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ON THE CHOICE OF METHOD IN MULTI-CRITERIA DECISION AIDING PROCESS CONCERNING EUROPEAN PROJECTS

Abstract

In this paper the problem of selecting the most appropriate multi-criteria decision aiding method for a particular application is considered. It is illustrated by a real-life example concerning applications for project co-financing by the European Union.

Making a proper decision on which method to choose is difficult because of the great diversity of MCDA techniques proposed so far within the literature. Thus, the systematic analysis of their assumptions and properties is required.

The paper presents the main strengths and weaknesses of particular decision aiding tools applicable to the problem of ordering European projects as well as chosen procedures aiming at facilitating the process of selecting an appropriate one.

Moreover, an extension of EXPROM II by stochastic dominance rules is proposed.

Keywords

Multi-criteria decision aiding methods, model choice algorithm, model selection process, EXPROM II with stochastic dominances.

Introduction

European regional policy is nowadays one of the most vital factors in strengthening the socio-economic development of Poland and other European Union countries, especially those that entered the EU in 2004 and 2007, whose economies have lagged far behind the economies of the old Member States of EU-15 and whose needs in the areas of environment, infrastructure, research and innovation, industry, services and SMEs are truly significant.

Regional policy helps to reduce disparities between countries, increase the regions' competitiveness and attractiveness, improve the employment prospects and support cross-border co-operation through financing specific projects for regions, towns and their inhabitants. In the previous programming period 2000-2006 over 233 billion EUR was earmarked for all regional

instruments for the 15 old Member States. Moreover, around 24 billion EUR was allocated for the 10 new Member States for the years 2004-2006, not to mention 22 billion EUR granted for pre-accession aid. In the present programming period 2007-2013 cohesion policy will benefit from total allocation of about 347 billion EUR, which represents nearly 36% of the entire Union's budget.

Because of the enormous amount of money devoted to the structural aid it is crucially important to allocate the means in the most effective way possible. And that depends among other things on the proper choice of projects that are going to be co-financed. In order to help the decision-makers in this challenging and difficult task, multi-criteria decision aiding techniques, which refers to making decision in the presence of multiple, usually conflicting criteria, should be applied as evaluation of the European projects requires taking into account many diverse aspects: economic, financial, ecological, technical, technological, social and legal.

Since many different MCDA methods are available and there are specific advantages, disadvantages and limitations of each of them, a detailed analysis must be carried out in order to choose an appropriate technique for a particular decision-making problem. Otherwise the solution may be misleading or unsatisfactory, useful methods may be rejected incurring losses in valuable time, energy and money and, last but not least, the potential users may be discouraged from applying MCDA methods to real-world problems [Gilliams et al., 2005].

The main aim of this paper is to examine and compare the applicability of various MCDA methods to the problem of ordering projects applying for co-financing from the EU. Decision concerning the usefulness of the selected methods will be taken on the basis of the analysis of the decision-making problem and the decision-making process as well as on the basis of the examination of the information constraints and the profile of the decision-makers.

Selecting the correct MCDA method is in itself an MCDA problem as there is a wide variety of criteria upon which the choice should be based. Therefore a number of procedures to assist both the analyst and the decision-maker to choose a suitable method for a specific decision problem has been presented in literature [Al-Shemmeri et al., 1997]. These procedures are helpful inasmuch as they can confirm or deny the results of the qualitative analysis mentioned above. While the confirmation of the outcome of the qualitative analysis indicates that the process of method selection reaches the end as the choice that has been made is appropriate, the denial implies rethinking of the problem.

In this paper the model choice algorithm of Gershon (see [Gershon, 1981]) and the model selection process of Teclé (see [Teclé, 1988]) will be applied in order to support the qualitative analysis.

1. Comparative analysis of selected MCDA methods

The choice of a correct tool to solve the decision-making problem depends on the type of the problem as well as on the goals of the decision-makers and the desired properties of the solution obtained. Sometimes ‘the simpler the method, the better’, but complicated decision problems may also require complex methods.

In the case of the European projects the approach based on the multi-attribute utility theory (see [Keeney, Raiffa, 1976]) may be implemented. Methods falling into this category assume that there exist global utility function to represent the decision-maker’s preferences and it can be built through aggregating variants’ partial utilities (according to each criterion). But the reduction of a multidimensional evaluation to a one-dimensional one via the formulation of global utility function is possible only when certain rigorous conditions* are met. Besides, it may lead to the complete compensation between criteria – the situation in which the variant evaluated low against one or even more criteria is ranked high because it has achieved high grades against the remaining criteria. In this approach a not very realistic assumption is accepted that the decision-maker’s preferences are given and fixed, i.e. they are expressed clearly and result in good ordering alternatives against criteria – the decision-maker is able to indicate, without any hesitation, even the smallest differences in utilities and confidently, consequently and precisely assign the scores to variants considered. In addition, determining an analytical form of the global utility function is usually very difficult and sometimes even infeasible – it happens frequently that the decision-maker is not able to provide information essential to build this function [Trzaskalik et al., 1998].

An interesting alternative is the approach based on the outranking relation and on the fundamental partial comparability axiom (see [Roy, 1990]) in which incomparability plays a key role [Martel, 1998]. The basic idea of this approach is as follows: alternative A outranks B if on a great part of the criteria A performs at least as good as B (concordance condition), while its worse performance is still acceptable on the other criteria (non-discordance condition). Indifference thresholds and preference thresholds are introduced in order to build outranking relations that represent decision-makers’ preferences and constitute partial relations of the global preferences. In this kind of approach there is place for incomparability, explained e.g. by the lack of sufficient

*For instance, the necessary and sufficient condition of applying an additive form of the utility function in the situation when the evaluations are deterministic is mutual preferential independence of the criteria. If the evaluations have the form of probability distributions the above mentioned condition is not sufficient – in that case the utility independence condition must be satisfied [Trzaskalik et al., 1998].

information to define preferential situation [Trzaskalik et al., 1998]. The procedures exploited according to this approach – among which the ELECTRE and PROMETHEE methods stand out – are usually less demanding for their users at the informational level and result in more balanced recommendations than those belonging to the first approach of a single criterion synthesis [Martel, 1998]. Since their assumptions correspond to reality they can definitely improve the procedure of appraising and selecting projects applying for co-financing from the European Union.

Although expected utility models and outranking relation models used to be often treated as competitors, it is possible to benefit from both approaches in the situation when the performances of various alternatives are evaluated in a probabilistic way (as it is in the case of the European projects because the number of experts participating in evaluation is greater than 1). Namely, stochastic dominance rules can be employed to establish preferences with respect to each criterion and the criteria aggregation method based on the outranking relation procedure can be used to obtain global preference [Martel, Zaráś, 1995]. Moreover, the concept of pseudo-criteria can be employed to distinguish situations of strict preference, weak preferences and indifference [Nowak, 2004]. As a matter of fact, applying this combined approach seems to be an appropriate solution in the case of appraisal of European projects.

In Table 1 the main advantages and drawbacks of various MCDA techniques in the context of the European projects selection are presented.

Table 1

Strengths and weaknesses of selected MCDA methods

No.	Method	Characteristics
1	Arithmetic mean of weighted sums of the scores of all experts participating in the Panel (see [Ministerstwo Gospodarki i Pracy, 2004])	The possibility of ranking the European projects with help of arithmetic mean of the weighted scores given by the members of the Panel of Experts – the procedure that was used in Poland in the period 2004-2006 – seems somewhat illusory in view of uncertainty, inaccuracy, instability and indefiniteness concerning data, evaluations and preferences characteristic for decision-making problems. Especially in the situation such as discussed here, when a large number of different stakeholders (e.g. various decision-makers, experts in the appropriate fields, consulting companies responsible for preparation of the projects, political parties, civic organisations, inhabitants and interest groups affected by the decision) with conflicting preferences are involved and costs and benefits of the alternatives are difficult to assess. Furthermore, this method – as others based on multi-attribute utility theory – allows for complete compensation between criteria, what can be even dangerous in the case of investment projects as poor performance on one criterion (e.g. technical feasibility) can be easily counterbalanced with a good one on another one. Moreover, it is

Table 1 contd.

No.	Method	Characteristics
		also imperfect as far as group decision-making is concerned since it does not take into account the distributions of the evaluations. On the other hand, this method is completely comprehensible for the potential users and because of that it is easy to implement. Besides, it requires neither a skilled analyst to operate the system nor specialized software
2	ELECTRE III with stochastic dominances (see [Nowak, 2004])	<p>This method requires from its users determination of indifference, preference and veto threshold as well as the weights of criteria. The thresholds values are easily interpretable and allows for better reflection of the decision-maker's preferences but their determination is time-consuming. Thanks to the veto thresholds the technique is partially compensatory (really bad score on one criterion cannot be compensated by a good score on another). Besides, it takes into consideration distributions of the evaluations thanks to applying the stochastic dominance rules. The final rankings are not transitive and the results are partial orders.</p> <p>The technique is complex and mathematically complicated – hence an analyst as well as specialized software are required. Because not every step of this method is understandable for the decision-makers it may be difficult to persuade them to apply it.</p> <p>Another drawbacks of this method are as follows:</p> <ul style="list-style-type: none"> – on one level a few projects can be classified, so in some cases we are not sure which projects should be co-financed, – the form of the final solution – final ranking without any points may be unconvincing for the potential users, – the possibility of incomparability occurrence – no potential beneficiary will accept the explanation that his/her application was rejected because it was incomparable with others
3	PROMETHEE II with stochastic dominances (see [Nowak, 2005])	<p>This method allows to discard the last three shortcomings of the ELECTRE III method with stochastic dominances as a complete pre-order of the projects is proposed to the decision-maker and points are assigned to the alternatives.</p> <p>The idea of calculation of the net flow for each project connected with this method is much preferable to the idea of distillation procedure connected with ELECTRE III from the point of view of participants of the decision-making process. They consider it as more user-friendly: easier to understand and to implement</p>
4	Modified BIPOLAR* with stochastic dominances (see [Górecka, 2008])	<p>Rankings obtained with help of both ELECTRE III and PROMETHEE II methods do not allow for stating whether highly ranked projects are really good or just the best of the weak ones. This problem can be solved by applying BIPOLAR method with modifications introduced by the author of this paper which enable ranking and sorting projects as well as determining their quality by taking into account what is good and undesirable from the decision-maker's point of view in the decision-making problem. At the same time, the problem of the projects' incomparability is eliminated</p>

Table 1 contd.

No.	Method	Characteristics
5	EXPROM II with stochastic dominances (see Appendix)	This method has similar strengths and weaknesses as PROMETHEE II with stochastic dominances but is based on the notion of ideal and anti-ideal solutions and enables the decision-maker to rank variants on a cardinal scale
6	SMART (see [Edwards, 1977; Edwards, Barron, 1994])	This method is a simple way to implement the multi-attribute utility theory by using the weighted linear averages. Its advantages and disadvantages are similar to those of arithmetic mean of weighted sums
7	TOPSIS (see [Hwang, Yoon, 1981])	According to this method the most preferred alternative should have a profile which is nearest to the ideal solution and farthest from the anti-ideal solution. It is slightly more complicated than, for example, SMART and potential users may not be completely aware of its consequences
8	AHP (see [Saaty, 1980])	Applying this method to European projects' selection is impossible as it requires from its users making comparisons between all the alternatives, and it happens that there are over 100 projects in the competition – in such situations pair-wise comparisons are infeasible

* The original version of BIPOLAR method was proposed by Konarzewska-Gubała. Detailed description of this technique is presented in [Konarzewska-Gubała, 1991].

To sum up, the following characteristics of the decision-making problem analysed and the following expectations of the decision-makers involved in the realisation of the EU regional policy should be taken into consideration in the process of selecting the most appropriate multi-criteria decision aiding method for the problem of choosing project applying for co-financing by the EU:

- the decision-making problem should be formulated as a problem of ordering a finite number of alternatives – it is indispensable to each beneficiary to be classified in the ranking and to know its own result,
- the problem is a group decision-making problem – experts engaged in the projects' appraisal individually and independently evaluate a finite number of competing projects and it is required to incorporate diverse individual views into a blended final decision,
- there should be a possibility to employ both quantitative and qualitative criteria,
- decision-makers are able to present the information about their preferences but they do not have much time for the interaction and cooperation with the analyst,
- participants of the decision-making process have very diverse educational background and their knowledge about multi-criteria decision aiding methods is usually limited,

- the decision aiding technique should not be too complicated so as to enable decision-makers to explain to the applicants how it works and elucidate the reasons of their projects rejection,
- the decision-making method should not be too simple so as to limit the possibilities of manipulating the results,
- it should be taken into account that experts appraising many projects during several days may not be consistent in their evaluations, especially in view of uncertainty and inaccuracy characteristic for the decision-making problem discussed,
- the possibility of a complete compensation occurrence should be removed – in the case of some criteria it may be hazardous and in the case of others, projects should fulfil the so-called “minimal quality”,
- there is no room for the incomparability of the alternatives – ranking should be complete as the explanation that the project has not been selected for co-financing because of the incomparability with the others will not be accepted by the applicants,
- the possibility that a few projects will be classified on the same place in the ranking should be limited as it may create problems with dividing the funds,
- the final solution should take the form in which the scoring points occur, otherwise it may be unconvincing for the applicants,
- it is desired that the decision aiding method enables to determine whether the highly ranked projects are really good or just better than the weak ones.

Taking into account all the above-mentioned information on the properties of the decision-making problem analysed, its participants and the selected MCDA techniques, the most suitable method to aid the decision-making process seem to be one based on the outranking relation combined with stochastic dominance rules, namely PROMETHEE II, EXPROM II or modified BIPOLAR technique. On the one hand, PROMETHEE II is the simplest and the most user-friendly of these three but, on the other hand, it allows only to determine the relative quality of the projects. This drawback does not occur in the modified BIPOLAR method from which the problem of projects incomparability has been also removed but this method is unfortunately more complicated than PROMETHEE II and it may be hard to explain to people without mathematical background. EXPROM II, in turn, enables to create cardinal rankings of the projects and it is only a bit more complex than PROMETHEE II – accordingly it seems that this is the method that should be recommended.

2. Model choice algorithm of Gershon

Gershon’s model contains 27 criteria as a basis of comparison between different MCDA methods. They are divided into 4 groups:

- criteria 1-5: compulsory binary criteria which remove candidate techniques from further consideration if they are not fulfilled, rated as 1 (if selected) or 0 (if not selected),
- criteria 6-12: non-obligatory binary criteria, rated as 1 (if satisfied) or 0 (if not satisfied),
- criteria 13-19: technique dependent criteria rated on a 0-10 subjective scale,
- criteria 20-27: application dependent criteria rated on a 0-10 subjective scale [Al-Shemmeri et al., 1997].

The procedure involves the selection of a subset of the criteria that are relevant to the problem, assignment of weights to the criteria in the subset and appraisal of the candidate methods with respect to predetermined criteria.

The decision situation was examined in the context of the problem of choosing European projects. Ten criteria were found to be irrelevant to the situation. Criteria 3 and 4 (continuous sets and dynamic problems) are not applicable, as the problem involves an explicit list of predefined alternatives and the input data is not changing. In turn, criterion 5 was eliminated because treating the problem as a stochastic one is neither required nor indispensable (although desired). Criteria 12, 18 and 20-23 were also eliminated because either they are meaningless for ordering the European projects or refer to conditions not encountered in this problem.

Compromise programming was applied* to rank the 7 methods listed in Table 2 and select the one that is closest to the ideal solution determined as follows: [1,1,1,1,1,10,10,10,10,10,10,10,10,10,10]. The distance metric to minimize was defined in the following way:

$$L_1 = \sum_{k=1}^n w_k \cdot \frac{f_k^* - f_k(a_i)}{f_k^* - f_k^{\min}},$$

where w_k is the weight, f_k^* is the optimal value of the criterion k , f_k^{\min} is the worst value attainable for criterion k and $f_k(a_i)$ is the evaluation of the i th technique with respect to the i th criterion.

As a result of detailed analysis of the selected MCDA techniques properties against chosen criteria and applying the model choice algorithm** it turned out that the method closest to the ideal solution is EXPROM II combined with stochastic dominances with a distance value of 9,8 as it is shown in Table 2. On the opposite end of the ranking ELECTRE III method with stochastic dominance rules was placed.

* It is possible to use other multi-criteria methods, e.g. ones based on the outranking relation such as PROMETHEE.

** The opinions expressed herein are those of the author and do not necessarily represent those of the decision-makers involved in the implementation of the EU regional policy.

Table 2

Model choice algorithm and evaluations of chosen multi-criteria methods

No.	Criterion	Eliminated	Weight	Weighted sum	ELECTRE III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
1	Handle qualitative criteria		x	1	1	1	1	1	1	1
2	Handle discrete sets		x	1	1	1	1	1	1	1
3	Handle continuous sets	x	x	x	x	x	x	x	x	x
4	Handle dynamic problems	x	x	x	x	x	x	x	x	x
5	Handle stochastic problems	x	x	x	x	x	x	x	x	x
6	Comparison to goal point		1	0	0	0	1	1	0	1
7	Comparison to aspiration level		1	1	0	0	0	0	1	0
8	Direct comparison		1	0	1	1	1	0	0	0
9	Strongly efficient solution	x	x	x	x	x	x	x	x	x
10	Complete ranking		7	1	1	1	1	1	1	1
11	Cardinal ranking		5	1	0	0	1	1	1	1
12	Ability to handle integer variables	x	x	x	x	x	x	x	x	x
13	Computer time required		1	10	8	9	9	9	10	10
14	Implementation time required		1	10	2	4	4	4	9	9

Table 2 contd.

No.	Criterion	Eliminated	Weight	Weighted sum	ELECTRE III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
15	Interaction time required		3	10	6	6	6	2	10	10
16	Decision maker's awareness		2	10	2	3	3	3	8	6
17	Consistency of results		4	10	8	10	10	10	10	10
18	Robustness of results	x	x	x	x	x	x	x	x	x
19	Handle group decision maker		4	4	10	10	10	10	4	4
20	Number of objectives	x	x	x	x	x	x	x	x	x
21	Number of systems	x	x	x	x	x	x	x	x	x
22	Number of constraints	x	x	x	x	x	x	x	x	x
23	Number of variables	x	x	x	x	x	x	x	x	x
24	Decision maker's level of knowledge		3	10	6	6	6	2	10	10
25	Time available for interaction		2	10	8	8	8	6	10	10
26	Desire for interaction		3	10	7	7	7	5	10	10
27	Confidence in preference structure		7	6	9	10	10	10	5	5
Distance metric				11.6	29.2	15.8	9.8	16	13.625	14.125

Source: Own calculation.

3. Model selection process of Tecle

This method consists of identifying a set of feasible MCDA techniques and evaluating them with respect to 49 criteria which are divided into 4 sets:

- criteria 1-13: problem related criteria,
- criteria 14-20: decision-maker or analyst related criteria,
- criteria 21-40: technique related criteria,
- criteria 41-49: solution related criteria.

They are presented in Table 3.

The main steps of the procedure are as follows:

- the desired objectives to be satisfied by the MCDA techniques are determined,
- the evaluation criteria relating methods' capabilities to the objectives are chosen,
- the MCDA techniques available for achieving the aims defined in the first step are selected,
- the methods' capabilities or the levels of performances of the techniques with respect to the successive evaluation criteria are determined according to the opinions and beliefs of the user,
- an evaluation matrix is constructed, whose elements represent the capabilities of competing techniques in terms of the selected criteria,
- the performances of the alternative MCDA methods specified in the third step are analysed [Al-Shemmeri et al., 1997].

Seven MCDA methods (the same as in the previous part of the paper) have been examined and appraised* for their performance in solving a multi-criteria European projects problem and only 23 out of the 49 criteria have been utilized in order to do that. The subset of the criteria and techniques which were selected for considered problem are presented in Table 3.

As far as the weighting coefficients are considered every criterion in each group was assigned a weight relative to its importance in that group as perceived by the user. It was done with help of 'resistance to change' grid proposed by Hinkle (see [Rogers, Bruen, 1998]). The calculated weights are presented in Table 3.

* The opinions expressed herein are those of the author and do not necessarily represent those of the decision-makers involved in the implementation of the EU regional policy.

Table 3 contd.

No.	Criterion	Selected	Weight	Weighted sum	ELECTR E III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
6	Applicability to real-world problems	x	19	5	7	10	10	10	4	4
7	Plausibility of algorithm									
8	Ease of use	x	14	10	1	3	2	2	9	8
9	Flexibility									
10	Instrumentality to achieve a solution									
11	Ease of coding	x	7	10	4	6	6	6	9	9
12	Computational burden	x	2	10	7	8	8	8	10	10
13	Interaction time required	x	14	10	6	6	6	2	10	10
14	CPU time required	x	2	10	8	9	9	9	10	10
15	Rapidity of convergence									
16	Parameters specifications required (like weights, trade-offs)	x	14	10	7	7	7	4	10	10
17	Amount of total information required									
18	Is algorithm an ad hoc procedure									
19	Is the technique interactive									
20	Need for specialized software	x	9	10	1	2	2	2	8	8

Table 3 contd.

No.	Criterion	Selected	Weight	Weighted sum	ELECTR E III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
Characteristics Describing the Solution Obtained										
1	Type of nondominance of solution (rank or numerical)	x	15	10	4	7	10	10	10	10
2	Strength of efficient solution									
3	Number of solutions obtained at each iteration									
4	Complete ranking	x	15	10	5	10	10	10	10	10
5	Cardinal ranking	x	5	10	0	0	10	10	10	10
6	Consistency of result	x	15	10	8	10	10	10	10	10
7	Robustness of result									
8	Confidence in the solution obtained	x	25	6	9	10	10	10	5	5
9	Usefulness of result to DM	x	25	7	4	7	10	10	7	7
Distance metric				95,33	197,94	119,01	92,85	123,83	110,90	112,45

Source: Own calculation.

After establishing the weights for the criteria the evaluation process was continued:

- criteria describing the problem were appraised using “yes” or “no” response, rated as 1 (if satisfied) or 0 (if not satisfied),
- criteria describing the decision-maker or analyst as well as the technique related criteria and solution related criteria were evaluated using a 0-10 subjective scale [Al-Shemmeri et al., 1997].

The evaluation matrix was analysed with help of compromise programming resulting in construction of the ranking of the methods according to ascending order of the values of the distance metric. The compromise programming was utilized as in Gershon’s algorithm instead of composite programming which was originally used by Tecle in his model.

The analysis carried out in this part of the paper confirmed that EXPROM II method together with stochastic dominance rules is the most suitable method for ordering projects applying for co-financing from the European Union. On the last place ELECTRE III method with stochastic dominances was classified, as it was in the former order.

Conclusions

Two algorithms were implemented to aid the process of selecting a suitable technique for ranking projects applying for co-financing from the European Union funds and in both cases EXPROM II with stochastic dominances was found to be the most preferred technique, which confirmed the results of the analysis carried out before applying these two algorithms and led to the conclusion that this method is appropriate for the considered decision-making problem. Consequently, reanalysis turned out to be unnecessary.

It is worth mentioning that on the second place the arithmetic mean of weighted sums was classified, mainly thanks to its simplicity.

In turn, the lowest ranked technique in both rankings was ELECTRE III with stochastic dominances. The properties of this method, especially its complexity and the form of solution obtained, make it practically useless when dealing with the problem of ordering European projects.

On the one hand, the huge diversity of MCDA methods is really helpful and may be seen as an advantage, on the other – it is rather a weakness as the selection of the right technique for a specific problem is becoming extremely difficult. The models described in the paper can be applied to any multi-criteria decision-making problem to support the process of selecting the appropriate MCDA technique.

The approach presented in the article, based on the qualitative analysis and fulfilling auxiliary function algorithms, could lead to the preparation of the catalogue of problem types and methods best suited to solve them, which could serve as a general guide for participants of the decision-making processes.

Appendix

APPLICATION OF THE EXPROM METHOD WITH STOCHASTIC DOMINANCES TO THE EUROPEAN PROJECTS' SELECTION

EXPROM is a modification and extension of the PROMETHEE method* proposed by Diakoulaki and Koumoutsos [1991]. It is based on the notion of ideal and anti-ideal solutions and enables the decision-maker to rank variants on a cardinal scale. Assuming that all criteria are to be maximized, the ideal and anti-ideal solutions' values are defined as follows:

- ideal variant: $f_k(a^*) = \max_{a_i \in A} f_k(a_i)$,
- anti-ideal variant: $f_k(a_*) = \min_{a_i \in A} f_k(a_i)$ **

where $A = \{a_1, a_2, \dots, a_m\}$ is finite set of m variants and $F = \{f_1, f_2, \dots, f_n\}$ is set of n criteria examined.

After introducing stochastic dominance rules to the EXPROM method the procedure of ordering projects consists of the following steps***:

1. Identification of stochastic dominances for all pairs of projects with respect to all criteria****. Because all criteria are measured on ordinal scale the ordinal stochastic dominance approach proposed in [Spector et al., 1996] is applied:

Definition 1: Ordinal First-Degree Stochastic Dominance (OFSD):

$$X_k^i \text{ OFSD } X_k^j \text{ if and only if } \sum_{l=1}^s p_{kl}^i \leq \sum_{l=1}^s p_{kl}^j \text{ for all } s = 1, \dots, z,$$

* The idea of the PROMETHEE methodology is presented in [Brans, Vincke, 1985] and a description of PROMETHEE techniques can be found in [Brans et al., 1986].

** The values can be also defined independently from the examined variants, representing – in the case of the ideal solution – some realistic goals and in the case of the anti-ideal solution – the situation that should be avoided.

*** The PROMETHEE method with stochastic dominances was proposed by Nowak. A detailed description of this method is presented in [Nowak, 2005].

**** According to the results of experiments presented in [Kahneman, Tversky, 1979] it is assumed that the decision-maker(s) is (are) risk-averse.

where:

X_k^i – distribution of the evaluations of project a_i with respect to criterion f_k ,
 p_{kl} – probability of obtaining given evaluation by the project in the case of criterion f_k .

Definition 2: Ordinal Second-Degree Stochastic Dominance (OSSD):

X_k^i OSSD X_k^j if and only if $\sum_{r=1}^s \sum_{l=1}^r p_{kl}^i \leq \sum_{r=1}^s \sum_{l=1}^r p_{kl}^j$ for all $s = 1, \dots, z$.

For modeling preferences the ordinal almost stochastic dominances are also utilized*:

Definition 3: Ordinal Almost First-Degree Stochastic Dominance (OAFSD):

$X_k^i \varepsilon_1^*$ – OAFSD X_k^j , if for $0 < \varepsilon_1^* < 0,5$

$$\sum \left(\sum_{l=1}^{s_1} p_{kl}^i - \sum_{l=1}^{s_1} p_{kl}^j \right) \leq \varepsilon_1^* \|X_k^i - X_k^j\| \text{ for all } s_1 = 1, \dots, z,$$

where: $s_1 = \left\{ s : \sum_{l=1}^s p_{kl}^j < \sum_{l=1}^s p_{kl}^i \right\}$, $\|X_k^i - X_k^j\| = \sum \left(\left| \sum_{l=1}^s p_{kl}^i - \sum_{l=1}^s p_{kl}^j \right| \right)$,

ε_1^* – allowed degree of OFSD rule violation, which reflects the decision-makers preferences; $\varepsilon_1^* \geq \varepsilon_1$, where ε_1 – the actual degree of OFSD rule violation.

Definition 4: Ordinal Almost Second-Degree Stochastic Dominance (OASSD):

$X_k^i \varepsilon_2^*$ – OASSD X_k^j , if for $0 < \varepsilon_2^* < 0,5$

$$\sum \left(\sum_{l=1}^{s_2} p_{kl}^i - \sum_{l=1}^{s_2} p_{kl}^j \right) \leq \varepsilon_2^* \|X_k^i - X_k^j\| \text{ for all } s_2 = 1, \dots, z \text{ and } \mu_k^i \geq \mu_k^j,$$

* Almost stochastic dominances were proposed by Leshno and Levy in [Leshno, Levy, 2002].

$$\text{where } s_2 = \left\{ s_1 : \sum_{r=1}^{s_1} \sum_{l=1}^r p_{kl}^j < \sum_{r=1}^{s_1} \sum_{l=1}^r p_{kl}^i \right\},$$

$$\|X_k^i - X_k^j\| = \sum \left(\left| \sum_{l=1}^s p_{kl}^i - \sum_{l=1}^s p_{kl}^j \right| \right),$$

μ_k^i and μ_k^j – average performances (expected values of the evaluation distributions) of the projects a_i and a_j on the criterion f_k ,

ε_2^* – allowed degree of OSSD rule violation, which reflects the decision-makers preferences; $\varepsilon_2^* \geq \varepsilon_2$, where ε_2 – the actual degree of OSSD rule violation.

2. Calculation of concordance indexes for each pair of projects (a_i, a_j) :

$$c(a_i, a_j) = \sum_{k=1}^n w_k \varphi_k(a_i, a_j)$$

where: $\sum_{k=1}^n w_k = 1,$

$$\varphi_k(a_i, a_j) = \begin{cases} 1 & \text{if } X_k^i \text{SD } X_k^j \text{ and } \mu_k^i > \mu_k^j + p_k[\mu_k^i], \\ \frac{\mu_k^i - q_k[\mu_k^i] - \mu_k^j}{p_k[\mu_k^i] - q_k[\mu_k^i]} & \text{if } X_k^i \text{SD } X_k^j \text{ and } \mu_k^j + q_k[\mu_k^i] < \mu_k^i \leq \mu_k^j + p_k[\mu_k^i], \\ 0 & \text{otherwise,} \end{cases}$$

w_k – coefficient of importance for criterion f_k ,

$q_k[\mu_k^i], p_k[\mu_k^i]$ – indifference and preference threshold for criterion f_k , respectively.

3. Calculation of discordance indexes for each pair of projects and for each criterion:

$$d_k(a_i, a_j) = \begin{cases} 1 & \text{if } X_k^j SD X_k^i \text{ and } \mu_k^j > \mu_k^i + v_k[\mu_k^i], \\ \frac{\mu_k^j - p_k[\mu_k^i] - \mu_k^i}{v_k[\mu_k^i] - p_k[\mu_k^i]} & \text{if } X_k^j SD X_k^i \text{ and } \mu_k^i + p_k[\mu_k^i] < \mu_k^j \leq \mu_k^i + v_k[\mu_k^i], \\ 0 & \text{otherwise} \end{cases}$$

where $v_k[\mu_k^i]$ – veto threshold for criterion f_k .

4. Calculation of credibility indexes for each pair of projects (a_i, a_j) :

$$\sigma(a_i, a_j) = c(a_i, a_j) \prod_{k \in D(a_i, a_j)} \frac{1 - d_k(a_i, a_j)}{1 - c(a_i, a_j)}$$

where: $D(a_i, a_j) = \{k : d_k(a_i, a_j) > c(a_i, a_j)\}$.

5. Determination of strict preference indexes for each pair of projects (a_i, a_j) :

$$\pi(a_i, a_j) = v(a_i, a_j) \cdot \sum_{k=1}^n w_k \pi_k(a_i, a_j),$$

where:

$$v(a_i, a_j) = \begin{cases} 1, & \text{if } \forall k : d_k(a_i, a_j) \leq c(a_i, a_j), \\ 0, & \text{if } \exists k : d_k(a_i, a_j) > c(a_i, a_j), \end{cases}$$

$$\pi_k(a_i, a_j) = \begin{cases} \frac{(\mu_k^i - \mu_k^j) - p_k[\mu_k^i]}{(\mu_k^* - \mu_{k*}) - p_k[\mu_k^i]} & \text{if } \varphi_k(a_i, a_j) = 1, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mu_k^* = \max_{a_i \in A} \mu_k^i \quad \text{and} \quad \mu_{k*} = \min_{a_i \in A} \mu_k^i.$$

The aim of the strict preference function $\pi_k(a_i, a_j)$ is to distinguish the state of the strict preference found to be valid for more than one pair of projects at a given criterion f_k . Their values belong to the interval $[0, 1]$ and $\pi_k(a_i, a_j) = 0$ denotes weak preference or indifference between two projects.

6. Calculation of total preference index for each pair of projects (a_i, a_j) :

$$\omega(a_i, a_j) = \min\{1; \sigma(a_i, a_j) + \pi(a_i, a_j)\}.$$

The total preference index gives an accurate measure of the intensity of preference of project a_i over a_j for all the criteria. It combines two aspects: subjective – expressed by the credibility index and referring only to the relation between two examined projects and objective – expressed by the strict preference index and representing the relation between two projects considered with regard to other projects examined.

7. Calculation of outgoing flow $\phi^+(a_i)$ and incoming flow $\phi^-(a_i)$ for each project:

$$\phi^+(a_i) = \frac{1}{m-1} \sum_{j=1}^m \omega(a_i, a_j)$$

$$\phi^-(a_i) = \frac{1}{m-1} \sum_{j=1}^m \omega(a_j, a_i)$$

In EXPROM I a final partial ranking is obtained as follows:

$$\left\{ \begin{array}{l} a_i P a_j, \quad gdy \quad \left\{ \begin{array}{l} \phi^+(a_i) > \phi^+(a_j) \quad i \quad \phi^-(a_i) < \phi^-(a_j) \quad or \\ \phi^+(a_i) = \phi^+(a_j) \quad i \quad \phi^-(a_i) < \phi^-(a_j) \quad or \\ \phi^+(a_i) > \phi^+(a_j) \quad i \quad \phi^-(a_i) = \phi^-(a_j); \end{array} \right. \\ a_i I a_j, \quad gdy \quad \phi^+(a_i) = \phi^+(a_j) \quad i \quad \phi^-(a_i) = \phi^-(a_j); \\ a_i R a_j, \quad gdy \quad \left\{ \begin{array}{l} \phi^+(a_i) > \phi^+(a_j) \quad i \quad \phi^-(a_i) > \phi^-(a_j) \quad or \\ \phi^+(a_i) < \phi^+(a_j) \quad i \quad \phi^-(a_i) < \phi^-(a_j); \end{array} \right. \end{array} \right.$$

where P , I and R stands for preference, indifference and incomparability respectively.

In EXPROM II a final complete ranking is constructed according to the descending order of the net flows $\phi(a_i)$, where $\phi(a_i) = \phi^+(a_i) - \phi^-(a_i)$.

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