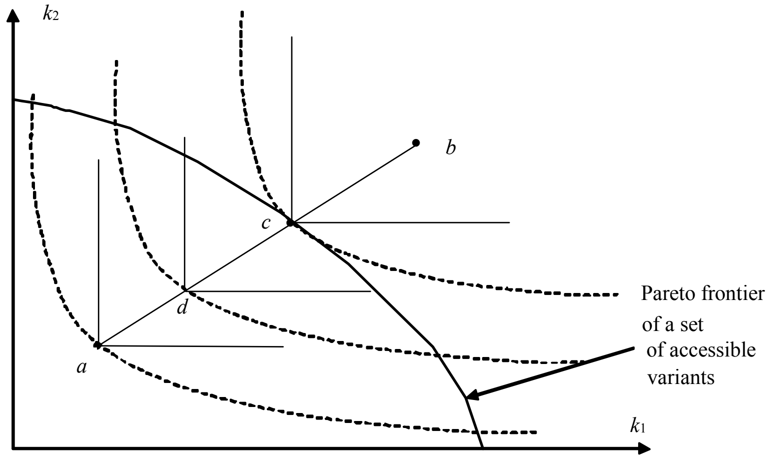

Figure 1. Indifference sets of a function measuring a buyer's satisfaction level

Figure 1 presents an example illustrating how the buyer satisfaction level is derived. Two maximized criteria are considered. The satisfaction level is defined by a scalar function defined in the space of the criteria. In a general case, it is a nonlinear utility function. Indifference sets of the function are presented by dashed lines in the space of criteria k_1, k_2. In the present version of the computer-based system, we assumed a specific variant of the function defined by the frontiers of the shifted positive cone drawn with thin continuous lines. In further research, other forms of the utility functions will be discussed including problems of its estimation and implementation in the system. The points presented in Figure 1 denote: *a* – reservation point, *b* – aspiration point indicated by a buyer after his multicriteria analysis, *c* – chosen preferred accessible variant. According to the scale assumed, all the points on the continuous lines point *d* have the satisfaction level equal to

$$s = (s_{up} - s_{lo})\cdot|a, d| \,/\, |a, c|.$$

Let the producer offer a variant characterized by a point in the space of criteria k_1, k_2. We can construct the respective line representing the indifference set, derive the point *d* and calculate the satisfaction level according to the above formula.

The maximized reputation criterion is calculated as:

$$y_{reputation} = \sum_{l \in L} s_l ,$$

where $y_{reputation}$ denotes a value of the reputation, L is the set of all buyers and s_l is the satisfaction level of the buyer $l \in L$.

The producer performs multicriteria analysis in the space of his own criteria, assuming reservation and aspiration points, respectively. The computer--based system derives and stores respective nondominated solutions. The producer can review the solutions generated and select the preferred one. The system derives the nondominated solution by solving the following optimization problem for the given reservation and aspiration points:

$$\max z + \varepsilon \sum_{i \in \bar{Y}} z_i$$

subject to the constraints due to the reference point method:

$$\begin{aligned}
& z \le z_i , \ \ \forall i \in \overline{Y}, \\
& z_i \le \gamma(y_i - r_i)/(a_i - r_i), \ \ \forall i \in \overline{Y}, \\
& z_i \le (y_i - r_i)/(a_i - r_i), \ \ \forall i \in \overline{Y}, \\
& z_i \le \beta(y_i - a_i)/(a_i - r_i) + 1, \ \ \forall i \in \overline{Y},
\end{aligned}$$

to the reputation criterion

$$y_{reputation} \le \sum_{l \in L} s_l ,$$

$$\begin{aligned}
& s_l \le (s_{up} - s_{lo}) f_{l_k} , \forall l \in L^+, k \in \overline{X}, \\
& f_{l_k} \le (x_k - \hat{r}_{l_k})/(\hat{x}_{l_k} - \hat{r}_{l_k}) + M(1 - v_l), \forall l \in L^+, k \in \overline{X}, \\
& (x_k - \hat{r}_{l_k})/(\hat{x}_{l_k} - \hat{r}_{l_k}) \ge -M(1 - v_l), \forall l \in L^+, k \in \overline{X}, \\
& s_l \ge \varepsilon^* - M(1 - v_l), \forall l \in L^+, \\
& v_l \le 0, \ \ \forall l \in L^-, \\
& f_{l_k} \le s_d + M v_l , \forall l \in L \ ,
\end{aligned}$$

to the profit criterion

$$y_{profit} \le \sum_{l \in L} w_l ,$$

$$w_l \le Mv_l, \forall l \in L,$$
$$w_l \le p_e x_e - p_u x_u + M(1 - v_l), \forall l \in L,$$
$$p_e x_e - p_u x_u - w_l + Mv_l \le M, \forall l \in L,$$

to the model constraints of the admissible variants of the product:

$$A\, x \le b.$$

In the above relations, w_l, v_l, f_{l_k} denote additional variables, $\bar{Y}$ is a set of indexes of the producer criteria, L^+ is a set of buyers for which there exists a product variant better than that defined by the reservation point, L^- the set of buyers for which such a variant does not exists. M is a large positive number, $\hat{x}_{l_k}, \hat{r}_{l_k}$ denote the components of the accessible solution selected by the buyer l, and of his reservation point, respectively. Not all buyers from the set L^+ can be interested in the variant offered by the producer. It has been assumed that a buyer is interested in the variant of the product if the level of his satisfaction is at least ε^* value greater than the level of his reservation point.

2. Analysis of some results

Computing experiments and a series of sessions have been performed with use of the system. In the first experiments, the system was intensively tested. Next, interactive sessions were carried on by a producer and by several buyers. It was interesting to check how preferences of buyers impacted the decisions of the producer maximizing his profit but also attaching an importance to the reputation of his product. On the other hand, the producer's decisions impacted the satisfaction levels of particular buyers. In one of the experiments presented here the users of the system played the roles of a producer and of 8 buyers.

Figure 2 illustrates the first stage of the algorithm. It presents the results of multicriteria analysis performed by one of the buyers. The selected reservation and aspiration points as well as the respective nondominated solutions are presented in the space of buyer's criteria: e (minimized cost), and u (maximized usefulness).

All the nondominated points shown in the figure have been derived by the reference point method, but only selected reservation points, aspiration points and the corresponding nondominated points are presented.

Each buyer, assuming different reservation and aspiration points, can derive a representation of the set of Pareto optimal variants. He is asked to select the preferred variant and the respective aspiration point.

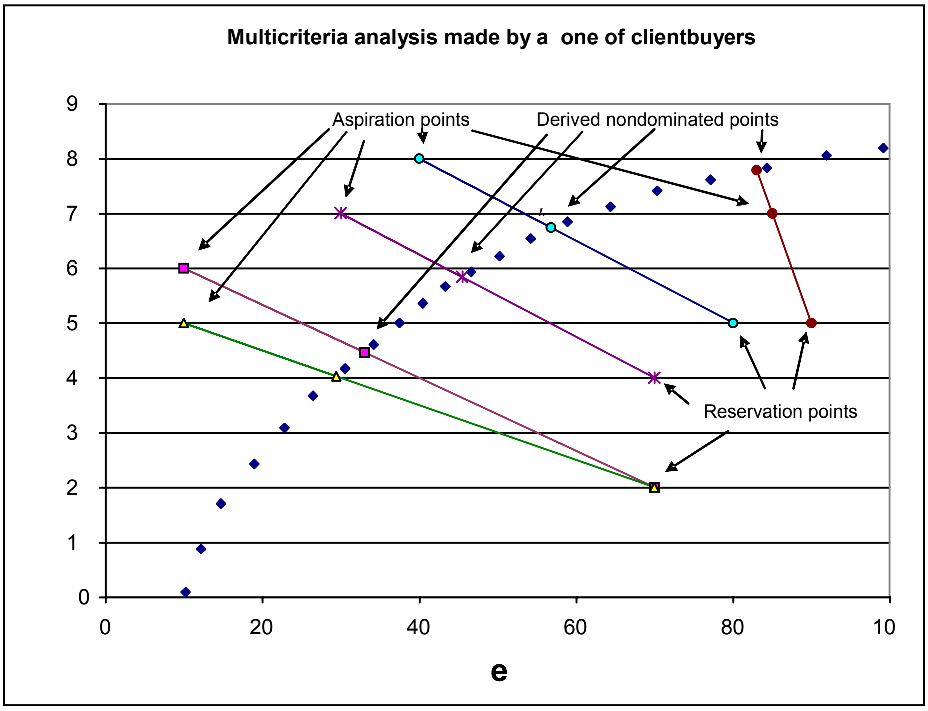

Figure 2. Illustration of multicriteria analysis made by one of the buyers.

The producer can start analysis when all the buyers have already selected their preferred variants. It is performed in the second stage of the algorithm. In the experiment, different preferences of the buyers have been assumed. They are represented by different reservation and aspiration points, and different preferred variants selected by each of the eighth buyers, as shown in Figure 3. Note that buyer 1 prefers a low cost (*e* – economic attributes) and a low usefulness (*u* criterion) of the product variant, while buyer 8 prefers a variant characterized by a high usefulness and a high cost.

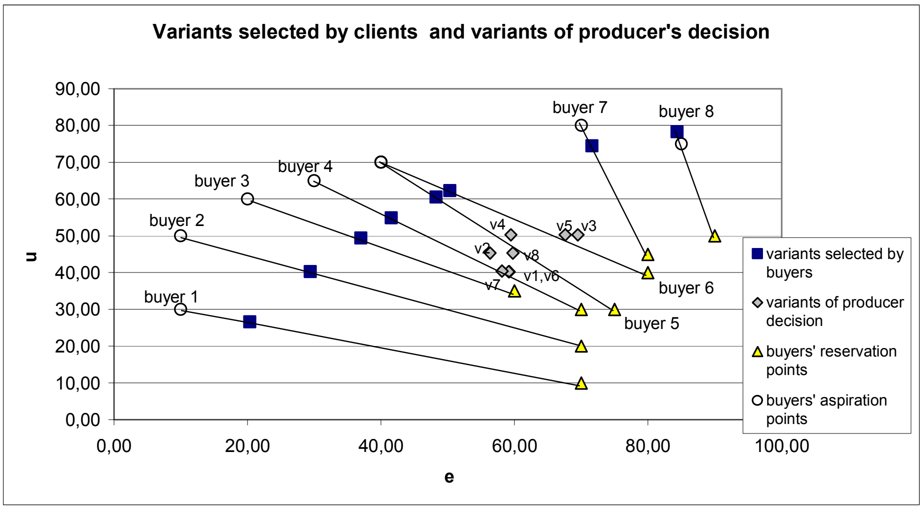

Figure 3. Results of a session with eight buyers

The producer has no access to information related to the particular buyers, their analysis, decisions or preferences. The computer-based system derives the values of the producer's criteria: the reputation of the variant among the buyers, and the profit, depending on the product variant considered by the producer to be offered to the buyers. Multicriteria analysis is performed by a producer using the reference point method analogously as in the case of buyers. The producer can make a representation of the set of Pareto solutions, can compare different nondominated variants and can select the preferred variant. Several nondominated variants derived by the system for different reservation and aspiration points in the space of producer's criteria are presented in Figure 4.

It was interesting to check the effects of a producer's preferences on his choice of the variant offered to buyers; which buyers accept the variant; what is their satisfaction levels and the resulting reputation criterion. A simulation was made assuming eighth different preferences of the producer resulting in eighth different variants of the producer's decisions.

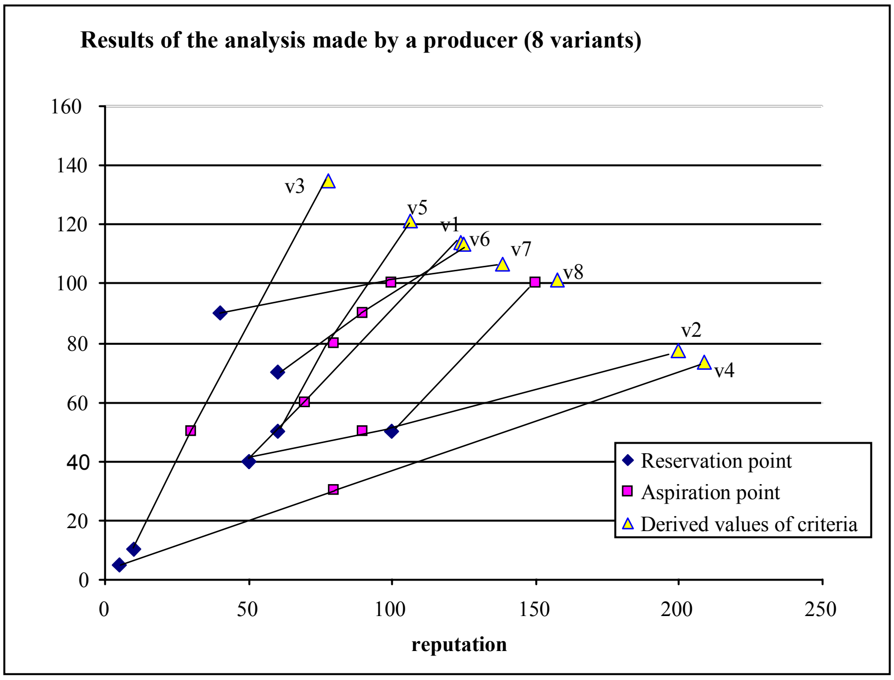

Figure 4. Different variants of producer's decision in the case of eight buyers

The variants differing with respect to reservation points, aspiration points and the respective nondominated solutions are shown in Table 1. One can see a variant with low reputation and a relatively high profit (variant 3), and on the other hand a variant with high reputation and a small profit (variant 4). The decision variables describing each variant are presented, i.e. *e* – economic attributes (cost criterion for buyers) and *u* – the usefulness. All the variants analysed are presented also in Figure 3, in the space of buyers' criteria: *e* and *u*. Therefore one can compare each variant of the producer's decision with the variants preferred by the buyers. In the last column of the table one can find the number of buyers accepting the given variant of the product.

Table 1

Reservation and aspiration points assumed by a producer and respective nondominated solutions (criteria and decision variables)

Variant	Analysis made by a producer				Decision variables		
	Criterion	Reservation point	Aspiration point	Derived values of criteria	e	u	Number of satisfied buyers
1	reputation profit	50 40	70 60	123,97 113,98	59,22	40,22	6
2	reputation profit	50 40	90 50	199,59 77,42	56,36	45,29	7
3	reputation profit	10 10	30 50	77,85 134,54	69,5	50,28	7
4	reputation profit	5 5	80 30	208,95 73,32	59,45	50,28	8
5	reputation profit	60 50	80 80	106,55 121,24	67,60	50,28	7
6	reputation profit	60 70	90 90	125,08 113,42	59,13	40,22	6
7	reputation profit	40 90	100 100	138,74 106,45	58,13	40,39	6
8	reputation profit	100 50	150 100	157,51 101,33	59,77	45,29	7

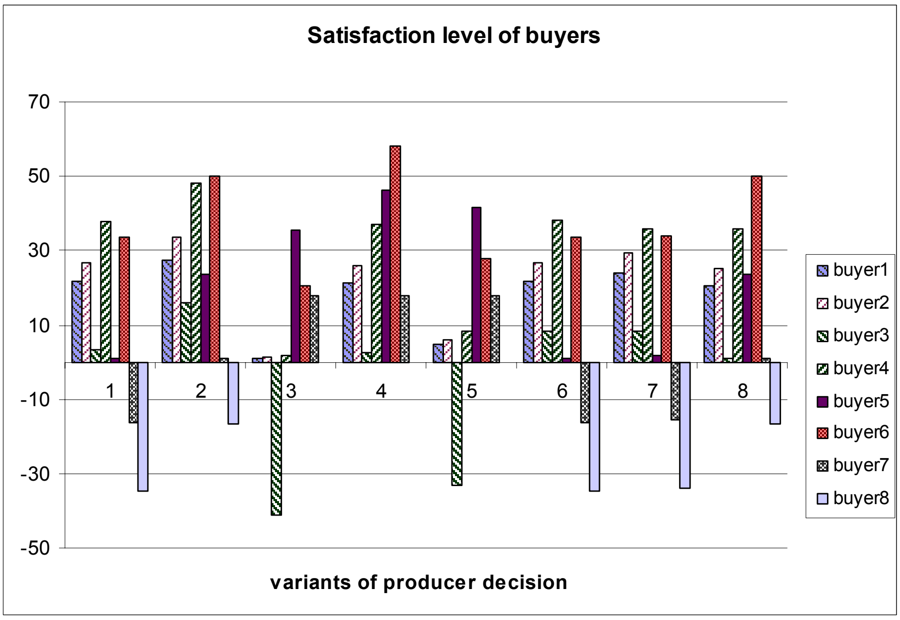

Figure 5. Buyers' satisfaction levels dependent on the producer's decision

Figure 5 shows satisfaction levels of buyers depending on the variants of product offered by the producer. Negative values of the level mean that the respective variant is not accepted by the respective buyers. Variants 1, 6, 7 are not accepted by buyers 7 and 8. Variant 3 and 5 are not accepted by buyer 3. Variant 3 gives the greatest profit to the producer in the set of variants analysed here. The greatest number of buyers is interested in variant 4. This variant has the greatest reputation among buyers but it gives the lowest profit to the producer.

Conclusions

A mathematical model describing the producer and buyers problem has been proposed. It includes formulations of optimization tasks solved during the multicriteria analysis conducted by the buyers and by the producer. The optimization tasks have been implemented in a specially designed multiagent computer-based system.

An original proposal for the derivation of satisfaction levels of individual buyers is presented. On this basis, the reputation can be calculated. It is one of the producer's criteria. It reconciles the producer's and buyers' interests. The buyer's satisfaction level is derived with use of the BATNA concept and with use of an assumed form of the buyer's utility function. In further research, different ways of the derivation will be analyzed. In particular, different forms of the utility function and interactive procedures for scaling the function with use of information obtained from buyers will be discussed.

The multiagent computer-based system enables buyers and producers to perform multicriteria analysis in two stages. An experiment with human users of the system, playing the roles of a producer and 8 buyers, illustrates the method. It is shown how the variant proposed to the buyers depends on the producer's preferences and how it is seen by the buyers.

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