Sydney C.K. Chu

James K. Ho

S.S. Lam

OPTIMIZED STAR PLOT AS DECISION AIDS: APPLICATIONS OF MAXIMUM RESOLUTION TOPOLOGY*

Abstract

The traditional star plot has been a longstanding means of presenting multivariate data. Its early-days use can be traced back to "star symbol plot" of automobile data for large rays to represent favorable characteristics. Another pioneering use in clinical data is a graphical way of summarizing patient's evolving responses. Since the early 1970s and with the coined name of Kiviat plot or graph, wide-spread use in visualizing computer and program performance has become industrial standard among software engineers. It is ever more so in the modern advance of computer graphics, transformed into popular evaluation tools such as 2/3-D Kiviat graph and 3-D Kiviat tube. Its importance amidst forward technological strides remains largely in its ease of visualization, qualitatively on the basis of the shape of a star plot.

In recent years we have staged a series of studies, by focusing of its analysis and topology, resulting in usefulness in the following extensions. First, a (canonical) star plot topology for high-dimensional data visualization is applied to data records of, specifically, multi-attribute dichotomies. Our project on data analysis of on-line auction markets provides such generic sample usage for dimensions identified in constructing a multi-attribute dichotomy to help discern relative empirical advantages to buyers and sellers.

The second stage, of data and optimization modeling aspects, bases on the deeper observation that the areas of the plot for the two parts of a dichotomy may be used quantitatively as an aggregate measure of their relative dominance. An optimization GP model is developed to determine a topology – the geometry and the arrangement of dimensions – that maximizes the resolution of this measure with respect to a given set of reference dichotomies. The outcome of this modeling phase

---

* This work is partially supported by the Hong Kong RGC Competitive Earmarked Research Grant (CERG) Award: HKU 7126/05E.

is what we call an MRT (or Maximum Resolution Topology), that in the sense of maximally discriminating its dichotomy of a set of multi-attribute data records, it is an overall best representation (accompanied by an "optimized" visualization).

The third stage is the coding of MRT construction integrated into a spreadsheet-style decision support system (MRT-DSS). Its ease of use has been promising and robust for diverse applications. Samples of these will conclude the paper as illustrations.

Keywords

Star plot, data visualization, multi-attribute dichotomy, optimization modeling, goal programming, maximum resolution topology (MRT), decision support system, applications for exploratory data analysis.

Introduction

The traditional star plot has been a long-standing means of presenting multivariate data [1]. Its early-days use can be traced back to "star symbol plot" of automobile data for large rays to represent favourable characteristics. Another pioneering use in clinical data is a graphical way of summarizing patient's evolving responses [4]. Since the early 1970s and with the coined name of Kiviat plot or graph [19; 20; 21], wide-spread use in visualizing computer and program performance has become industrial standard among software engineers [25]. It is ever more so in the modern advance of computer graphics, transformed into popular evaluation tools such as 2-D and 3-D Kiviat graphs and 3-D Kiviat tube [5]. One particularly important use in computer hardware/software system monitoring is its real-time visualization of parallel programs and performance [6; 7]. Other applications abound, for example, in petroleum industry and geology [26]. Its importance amidst forward technological strides remains largely in its ease of visualization [22], qualitatively on the basis of the shape of a star plot [25].

The rest of this paper is organized as follows. Section 1 reviews the background of our first stage work on extending a star plot to a topological model. Section 2 reports our second stage work on goal programming (GP) formulation. Section 3 provides details of our third stage work on the maximum resolution topology (MRT) DSS and its applications. Section 4 gives concluding remarks with respect to our current spectrum of application projects and future studies.

1. Visualization and resolution

1.1. Star plot and visualization

Our visualization (and pre-optimization) model is based on "canonical" star plot for displaying multivariate data with an arbitrary number of dimensions [1]. Each data point is plotted as a star-shaped figure (or star graph) with one ray for each dimension. As the resulting shapes depend on the configuration of the dimensions, we subsequently analyse the observations along the dimensions in an effort to present a visual model (see a generic example in Figure 1 of the shape of on-line auction markets [8]).

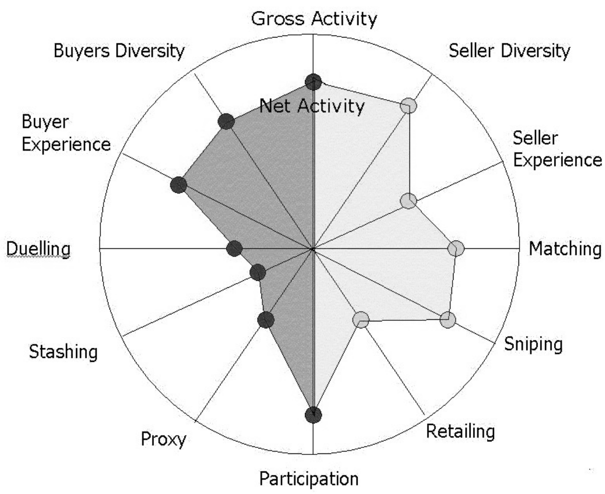

Figure 1. Star plot visualization for on-line auction market

This further investigation leads to generalization from conventional star plot for visualization to a topological model with extended features of dichotomous resolution.

1.2. Topology and resolution

A topological model for a high dimensional data set is a simultaneous graphical display of all its relevant attributes, which provides a geometrical shape as a descriptive, visual statistics of the underlying construct engendering the data. In particular, when various dimensions can be identified to form a multi-attribute dichotomy, the area spanned by the two halves of the topological model can be used as a measure of the relative dominance of the two parts of the dichotomy. Using a reference subset of prejudged cases, the configuration of the dimensions and the angles among them can be optimized in a GP [24] model for a topology that maximizes the resolution of such dichotomies.

In recent years we have staged such a series of studies, by focusing on its analysis and topology, resulting in usefulness in the following extensions. First, a (canonical) star plot topology for high-dimensional data visualization is applied to data records of, specifically, multi-attribute dichotomies. Our project on data analysis of on-line auction markets provides such generic sample usage for dimensions identified in constructing a multi-attribute dichotomy to help discern relative empirical advantages to buyers and sellers. (See Figure 1 above for the dimensions on the right being favourable for buyers versus those on the left for sellers).

The second stage, of data and optimization modelling aspects, bases on the deeper observation that the areas of the plot for the two parts of a dichotomy may be used quantitatively as an aggregate measure of their relative dominance. A GP model is developed to determine the topology – the geometry and the arrangement of dimensions – that maximizes the resolution of this measure with respect to a given set of reference dichotomies. The outcome of this modelling phase is what we call an MRT (for Maximum Resolution Topology), that in the sense of maximally discriminating its dichotomy of a set of multi-attribute data records, it is an overall best representation, accompanied by an "optimized" visualization [2]. (See Figure 2 below for such an "optimized" star graph [17] for the same generic example as in Figure 1).

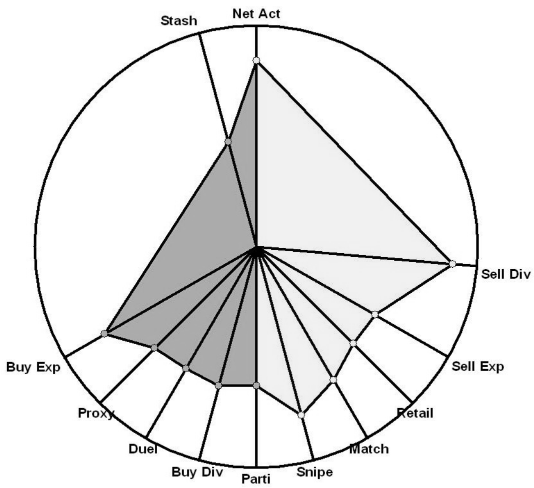

Figure 2. Maximum resolution topology for on-line auction market

The third stage is the coding of MRT construction integrated into a spreadsheet-style decision support system (MRT-DSS). Its ease of use has been promising and applications abound in diverse fields, including diffusion of innovation [11], investment climate and business environment [12], marketing research and customer relations management [13], and medical diagnostics. The implementation of the MRT optimization model as an easy-to-use, spreadsheet-based DSS provides a robust tool for such applications.

2. Modelling and optimization

Subject to the constraints of preserving the prejudged dominance in the reference subset of dichotomies, an optimal topology – the arrangement of dimensions on, and geometry of, the star graph (i.e. configuration of attributes and angles between adjacent pairs) is sought that maximizes the discriminating power, or *resolution*, as measured by the sum of absolute differences in right and left areas for the reference subset. Such an optimal configuration will be called a *maximum resolution topology* (MRT). For any given configuration of the attributes, maximization of the discriminating power can be formulated as a linear program (LP). However, LP produces extreme-point solutions, which may reduce some of the angles between attributes to zero, thus collapsing the corresponding sector areas of the graph. To avoid such degeneration, maximization with bounded deviation of the angles is modelled as a goal program in [16], as summarized below.

2.1. Goal programming formulation

In the star graph for a multi-attribute dichotomy, denote the *angles* between attributes x_{i-1} and x_i by α_i, $i \in I$, and those between attributes y_{j-1} and y_j by β_j, $j \in J$, for all $|K|$ records in the reference set. The sector areas are similarly denoted by A_i A_i and B_j. Let the *weights* be

$$a_i \equiv \mathrm{Sin}\alpha_i, \quad i \in I; \quad \text{and} \quad b_j \equiv \mathrm{Sin}\beta_j, \quad j \in J$$

And we have $0 \le a_i \le 1$, $i \in I$ and $0 \le b_j \le 1$, $j \in J$. Hence for the k^{th} record, the sector area between attributes x_{i-1} and x_i, and that between attributes y_{j-1} and y_j, are given respectively by

$$A_i^k (= A_i^k(x)) \equiv \frac{1}{2} x_{i-1}^k x_i^k \mathrm{Sin}\ \alpha_i = \frac{x_{i-1} x_i}{2} a_i, i \in I$$

$$B_j^k (= B_j^k(y)) \equiv \frac{1}{2} y_{j-1}^k y_j^k \mathrm{Sin}\ \beta_j = \frac{y_{j-1} y_j}{2} b_j, j \in J$$

With $A^k = \Sigma A_i^k$ denoting the area of the right part of the dichotomy, and $B^k = \Sigma B_j^k$ that of the left part, of record k, $k \in K$, the $|K|$ records are partitioned according to prejudgment in the reference set, such that $K \equiv K^+ \cup K^-$, for "right" and "left" dominance, respectively.

The MRT-GP with decision variables a_i, $i \in I$ and b_j, $j \in J$ is then given by [16]

$$\begin{array}{ll} \text{Max} & \text{TtlMRT} \\ \text{Min} & \text{TtlDev} \\ \text{Min} & \text{TtlVar} \end{array}$$

Subject to

$$A^k - B^k = D^k, \quad k \in K^+ \quad \text{and} \quad B^k - A^k = E^k, \quad k \in K^-$$
$$\sum_{i \in I} a_i = 1; \quad \sum_{j \in J} b_j = 1;$$
$$a_i \geq 0, i \in I; \quad b_j \geq 0, j \in J$$
$$\text{TtlMRT} = \sum_{k \in K^+} D^k + \sum_{k \in K^-} E^k$$
$$\text{TtlDev} = \frac{1}{|I|} \sum_{i \in I} \left[\text{PDevX}(i) + \text{NDevX}(i)\right] + \frac{1}{|J|} \sum_{j \in J} \left[\text{PDevY}(j) + \text{NDevY}(j)\right]$$
$$\text{TtlVar} = \frac{1}{|I|} \sum_{i \in I} \left[\text{PVarX}(i) + \text{NVarX}(i)\right] + \frac{1}{|J|} \sum_{j \in J} \left[\text{PVarY}(j) + \text{NVarY}(j)\right]$$

(Deviation bounds)

$$\frac{1}{|I|} - F * \frac{1}{|I|} \leq a_i \leq \frac{1}{|I|} + F * \left(1 - \frac{1}{|I|}\right), \quad i \in I; \qquad \frac{1}{|J|} - F * \frac{1}{|J|} \leq b_j \leq \frac{1}{|J|} + F * \left(1 - \frac{1}{|J|}\right), \quad j \in J$$

(Deviation goal constraints)

$$a_i - \text{PDevX}(i) + \text{NDevX}(i) = \frac{1}{|I|}, \quad i \in I; \qquad b_j - \text{PDevY}(j) + \text{NDevY}(j) = \frac{1}{|J|}, \quad j \in J$$

(Variation/smoothing goal constraints)

$$a_i - a_{i+1} - \text{PVarX}(i) + \text{NVarX}(i) = 0, \quad i \in I \quad (a_{m+1} \equiv a_0)$$
$$b_j - b_{j+1} - \text{PVarY}(j) + \text{NVarY}(j) = 0, \quad j \in I \quad (b_{n+1} \equiv b_0)$$

The $0 < F < 1$ parameter in the MRT-GP formulation above is the fraction of deviation from equal weights allowed for the weight variables a and b. PDev, Ndev, PVar, NVar stand for positive deviation, negative deviation, positive variation, negative variation, respectively, in conventional GP terminology [24].

2.2. Sample illustration of MRT computation

Table 1 records two numerical instances (34 eBay and 20 eGDI data records) of MRT-GP computation. The canonical star plot values are the equal-weight (i.e. angle) total area difference, given by the definition:

$$\sum_{k\in K^+}[A^k - B^k] + \sum_{k\in K^-}[B^k - A^k]\left|\left\{a_i = \frac{1}{|I|}, \forall i; b_j = \frac{1}{|J|}, \forall j\right\}\right. = 1.8682$$

for eBay dataset and 0.6725 for eGDI dataset. The respective optimal values for the first objective function TtlMRT are increased to 3.1498 and 1.1725. Hence there are gains in discriminating power of 68.60% and 74.35%, respectively. As MRT consider both configuration of attributes *and* angles, we illustrate in Table 1 the less desirable effects of optimizing only *one* of the two aspects of the resolution topology. Numerically, it can be seen that GP optimization with respect to the angles as decision variables accounts for the substantially larger share of improved resolution in both instances.

Table 1

Sample performance of MRT-GP results

	Canonical star plot	Configuration ONLY: attributes permutation	Weight ONLY: angles optimization	Optimized MRT
34 eBay data records	1.8682	1.8684 (0.01%)	2.7100 (45.06%)	3.1498 (68.60%)
20 eGDI data records	0.6725	0.7125 (5.95%)	0.7499 (11.51%)	1.1725 (74.35%)

3. Applications of maximum resolution topology

3.1. Decision support system: MRT-DSS

To facilitate the computation of a maximum resolution topology (MRT) for a given set of data from a multi-attribute dichotomy, an easy-to-use decision support system (DSS) has been built on Excel spreadsheet software. Such an MRT-DSS system has both its front end and report routine integrated in the same Excel spreadsheet workfile, into which the input data records can be placed (for example, imported from a database); and outputs of values and MRT-star plots displayed. The functions of MRT-DSS are grouped in a pull-down menu as shown in Figure 3.

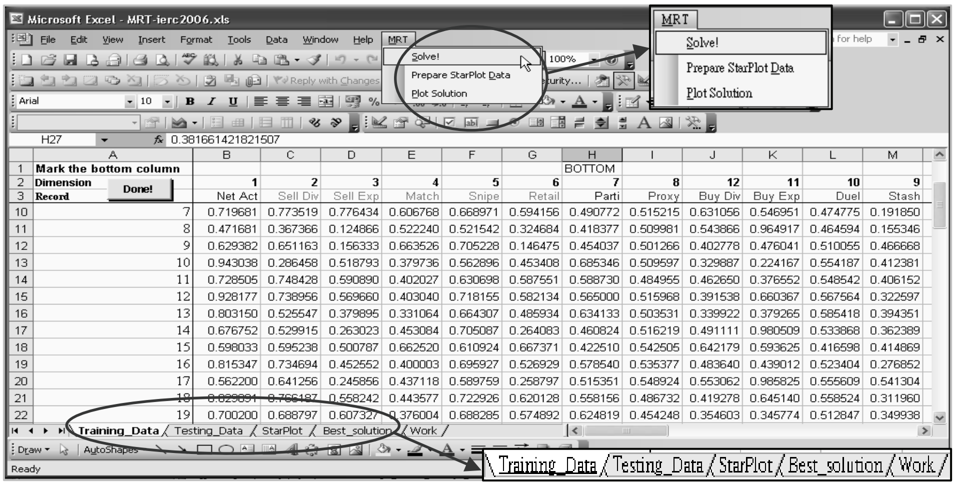

Figure 3. Worksheets and menu of MRT-DSS

To find the solution, the user only needs to copy and paste the records of training data (the “reference set”) to the ‘Training Data’ worksheet and click the ‘Solve!’ item button on the ‘MRT’ menu. MRT-DSS will permute over all possible configurations and dynamically generate the input data for each configuration. The training data will be passed to an LP solver (LINGO [23]) to find the solution based on the MRT-GP model. MRT-DSS will store the solution of each configuration on the ‘Work’ worksheet, as well as the best solution on the ‘Best solution’ worksheet. It will also keep the optimal MRT configuration and angles in the ‘StarPlot’ worksheet for preparing the test data for plotting.

By completing the training of MRT-DSS and obtaining the optimal configuration, the system can then be used to evaluate new cases of the model. With data copied to the ‘Testing Data’ worksheet, the ‘Prepare StarPlot Data’ item dichotic on the ‘MRT’ menu is selected. MRT-DSS will arrange and store the data in the ‘StarPlot’ worksheet. It will also compute for each test case the areas of the right (A) and left (B) parts of the dichotomy and their difference (A-B) as shown in Figure 4. The user can easily evaluate the test cases based on these numerical results. To visualize and further analyse a particular data record, the user can choose the ‘Plot Solution’ item on the ‘MRT’ menu to draw its StarPlot diagram under the maximum resolution topology. By inspecting and comparing records under the optimal configuration and angles in the diagrams, and by studying the right-left differentials provided by MRT-DSS, substantial topological analysis can be performed for insight into the model under study.

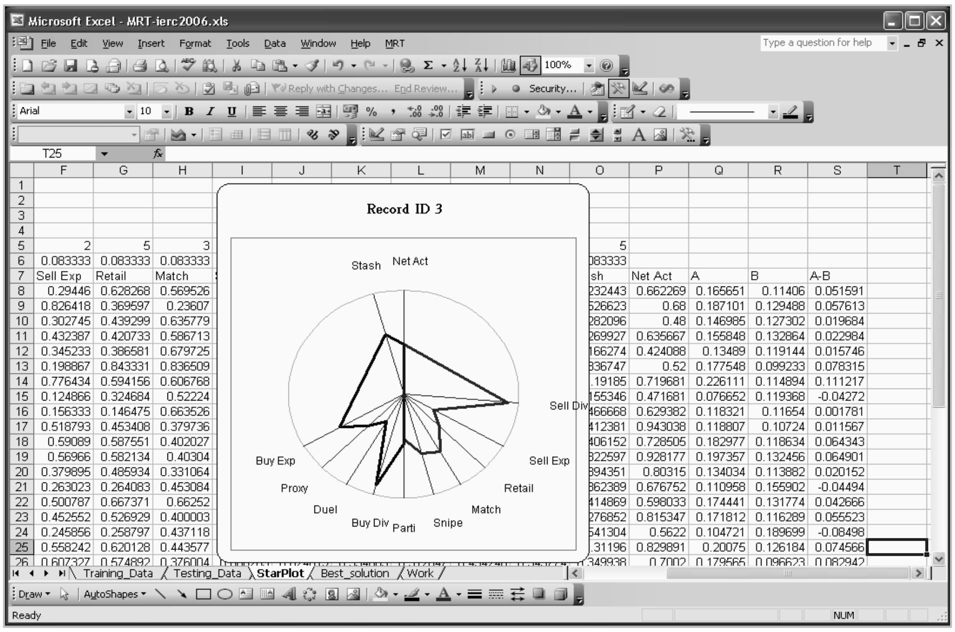

Figure 4. Using MRT-DSS to evaluate and visualize test dataset

3.2. Applications of MRT-DSS

To demonstrate the robust application of MRT-DSS in diverse fields, the results of various published studies are summarized below.

3.2.1. Comparative study of on-line auction markets

As exploratory data mining, 34 data sets with the bidding records of approximately 500 auctions each are used as the reference subset. The market categories comprise automobiles, business software, camcorders, coffee tables, coins, cordless phones, desktop PCs, digital cameras, gift certificates, handbags, laptop PCs, lodging, necklaces, personal digital devices, rings, travel and vacation packages, and wristwatches. MRT-DSS produced the optimal configuration of the twelve dimensions in the topological model for on-line auctions as shown in Figure 2. In [17], a comparative study of four markets in five countries was conducted. The results are summarized here for illustrative purposes. The countries are Australia (AU), Canada (CA), France (FR), United Kingdom (UK), and United States (US). The markets are: classical CDs, Star Wars toys and games; diamond rings, and digital cameras. The star plots for the twenty cases are shown in Figure 5.

The seven cases boxed in Figure 5 are classified as "right-dominant" (buyer's market). The other thirteen cases are classified "left-dominant" (seller's market). It was observed that on-line auction markets for digital cameras, a hi-tech product of common value, tend to be favourable to buyers across nations. By contrast, the other markets of particular items catering to more subjective preferences and tastes tend to favour sellers on eBay.

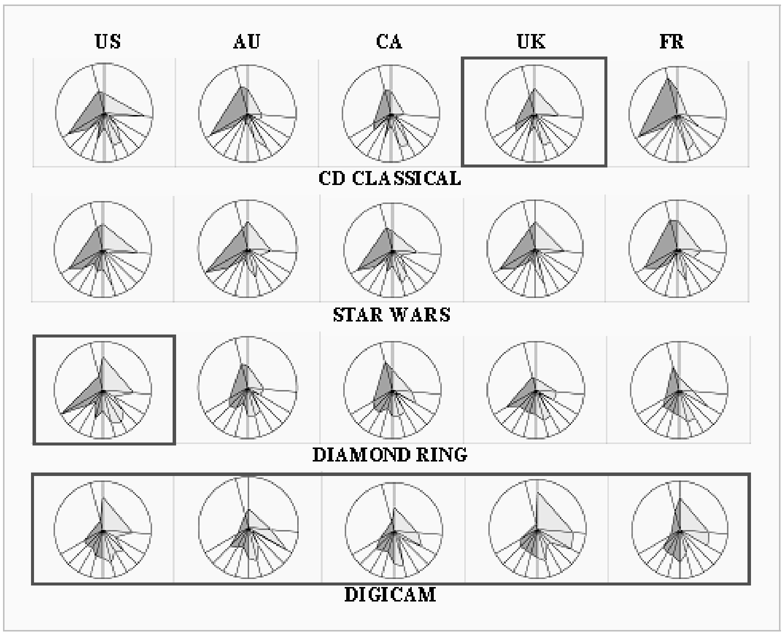

Figure 5. MRT plots of 20 markets in comparative study

3.2.2. Global Diffusion of the Internet

The Global Diffusion of the Internet (GDI) framework has been used to study the progress in the adoption of this communication medium in close to 30 countries since 1997. Six dimensions are used to measure the essential features of the status of the Internet in a country. Collectively, they cover the bundle of requisite technologies, from infrastructure to end use applications, to capture the multifaceted evolution of the Internet experience in different countries. They also fall into a supply-demand dichotomy:

Demand Attributes	Supply Attributes
Pervasiveness	Organizational Infrastructure
Sectoral Absorption	Connectivity Infrastructure
Sophistication of Use	Geographic Dispersion

Using a reference subset of 24 cases in 1999, an MRT is obtained and used to calibrate subsequent cases [11]. For example, China, India, Pakistan, and Turkey in 2000 are plotted in Figure 6, showing that China was demand-dominant while the other three were supply-dominant in GDI.

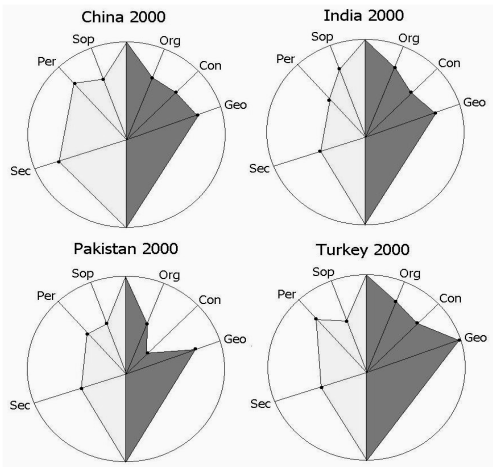

Figure 6. MRT plot for GDI of four countries in 2000

3.2.3. Investment climate indicators

The Investment Climate Surveys conducted by the World Bank measure specific constraints facing firms in over 50 countries, and relate them to measures of firm performance, growth, and investment. By casting ICI as a multi-attribute dichotomy, an MRT is sought to classify countries as to whether the investment climate is constrained primarily by policies or by resources [12].

Policy Attributes	*Resource Attributes*
Policy Uncertainty	Electricity
Regulation	Finance
Tax rate	Labour Skills

Based on the initial plots, the five countries with the highest policy dominance in constraints on investment climate are Belarus, Moldova, Romania, Poland, and Brazil. The five with the highest resource dominance in constraints on investment climate are Bangladesh, Senegal, Zambia, Uganda, and Eritrea. These are used as the reference subset to derive an optimal MRT. The resulting plots for four populous countries are shown in Figure 7.

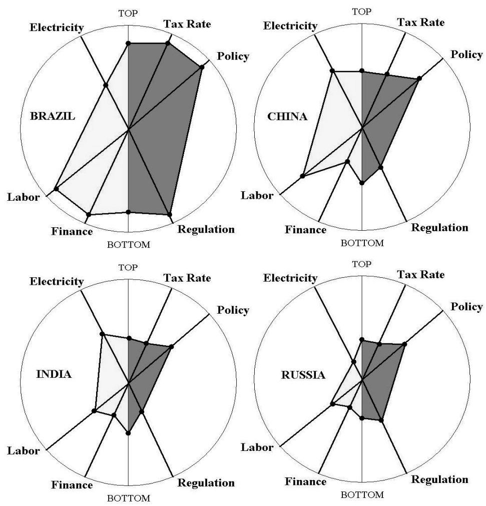

Figure 7. MRT plot for ICI of four countries

3.2.4. Customer relations management

In customer relations management (CRM), consumers are often surveyed for their attitude toward various attributes of products and services for insight into their perception of price, quality and value. The results of analysis can be used in market segmentation for more effective design and promotion of future offerings. In [13] a survey was presented that asked customers to rate the importance of the attributes directly. The maximum resolution dichotomy (or MRD) methodology was applied as a DSS tool in market research to classify customers according to the MRT of whether they tend to be cost- or benefit-focused.

We consider customer attitude surveys that ask for the rating, on some given scale of relative importance, of various cost and benefit attributes of a product, service, or experience. Typically, a number of questions relate to different aspects of cost, including direct pricing, and indirect opportunity costs. The other questions, not necessarily in the same number, relate to benefits in terms of perceived quality, and customer satisfaction. A high importance rating of a cost attribute means the customer is cost-conscious in that regard, and a high actual cost is likely to detract from overall satisfaction. Conversely, a high importance rating of a benefit attribute means the customer is particular about its value and is likely to pay for improved quality there.

As an example, we use the case of a major league sports team and a survey of its fans' attitude toward the following attributes of their entertainment experience at a home game.

On a scale of 1 to 7 (most important), how do you rate the importance of:
Question 1: the view of the game from your seat?
Question 2: amenities available in the stadium?
Question 3: the team's performance in the games?
Question 4: cost of tickets?
Question 5: cost of transportation and/or parking?
Question 6: cost of concessions?

Of the 369 respondents, 58% were classified as benefit-focused, 33.3% as cost-focused, and 8.7% as cost-benefit neutral. The ratio of benefit-focused respondents to cost-focused respondents is 1.74. Management may use this indicator to, for example, devote more effort into quality improvement instead of price reduction.

3.2.5. On-line auction markets in tourism

Since the value of an empty seat on an airliner vanishes once the flight takes off, just as an empty hotel room by check-out time, most travel- and tourism-related services can be regarded as perishable commodities. Because of that, the concept of dynamic pricing to balance supply and demand in a free market appeals to both economists and consumers. To the former, this means market efficiency. To the latter, it suggests potential bargains as expiration looms. However, suppliers must overcome the stigma of brand dilution if they let know that lower than set prices are acceptable. In [14], a comprehensive, though by no means exhaustive survey found on-line auction markets in travel and tourism in a fledgling stage of development. On eBay.com, for example, there were around 2000 active listings in December 2006, while the comparable number was over 190,000 for Antiques, and over 390,000 for Consumer Electronics. Yet there is already significant data available for formal analysis to track the growth of markets in travel and tourism.

The MRD methodology is applied to the Travel categories on eBay in the early 2006. The sub-category of Vacation Packages and Lodging are used. Under Lodging, we have Vacation Rental, Hotel, and Bed and Breakfast. The last two are grouped as one.

As quantitative measures, we can use an index:

$$I_B = (\text{Right Area} - \text{Left Area}) / (\text{Right Area} + \text{Left Area})$$

which is positive for a buyer's market, and negative for a seller's market. This index ranges from -0.29 to +0.43 with an average of 0.11 for the 34 cases in the reference set. For the three Travel cases, we have the indices below:

Market	**VacRental**	**Hotel+B&B**	**VacPack**
I_B	0.18	0.21	0.64

Our results show that actual conditions are also favourable to buyers, and to extents that are above the reference average index of 0.12. In particular, Vacation Packages, with an index of 0.64 is remarkable.

3.2.6. Inter-brand comparisons

In [15], the MRD methodology is applied to four brands of digital cameras on eBay in the first quarter of 2006. The specific auction categories used were:

Cameras & Photo> Digital Cameras> Point & Shoot> 4.0 to 4.9 Mega-pixels> *Brand*; where *Brand* was Nikon, Sony, Kodak, and Canon, respectively.

It should be apparent that with the high number of dimensions and their categorical measures, inter-brand comparison of subtle differences will be rather difficult. This is where our maximum resolution dichotomy methodology can become useful for gaining further insight into such market data. For the four brands of digital cameras, we have the indices below:

Brand	**Sony**	**Canon**	**Kodak**	**Nikon**
I_B	0.20	0.39	0.53	0.55

All four MRDs exhibit a right-dominant topology, indicating that market conditions are favourable to buyers for these brands. On a relative scale, Nikon ranks the highest at 0.55, closely followed by Kodak at 0.53. Even at 0.39 and 0.20, respectively, both Canon and Sony are significantly above the average of 0.1 for the reference set used to derive the MRT. This is consistent with more general and extensive observations that for hi-tech consumer products of so--called common values, eBay auctions tend to be buyer's markets. Cost of entry and operation being low compared to brick-and-mortar stores, competition is keen among sellers. They must compete on price and service to establish credibility and gain market share.

Concluding remarks

Our series of studies ([8; 9; 10; 16; 3; 17; 18] in this order) have progressed from an initial key study on topology analysis of on-line auction markets [8], to optimization modelling towards maximum resolution topology for multi-attribute dichotomies [16], to its spreadsheet MRT-DSS implementation [18], and finally to a wide spectrum of applications [11; 12; 13; 14; 15]. A concise summary from the data mining applications perspective of this tool is given below.

The potential usefulness of MRT-DSS has been demonstrated in diverse applications. In customer relations management and marketing research, it is of common interest to gain insight into consumers' attitude toward the costs and benefits associated with a product, service, or experience. Surveys on the multitude of attributes lead to high-dimensional data, for which meaningful aggregate measures remain a major challenge in data mining. As a contribution to the modelling of market segments, our methodology helps to rationally classify customers as to whether they are benefit-focused or cost--focused. The cost-benefit survey framework is cast as a multi-attribute dichotomy, with a cost side and a benefit side [13]. Similarly, as a contribution to the rational classification of countries as to whether their investment climate is constrained primarily by policies or by resources, the Investment Climate Indicators (ICI) framework used by the World Bank is cast as a multi-attribute dichotomy, with a policy-constraints side and a resource-constraints side. As an extension of the star plot to display multi-dimensional data, the areas spanned by the policy-side and resource-side attributes of a data instance in ICI suggest an aggregate measure of the relative dominance of the corresponding parts [12]. Future work includes expounding the application of this dichotomy as a significant output of data mining for CRM and ICI, as well as refining the selection of the reference set by, for example, incorporating results from focus group studies, as well as expert judgment from researchers of specific markets and economies.

The inter-brand comparative study of on-line auction markets was conducted for four brands of digital cameras. Potential sellers and buyers may ask whether eBay auctions are good places to sell and buy digital cameras, respectively. The auctioneer (eBay) may wish to determine how such markets are performing, and whether to apply strategies such as discounted fees to promote activity and participation. Do answers to such questions apply broadly to all brands, or do they depend on the brand? Our findings help to discern aggregate, operational characteristics for the market among brands [15]. Similarly, while travel- and tourism-related markets are still in a fledgling stage of development, our study showed that actual conditions are also favourable to buyers, and that further progress and growth will depend on the reaction of sellers, and their willingness to put their offerings to the test of the free market. Our results demonstrated the potential usefulness of our approach as a tool in data mining for future study and analysis of on-line auctions [14]. Indeed, in recognition of the potential wealth of information in its transactional data, eBay.com itself has begun marketing both the content and analysis of its databases commercially. Third-party vendors, known as eBay market data resellers, are also appearing to offer services in related market analysis. Our MRT-DSS is an addition to data mining tools that may be particularly useful in this context. Finally, it should be remarked that our example applications, while diverse, are quite generic. Other potential cases abound, notably in medical diagnostics, risk profiling in both financial and security issues, just to name a few.

References

[1] Chambers J., Cleveland W., Kleiner B., Tukey P.: *Graphical Methods for Data Analysis*. Wadsworth Press, Belmont, CA 1983.

[2] Chu S.C.K., Ho J.K.: *Topological and Optimization Modeling for Internet Data of Online Auction Markets*. Proceedings of ITI2005 (27th International Conference on Information Technology Interfaces). Cavtat, Croatia June 20-23, 2005, 5.

[3] Chu S.C.K., Ho J.K., Lam S.S.: *Optimization Modeling DSS for Maximum Resolution Topology of Internet Data. Institute of Industrial Engineering Annual Conference*. CD-Rom Proceedings. Orlando, FL May 20-24, 2006, 6.

[4] Friedman H.P., Farrell E.J., Goldwyn R.M., Miller M., Siegel J.H.: *A Graphical Way of Describing Changing Multivariate Patterns*. Proceedings of the Computer Science and Statistics 6th Annual Symposium on the Interface. University of California, Berkeley October 16-17, 1972, pp. 56-59.

[5] Hackstadt S.T. and Malony A.D.: *Visualization Parallel Programs and Performance*. "IEEE Computer Graphs and Applications" 1995, Vol. 15(4), pp. 12--14.

[6] Heath M.T. and Etheridge J.A.: *Visualizing the Performance of Parallel Programs*. "IEEE Software" 1991, Vol. 8(5), pp. 29-39.

[7] Heath M.T., Malony A.D., Rover D.T.: *The Visual Display of Parallel Performance Data*. "Computer" 1995, Vol. 28(11), pp. 21-28.

[8] Ho J.K.: *Topological Analysis of Online Auction Markets*. "International Journal of Electronic Markets" 2004, Vol. 14(3), pp. 202-213.

[9] Ho J.K.: *Topology of Online Auction Markets*. In: *Competitive Bidding and Auctions II*. K.K. Lai et al. (eds). Global-Link Publisher, Hong Kong 2004, pp. 1-9.

[10] Ho J.K.: *A Global Comparative Study of Online Auction Markets*. In: *Shaping Business Strategy in a Networked World*. J. Chen (ed.). Proceedings of ICEB2004, Beijing, China December 5-9, 2004, pp. 239-244.

[11] Ho J.K.: *Maximum Resolution Dichotomy for Global Diffusion of the Internet*. "Communications of the Association of Information Systems" 2005, Vol. 16, pp. 797-809.

[12] Ho J.K.: *Maximum Resolution Dichotomy for Investment Climate Indicators*. "International Journal of Business Environment" 2006, Vol. 1(1), pp. 126-135.

[13] Ho J.K.: *Maximum Resolution Dichotomy for Customer Relations Management*. In *Data Mining VII*. A. Zanansi, C. Brebbia, and N. Ebecken (eds). WIT Press, Southampton 2006, pp. 279-288.

[14] Ho J.K.: *Online Auction Markets in Tourism*. "Information Technology and Tourism" 2008, Vol. 10(1), pp. 19-29.

[15] Ho J.K.: *Inter-Brand Comparison of Online Auction Markets*. "Electronic Commerce Research" 2008, Vol. 8, pp. 103-114.

[16] Ho J.K., Chu S.C.K.: *Maximum Resolution Topology for Multi-attribute Dichotomies*. "Informatica" 2005, Vol. 16(4): pp. 557-570.

[17] Ho J.K., Chu S.C.K., Lam S.S.: *Maximum Resolution Topology for Online Auction Markets*. "International Journal of Electronic Markets" 2007, Vol. 17(2), pp. 164-174.

[18] Ho J.K., Chu S.C.K., Lam S.S.: *Optimization Model and DSS for Maximum Resolution Dichotomies*. In: *ICINCO 2007, Proceedings of the Fourth International Conference on Informatics in Control, Automation and Robotics, Intelligent Control Systems and Optimization*. J. Zaytoon, J.-L. Ferrier, J. Andrade-Cetto, J. Filipe (eds). Angers, France May 9-12, 2007, pp 355-358.

[19] Kolence K.W.: *The Software Empiricist*. "ACM SIGMETRICS Performance Evaluation Review" 1973, Vol. 2(2), pp. 31-36.

[20] Kolence K.W., Kiviat P.J.: *Software Unit Profiles and Kiviat Figures*. "ACM SIGMETRICS Performance Evaluation Review" 1973, Vol. 2(3), pp. 2-12.

[21] Morris M.F.: *Kiviat Graphs*: *Conventions and Figures of Merit*. "ACM SIGMETRICS Performance Evaluation Review" 1974, Vol. 3(3), pp. 2-8.

[22] Muller W., Schumann H.: *Visualization Methods for Time-Dependent Data: An Overview*. Proceedings of 2003 Winter Simulation Conference. Vol. 1. New Orleans Dec 7-10, 2003, pp. 737-745.

[23] Schrage L.: *Optimization Modeling with LINGO, 3/e.* Lindo Systems Inc., Chicago, IL 1999.

[24] Scniederjans M.J.: *Goal Programming Methodology and Applications 1995*. Kluwer Publishers, Boston 1995.

[25] Spence R.: *Information Visualization 2001*. Pearson Education Ltd., ACM Press, Harlow.

[26] Wever H.E.: *Petroleum and Source Rock Characterization Based on C7 Star Plot Results: Examples from Egypt*. "American Association of Petroleum Geologists (AAPG)" 2000, Vol. 84(7), pp.1041-1054.