Rafael Caballero

Trinidad Gómez

Mónica Hernández

María Amparo León

GOAL PROGRAMMING WITH LINEAR FRACTIONAL CRITERIA: AN APPLICATION TO A FOREST PROBLEM*

INTRODUCTION

In a goal programming (GP) problem, when the goals are linear fractional functions, the formulation of the goal programming problem to be solved is quite complex because of non-linear constraints. As Awerbuch et al. showed [1], solving this kind of problem is not as easy as it might appear. These authors offer an example showing that direct linearization of the problem is not suitable for solving it. In the literature, there are very few references to goal programming with fractional goals, except for the papers of Hannan [9; 10], Soyster and Lev [15], and an article by Kornbluth and Steuer [13].

In this work, we first review the difficulties encountered when solving a linear fractional goal programming problem, and suggest the use of an associated linear problem. Following this, a theoretical study is presented of the relationships that exist between the solutions of the linear problem associated with the true linear fractional goal programming problem. We show that the linear problem can be used as a search strategy for solutions that satisfy all goals, but that it is unsuitable when such solutions do not exist.

In the third section we present a linear fractional goal programming model for solving a timber harvest scheduling problem in order to obtain a balanced age class distribution of a forest plantation in Cuba. The forest area

* The authors wish to express their gratitude to the anonymous referees for their valuable and helpful comments. This research has been partially founded by research projects of Andalusian Regional Government and Spanish Ministry of Science and Education.

of Cuba has been severely reduced due to indiscriminate exploitation and natural disasters (fires, hurricanes etc.). Thus, in this particular case, the main goal is to organize and regulate the forest. This involves a significant change from its current distribution by ages to obtain a more even-aged structure over a planning horizon of 25 years. This has been formalized as fractional goals which take into account the dynamic aspect of the problem and ensure attaining a balanced age class distribution in a progressive and flexible way.

In Section 4 the model is applied to a specific case, the *San Juan and Martínez Management Unit*, which belongs to the *Integral Pinar del Río* forestry company. We then analyse the results obtained. Finally, in Section 5 we draw some conclusions followed by the references in Section 6.

1. GOAL PROGRAMMING IN MULTIOBJECTIVE LINEAR FRACTIONAL PROGRAMMING

The problem we deal with has p linear fractional objectives and a constraints set that is a convex polyhedron. Without loss of generality, we assume that the decision-maker imposes a minimum target value for each objective. Thus, the unwanted deviation variables are the negative ones, that is, n_i for $i = 1, ..., p$. Using the Lexicographic GP approach and assuming that the priority levels are imposed in such a way that in a given level N_s we find k goals numbered from 1 to k, (in the case of Weighted GP, $k = p$), in this level N_s, the problem to be solved is as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{k} n_i \\ s.t \quad & Ax \leq b \\ & \frac{f_i(x)}{g_i(x)} + n_i - p_i = u_i \quad i = 1,...,k \\ & x, n_i, p_i \geq 0 \qquad i = 1,...,k \end{aligned} \tag{1}$$

where $A \in M_{mxn}(\mathbb{R})$ and $b \in \mathbb{R}^m$. Let $X = \{x \in \mathbb{R}^n \,/\, Ax \le b,\ x \ge 0\}$, which includes the goals in the previous levels[1]; and $f_i(x) = c_i^t x + \alpha_i$, $g_i(x) = d_i^t x + \beta_i$ where $c,d \in \mathbb{R}^n$, α_i, $\beta_i \in \mathbb{R}$. In addition, we assume that the denominators $g_i(x)$, $i = 1,\dots, p$ are strictly positive for every $x \in X$.

If the solution of problem (1), $(x^*, n_i^*, p_i^*)_{i=1,\dots,k}$ is such that $\sum_{i=1}^{k} n_i^* = 0$, then x^* is a solution that satisfies all the goals in this priority level. However, it is obvious that in principle this is not an easy problem to solve due to the non--linear constraints corresponding to the goals we aim to satisfy.

If we multiply all these constraints on both sides by the factor $g_i(x)$ – which is always positive in X for every i per hypothesis – we obtain the formulation of the following linear programming problem, since $f_i(x)$ and $g_i(x)$ are all linear functions:

$$\begin{aligned} &\min \ \sum_{i=1}^{k} n_i' \\ &s.t \quad x \in X \\ &\qquad f_i(x) - g_i(x) \cdot u_i + n_i' - p_i' = 0 \\ &\qquad n_i', p_i' \ge 0 \qquad i = 1,\dots,k \end{aligned} \tag{2}$$

The relationship between these two problems has already been studied in the literature, which concludes that the use of (2) to solve (1) is not always a valid approach, since the solutions of the two problems might not be identical, even when there are unique solutions. A test problem can be used to verify whether the solutions for the two problems are identical or not (see [1; 15; 10]).

However, by setting aside the aim of solving problem (1) directly, we can focus on the search of points in X that verify all the goals imposed. Although it is true that the two problems are not equivalent, and therefore we cannot use (2) for solving (1) in every case, we can prove that (2) can be used for deducing the existence of solutions that satisfy all the problem's goals.

---

[1] The constraints imposing the achievement of the goals in the previous levels are of the form $f_j(x)/g_j(x) \ge u_j$ for those j belonging to previous levels. However, these constraints become linear constraints, since they are equivalent to $f_j(x) - u_j g_j(x) \ge 0$, where $f_j(x)$ and $g_j(x)$ are linear functions, so they can be included in the formulation $Ax \le b$.

Theorem 1

Given problems (1) and (2) as described earlier, the following statements are valid:

i. If, when solving (2), the solution is $(x^*, n_i'^*, p_i'^*)_{i=1,\dots,k}$ such that $\Sigma_i n_i'^* = 0$, then there is at least one point of X that verifies the fractional goals imposed by the decision-maker for the current priority level and this solution coincides with x^*.

ii. If, when solving (2), the solution is $(x^*, n_i'^*, p_i'^*)_{i=1,\dots,k}$ such that $\Sigma_i n_i'^* > 0$, then, there is **no** solution that satisfies all the goals of the linear fractional problem for the current priority level.

Proof. See [3].

Although this theorem does not attempt to show the equivalence between problems (1) and (2), it is obvious that there is a strong relationship between them. This is sufficient to allow us to use (2) (which is a linear problem) for finding a solution that will satisfy all the goals of the linear fractional goal programming problem.

In the examples provided by Awerbuch et al. [1], Hannan [9; 10], and Soyster and Lev [15] to show the lack of equivalence between (1) and (2), the statements of Theorem 1 are verified.

Finally, we establish a result similar to that of Theorem 1 using the Minimax GP approach. In this instance the problem to solve is as follows:

$$\begin{aligned} \min \quad & d \\ s.t \quad & Ax \leq b \\ & \varphi_i(x) + n_i - p_i = u_i \quad i = 1,\dots,k \\ & \lambda_i n_i \leq d \qquad i = 1,\dots,k \\ & x, n_i, p_i \geq 0 \qquad i = 1,\dots,k \end{aligned} \tag{3}$$

Once again, we are dealing with the non-linearity of the constraints associated with the goals because the initial functions are fractional functions. Again, we deal with this drawback by solving the linear problem associated with the original problem:

$$
\begin{aligned}
\min \quad & d' \\
s.t \quad & Ax \le b \\
& f_i(x) - g_i(x) \cdot u_i + n'_i - p'_i = 0 \quad i = 1,\dots,k \\
& \lambda_i n'_i \le d' \quad i = 1,\dots,k \\
& x, n'_i, p'_i \ge 0 \quad i = 1,\dots,k
\end{aligned} \tag{4}
$$

As shown for Theorem 1, it is possible to prove that solving this linear problem is sufficient to guarantee the existence − or non-existence − of a solution that will satisfy all the problem's goals using the minimax approach. However, the two problems − (3) and (4) − are not equivalent problems.

Theorem 2

Given problems (3) and (4) as described earlier, the following statements are valid:

i. If, when solving (4), the solution is $(x^*, d'^*, n_i'^*, p_i'^*)_{i=1,\dots,k}$ with $d'^* = 0$, then there is at least one point of X that verifies the fractional goals imposed by the decision-maker and this solution coincides with x^*.
ii. If, when solving (4), the solution $(x^*, d'^*, n_i'^*, p_i'^*)_{i=1,\dots,k}$ is such that $d'^* > 0$ then there is no solution that satisfies all the goals of the original problem.

Proof. See [3].

2. THE APPLICATION

2.1. The problem

Decision making in forest planning has currently become a multi-dimensional decision context, concerned with multiple and sustainable use of the forests. They are not envisaged simply as a source of goods and services; rather, the preservation of biodiversity and environmental protection are also factors to be taken into account. In fact, the term *sustainability* goes beyond the steady supply of timber products in order to include other goods and services provided by forestry systems [6]. Therefore, we need multiple criteria decision-making models to the management of any forest system.

Field [7] was a pioneer in this area who analysed a forest planning problem using a multicriteria framework. From this time onwards many other works applying multicriteria techniques to forestry problems were published. Kao and Brodie [12], Field et al. [8] and Hotvedt [11] are some of the authors who have used Goal Programming for timber production planning. On the other hand, Díaz-Balteiro and Romero [4; 5] designed a multigoal programming model and obtained the best-compromise solutions validated in terms of optimal utility.

The characteristics common to all these models is that they have been applied to European and North-American even-aged forests.

In this application we deal with a very different forest management problem, placed in the Cuban context. The forestry area of Cuba has suffered dramatically due to indiscriminate exploitation and natural disasters (fires, hurricanes etc.). This situation, together with the fear of greater ecological disasters, has given rise to conservationist policies which lead to old growth forests with subsequent financial losses and other problems. This also means that Cuban forests have a highly uneven-aged distribution and thus an important objective in the Cuban context will be to plan a redistribution of the forest into even-aged stands.

However, Cuba is making great efforts in reforesting and caring for its natural forests. Some Cuban forestry companies have focused on achieving an even-aged structure for their plantations. In this line, the Pinar del Río University has been authorized to carry out this kind of work with the forestry companies of the region. The present study is framed within this approach, and is preceded by the work of León et al. [14], where a Goal Programming model with linear goals was formulated into the management planning of a *Pinus Caribaea* plantation in this province. The authors obtained several solutions which satisfied the target values, but not all of them ensured a balanced even-aged distribution over the planning horizon. Thus, we propose fractional programming as a good alternative to take into account the dynamic aspect of the problem in order to achieve that the area covered by each age class must be the same by the planning horizon.

Therefore, we have designed a lexicographic GP model with fractional goals to regulate a pure plantation of *Pinus Caribaea* in Pinar del Río (Cuba).

2.2. The model

The model is initially formalized in a general way and then applied to the specific case of a Cuban plantation which belongs to the Integral Pinar del Río forestry company.

Let us assume that the plantation area to reorganize is managed for wood production and is classified according to productivity (site class) and by age of the stands (age class). Thus, the starting situation is given by the following matrix:

$$\mathbf{S}^0 = \begin{pmatrix} s_{11}^0 & s_{12}^0 & \cdots & s_{1I}^0 \\ s_{21}^0 & s_{22}^0 & \cdots & s_{2I}^0 \\ \cdots & \cdots & \cdots & \cdots \\ \\ s_{H1}^0 & s_{H2}^0 & \cdots & s_{HI}^0 \end{pmatrix}$$

where s_{hi}^0 is the total number of hectares of the site class h ($h = 1, 2,\ldots, H$) within the age class i ($i = 1, 2,\ldots, I$) at the starting point. The sum of the column elements of the matrix shows the available area at the starting point in each age class ($\mathbf{S}_i^0 = \sum_{h=1}^{H} s_{hi}^0$), whereas the sum by rows gives the available area in each site class ($\mathbf{S}_h^0 = \sum_{i=1}^{I} s_{hi}^0$).

The planning horizon *(T)* has been divided into periods, so that, when a period has elapsed, the trees in age class i become age class $i + 1$. Thus, if t is the number of years in each class (for reasons of simplicity we assume this number is constant), the number of periods under consideration, denoted by P, is equal to the number of years of the planning horizon divided by t.

The decision variables of our model represent the number of hectares of a specific site class h ($h = 1, 2,\ldots,H$) and age class ($i = 1, 2,\ldots,I$) with a forest intermediate treatment or a final cutting j ($j = 1, 2,\ldots, J$) at period p ($p = 1, 2,\ldots,P$), denoted by x_{hij}^p. The forest treatment to apply depends on age, and so the value of the subscript j depends on the value of i, $j \in N(i)$, where $N(i) = \{j/(i,j) \in N\}$ and $N = \{(i,j)/j$ is the forest treatment corresponding to age class $i\}$. Clearcutting is denoted by J, the last value of the subscript j.

Due to the evolution of the forest, s_{hi}^{p} depends on the area of the previous period in the following way:

$$s_{h1}^{p} = \sum_{i=1}^{I} x_{hiJ}^{p} , \qquad h = 1, 2, \ldots, H$$

$$s_{hi}^{p} = s_{h(i-1)}^{(p-1)} - x_{h(i-1)J}^{p} , \qquad i = 2, \ldots, I\text{-}1; \quad h = 1, 2, \ldots, H$$

$$s_{hI}^{p} = s_{h(I-1)}^{(p-1)} - x_{h(I-1)J}^{p} + s_{hI}^{(p-1)} - x_{hIJ}^{p} , \qquad h = 1, 2, \ldots, H$$

In our context, the following premises summarize the wishes of the decision-maker:

- Timber production should be such that non declining yield occurs, for each period into which the time horizon is divided.
- Whenever possible avoid clearcutting at early ages.
- The area covered by each age class should be roughly the same by the end of the planning horizon.
- The Net Present Value (NPV) must be higher than a certain threshold throughout the planning period.

We formalize the previous premises as goals, that is, as soft constraints and thus our model becomes a Goal Programming problem. The preferences regarding the satisfaction of the goals are modelled by using the lexicographic approach according to their priority and taking into account that they are the same in each period p ($p = 1, 2, \ldots, P$).

First priority level. The area to which clearcutting is applied *($j = J$)* should be kept to ecologically acceptable levels. Thus, the area which ensures the perpetuation of the forest harvest in site class h, for period p, Se_{h}^{p} should not be exceeded. This area Se_{h}^{p} is given by the total area in site class h divided by the rotation age and multiplied by the number of years in each class. Therefore, in each period, we have the following H goals:

$$\sum_{i=1}^{I} x_{hiJ}^{p} + n1_{h}^{p} - p1_{h}^{p} = Se_{h}^{p} \quad h = 1, \ldots, H$$

where $n1_{h}^{p}$ *and* $p1_{h}^{p}$ are the negative and positive deviation variables and for each site class the positive ones are unwanted.

In addition, given that all goals have the same relevance, the function to be minimized in this level is the sum of the positive deviation variables multiplied by a normalizing coefficient in order to prevent bias.

Second priority level. We aim at keeping harvest levels to the maximum sustained yield. Thus, if V^p represents the volume harvested at period p and $v^p{}_{hij}$ is the volume per hectare harvested from each site class, age, forest treatment and period, this goal can be expressed by the following equation:

$$\sum_{h=1}^{H} \sum_{(i,j)\in N} v_{hij}^{p} x_{hij}^{p} + n2^{p} - p2^{p} = V^{p}$$

As before, the positive deviation variable is the one to be minimized.

Third priority level. We try to regulate the forest without having to sacrifice young stands in the process, so no stand under age class I-1 should be cut. Consequently, this goal is formulated as follows:

$$\sum_{h=1}^{H} \sum_{i=1}^{I-2} x_{hiJ}^{p} + n3^{p} - p3^{p} = 0$$

where the positive deviation variable is the one to be minimized.

Fourth priority level. The area covered by each age class should be roughly the same by the end of the planning horizon. This is expressed by a goal establishing that the ratio between the number of hectares in the first age class and the last age class in each period must be above a target value. Thus, this is a fractional goal formulated as follows:

$$\frac{\mathbf{S}_{1}^{p}}{\mathbf{S}_{I}^{p}} + n4^{p} - p4^{p} = \frac{1}{P} p \quad p = 1,\dots, P$$

where $\mathbf{S}_{1}^{p} = \sum_{h=1}^{H} \sum_{i=1}^{I} x_{hiJ}^{p}$ and $\mathbf{S}_{I}^{p} = \sum_{h=1}^{H} s_{h(I-1)}^{(p-1)} - x_{h(I-1)J}^{p} + s_{hI}^{(p-1)} - x_{hIJ}^{p}$.

The target values increase within each period in such a way that in the last period the target value is 1. If this last value is reached, a balanced age class distribution by the end of the last planning period is ensured. In this case, the unwanted deviation variable is the negative one.

Fifth priority level. Finally, the following goal reflects the economic objective of the model. We want to exceed a value requested by the decision-makers in each period NPV^p,

$$\sum_{h=1}^{H} \sum_{(i,j)\in N} NPV_{hij}^{p} x_{hij}^{p} + n5^{p} - p5^{p} = NPV^{p}$$

where NPV_{hij}^{p} is the Net Present Value per each hectare harvested from site class h, age class i, and treatment j at period p. The negative deviation variable is the one to be minimized.

These priority levels are applied to each period of the planning horizon. Therefore, the objective function of the model is as follows:

$$LexMin\left(f^1,...,f^P\right) =$$

$$= \left(\left\{ \sum_{h=1}^{H} \frac{p1_h^1}{Se_h^1}, p2^1, p3^1, n4^1, n5^1 \right\}, ..., \left\{ \sum_{h=1}^{H} \frac{p1_h^P}{Se_h^P}, p2^P, p3^P, n4^P, n5^P \right\} \right)$$

On the other hand, the feasible set of the model is defined by the following constraints.

We have area accounting constraints per site class and per age class during each period p ($p = 1, 2, \ldots, P$):

$$\sum_{j \in N(i)} x_{hij}^p \le s_{hi}^{(p-1)}; \quad h = 1,2,...,H; \quad i = 1,...,I; \; p = 1,2,...,P$$

We also impose constraints to control some of the model's key values. We establish constraints to control the lower bound of the total cutting area and thus guarantee the regeneration of the stands:

$$\sum_{i=1}^{I} x_{hiJ}^p \ge \beta \, Se_h^p \quad h = 1,2,...H; \, p = 1,2,..., P; \quad 0 \le \beta \le 1$$

Finally, we establish lower bounds for *Net Present Value*:

$$\sum_{h}^{H} \sum_{(i,j) \in N} NPV_{hij}^p x_{hij}^p \ge \gamma \, NPV^p \quad p = 1,2,..., P; \quad 0 \le \gamma < 1$$

The values of the parameters β and γ are calculated when the model is applied to a particular situation and depend on the decision-makers' requests.

2.3. Results and discussion

This model has been applied to the San Juan y Martínez Management Unit which has 3,984.3 hectares of *Pinus Caribaea*. The initial forest configuration is as follows:

$$\mathbf{S}^0 = \begin{pmatrix} 0.0 & 0.0 & 198.0 & 188.0 & 83.2 \\ 32.2 & 344.6 & 405.9 & 79.0 & 759.6 \\ 33.5 & 236.8 & 266.7 & 102.0 & 692.4 \\ 30.6 & 78.9 & 130.5 & 174.4 & 148.0 \end{pmatrix}$$

As indicated, the sum by columns corresponds to the number of hectares available in each age class at the starting situation, $\mathbf{S}_i^0 (i = 1,2,...,5)$

$$(96.3 \quad 660.3 \quad 1{,}001.1 \quad 543.4 \quad 1{,}683.2)$$

and the sum by rows refers to the availability of each site class, $\mathbf{S}_h^0 (h = 1,2,...4)$

$$(469.2 \quad 1{,}621.3 \quad 1{,}331.4 \quad 562.4)$$

There are four site classes in this plantation ($H = 4$) and five age classes ($I = 5$). The planning horizon, T, coincides with the rotation length and, in our context, is equal to 25 years [14]. The time unit for each planning period is 5 years and thus, we have a total of five periods ($P = 5$).

Besides applying clearcutting (treatment 4) in all age classes, the other intermediate treatments to be applied by age class are as follows: thinning 1 ($j = 1$) in age class 2, thinning 2 ($j = 2$) in age class 3 and thinning 3 ($j = 3$) in age class 4. Therefore, the problem has a total of 160 decision variables.

For the first priority level, the target values are given by $Se_h^p = Se_h^0 = \frac{1}{5} \mathbf{S}_h^0 \;\; h = 1,...,4$.

Regarding the second priority level, V^p is 138,328 m^3 for every period. For the third and fourth priority levels, the target values have already been specified in the model. Finally, for the fifth priority level, and in line with the decision-makers' requests, the minimum desired level of NPV is 790 000 pesos[2] for the first two periods and 760 000 pesos for the last three.

On the other hand, also in line with the decision-makers' requests, in order to guarantee the regeneration of stands, the value of parameter β takes the value 0.9.

Similarly, the value of γ is set to 0.9 to guarantee that the values of NPV in each period are always more than or equal to 90% of the set target values.

---

[2] 25 Cuban pesos ≅ 1 $.

The resolution of the problem was done with the program *PFLMO* [2] using the resolution method described in Section 2. Given the high level of initial uneven age class distribution in the plantation we were forced to relax the target values of the fractional goal for period 3, from 0.6 to 0.5, which had no effect on the final equilibrium achieved. After this adjustment, *PFLMO* found solutions that satisfied all the goals, and therefore even-aged solutions by the end of the planning horizon.

Once the existence of solutions verifying all the target values was established, the efficiency of the solution obtained was restored. In this case the restoration technique used was the Interactive Restoration method that allows the decision-makers to work with several options at this new stage of problem resolution. The decision-makers chose *NPV*, the economic objective, to be the one to maximize within the set of solutions verifying all the problem goals. Thus, the solution obtained after restoration achieved a balance age class distribution by the end of the planning horizon and satisfied all the ecological goals of the problem while yielding the greatest *NPV* for the company.

The solution obtained is shown in Appendix 1 as Solution 1 (Table 2). As shown in Table 1, the *NPV* for the company is 4 151 784 pesos – which is quite high if we take into account that the target value was around 3 860 000 pesos. However, the decision-makers did not consider this solution to be ecologically acceptable because it meant applying clearcutting to a large number of hectares of age class 4 in all the periods.

The decision-makers wanted to impose a stricter constraint on clear-cutting in age class 4. Therefore, we choose a solution that satisfied the goals and such that only a maximum of 15% of the total age class 4 area available for each site class and in each period was available for clearcutting. The solution obtained in this case is given in Appendix 1 as Solution 2 (Table 3). Total *NPV* is 4 067 495 pesos and the total number of hectares cut in age class 4 is 109.26 (Table 1).

The decision-makers also wanted to obtain the solution which, while satisfying all the target values, involved the least amount of cutting of age class 4, in order to compare such a solution with the previous ones. This solution is shown in Appendix 1 as Solution 3 (Table 4). In this case, the clearcutting of age class 4 stands is only done during the first period and, as shown in Table 1, only a very small percentage of the total age class 4 area is involved, i.e. 0.6%. However, the *NPV* obtained with this solution is lower than in previous solutions, i.e. 4 000 371 pesos.

Table 1

Comparison between solutions

	Cutting in age 4 (ha.)	*NPV (pesos)*
SOLUTION 1	791.94	4 151 784
SOLUTION 2	398.77	4 067 495
SOLUTION 3	1.26	4 000 371

Bearing these solutions in mind, the decision-makers evaluated the different alternatives provided and chose Solution 2. This solution satisfies all the target values and only 15% of age class 4 underwent clearcutting. In addition, the *NPV* in this solution is 4 067 495 pesos. The decision-makers were fully satisfied with this solution and so the resolution process ended.

Figure 1 shows the evolution of each age class during the different planning periods for the solution chosen by the decision-makers. As we can see, the area covered by each age class has been balanced by the last period of the planning horizon.

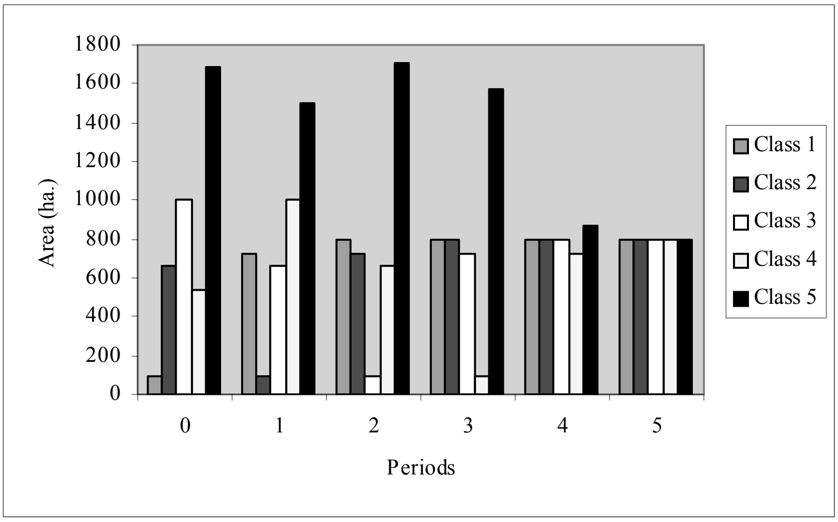

Fig. 1. Evolution of each age class during the planning periods

CONCLUSIONS

This work aims at providing a simple and useful technique for solving a goal programming problem with linear fractional criteria when there are points in the feasible set that satisfy all the goals. The theoretical results demonstrate that we can use an associated linear problem to check whether the problem has solutions that satisfy the goals and, in this case, to find them.

In addition, as an application, we have proposed a model to calculate the area to be harvested in a plantation in Cuba, in each site class during each period while maximizing profits, without having a harmful impact on the ecosystem. It ensures a balanced age class distribution in the plantation by the end of the planning horizon, which fully satisfies the wishes of the decision-makers, thereby solving the company's requirements.

The fractional goal models the decision-makers' desire for a balance age class distribution in a way that takes into account the dynamic aspect of the problem, also ensuring that those solutions which satisfy the goals fulfil this desire. All this is achieved without giving up the financial objective. Thus, the model we offer not only achieves an even-aged distribution of the forest, but also enables its efficient exploitation.

Furthermore, the model allows us to calculate the number of hectares undergoing different treatments (indicating the timber volume to be extracted in each planning period), to know the Net Present Value generated by such management planning, and also to reduce clearcutting during the planning horizon.

APPENDIX 1

The selected solutions are shown below. The first column (named FRACT) shows the value of each solution for the fractional goal at each period. Columns 2-5 show the hectares undergoing different management treatments. In column 6 we specifically show the number of hectares for clear cutting in age class 4 for each period. Finally, we show the *NPV* generated by the solutions in each period, expressed in Cuban pesos.

Table 2

Solution 1

	FRACT	T1 (ha.)	T2 (ha.)	T3 (ha.)	T4 (ha.)	T4 age 4 (ha.)	NPV (pesos)
P.1	0.51309	660.3	1001.1	335.5131	755.0499	207.8869	857,945
P.2	0.47551	96.3	660.3	738.5976	796.8596	93.84	848,004
P.3	0.5177	670.594	96.3	581.314	796.8601	78.9861	787,365
P.4	0.95014	703.02	755.05	2.9515	796.86	93.3485	792,759
P.5	1	703.02	796.86	437.1714	796.8604	317.879	865,711

Table 3

Solution 2

	FRACT	T1 (ha.)	T2 (ha.)	T3 (ha.)	T4 (ha.)	T4 age 4 (ha.)	NPV (pesos)
P.1	0.486205	660.3	1001.1	431.83	728.422	81.51	854,400
P.2	0.468075	30.6	660.3	532.183	796.86	110.16	823,388
P.3	0.508897	643.966	96.3	576.905	796.8598	83.3948	783,699
P.4	0.920908	423.7923	643.966	81.855	796.86	14.445	783,686
P.5	1	112.48	703.02	382.6762	796.8599	109.2633	822,322

Table 4

Solution 3

	FRACT	T1 (ha.)	T2 (ha.)	T3 (ha.)	T4 (ha.)	T4 age 4 (ha.)	NPV (pesos)
P.1	0.47513	660.3	1001.1	456.05	717.174	1.256	852,291
P.2	0.465003	30.6	486.889	397.2	796.86	0	795,441
P.3	0.505267	171.994	96.3	660.3	796.86	0	772,549
P.4	0.909091	357.815	632.718	96.3	796.86	0	780,418
P.5	1	112.48	501.481	340.884	796.86	0	799,672

REFERENCES

1. Awerbuch S., Ecker, J.G., Wallace W.A.: A Note: Hidden Nonlinearities in the Application of Goal Programming. "Management Science" 1976, 22, pp. 918--920.
2. Caballero R., Hernández M.: *PFLMO* (Programación Fraccional Lineal Multiobjetivo). Computer Software. R.P.I.: MA903-2003.
3. Caballero R., Hernández M.: Restoration of Efficiency in a Goal Programming Problem with Linear Fractional Criteria. "European Journal of Operational Research" 2005 (in press).
4. Díaz-Balteiro L., Romero C.: Modeling Timber Harvest Scheduling Problems with Multiple Criteria: An Application in Spain. "Forest Science" 1998, 44, pp. 47--57.
5. Díaz-Balteiro L., Romero C.: Forest Management Optimisation Models when Carbon Captured Is Considered: A Goal Programming Approach. "Forest Ecology and Management" 2003, 174, pp. 447-457.
6. Diaz-Balteiro L., Romero C.: Sustainability of Forest Management Plans; A Discrete Goal Programming Approach. "Journal of Environmental Management" 2004, 71, pp. 351-359.
7. Field D.B.: Goal Programming for Forest Management. "Forest Science" 1973, 19, pp. 125-135.
8. Field R., Dress P.E., Fortson J.C.: Complementary Linear and Goal Programming Procedures for Timber Harvest Scheduling. "Forest Science" 1980, 26(1), pp. 121--133.
9. Hannan E.L.: Effects of Substituting a Linear Goal for a Gractional Goal in the Goal Programming Problem. "Management Science" 1977, 24 (1), pp. 105--107.
10. Hannan E.L.: An Interpretation of Fractional Objectives in Goal Programming as Related to Papers by Awerbuch et al. and Hannan. "Management Science" 1981, 27 (7), pp. 847-848.
11. Hotvedt J.E.: Application of Linear Goal Programming to Forest Harvest Scheduling. "Southern Journal of Agricultural Economics" 1983, 15, pp. 103--108.
12. Kao C., Brodie J.D.: Goal Programming for Reconciling Economic, Even Flow, and Regulation Objectives in Forest Harvest Scheduling. "Can. J. Forest Res" 1979, 9, pp. 525-531.
13. Kornbluth J.S.H., Steuer R.E.: Goal Programming with Linear Fractional Criteria. "European Journal of Operational Research" 1981, 8, pp. 58-65.
14. León M.A., Caballero R., Gómez T., Molina J.: Modelización de los problemas de ordenación forestal con múltiples criterios. Una aplicación a la economía forestal cubana. "Estudios de Economía Aplicada" 2003, 21-2, pp. 339-360.
15. Soyster A.L., Lev B.: An Interpretation of Fractional Objectives in Goal Programming as Related to Papers by Awerbuch et al. and Hannan. "Management Science" 1978, 24, pp. 1546-1549.