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MULTICRITERIA DECISION AID UNDER UNCERTAINTY

INTRODUCTION

Decision making under uncertainty is a very important area of decision theory. Uncertainty implies that in certain situations a person does not have the information which quantitatively and qualitatively is appropriate to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristics [27]. Thus uncertainty relates to a state of the human mind, i.e., lack of complete knowledge about something [22].

In earlier works the term "risk" was applied to the situations in which probabilities of outcomes are known objectively. Nowadays the term "risk" means a possibility of something bad happening [5]. The term "uncertainty" is applied to the problems in which alternatives with several possible outcomes exist.

The sources of uncertainty may be divided into two main groups: internal and external. Internal sources of uncertainty are created by imprecision of human judgment as regards the specification of preferences or values or the assessment of consequences of actions [22]. In the MCDA approach we find a wide range of methods and techniques suitable to deal with uncertainty created by internal factors: sensitivity analysis (e.g. [19]), fuzzy set approach (e.g. [12; 2]), rough set approach (e.g. [8; 9]).

External uncertainty refers to lack of knowledge about the consequences of our choices [22]. For these types of problems the following methods are applied: probabilistic models and expected utility (e.g. [11; 1; 20]), pairwise comparisons based on stochastic dominance (e.g. [4; 15]). Risk measures as surrogate criteria are also applied (e.g. [16; 21; 10]). In problems where we have to take into account external uncertainty the scenario planning approach may be applied (e.g. [13; 6; 18; 23]).

1. SCENARIO PLANNING

Scenario planning was developed as a technique for facilitating the process of identifying uncertain and uncontrollable factors which may influence the consequences of decisions in the strategic management context. Scenario analysis is widely accepted as an important component of strategic planning. Scenario planning may be regarded as a process of organizational learning, distinguished by the emphasis on the explicit and ongoing consideration of multiple futures. The following five principles should guide a scenario construction:

- at least two scenarios are required to reflect uncertainty,
- each scenario must be plausible, that is, it can be seen to evolve in a logical manner from the past and present,
- each scenario must be internally consistent,
- scenarios must be relevant to the DM's concerns and must provide a useful comprehensive and challenging framework against which the DM can develop and test strategies and action plans,
- the scenarios must produce a novel perspective on the issues of concern to the DM [24].

Most users of scenario planning avoid formal evaluations, preferring to leave the selection of strategy to informal judgment [22, p. 461]. There are few papers whose authors deal with the scenario planning and multi-criteria decision analysis (e.g. [13; 6; 25; 18; 23; 22]).

In this paper we propose three multi-criteria decision aiding procedures under uncertainty based on the scenario planning approach.

2. PROPOSED METHODS FOR DECISION AIDING

2.1. Problem formulation

We consider the "traditionally understood" problem of decision making under uncertainty and therefore we assume that we don't know the probabilities of the states of nature. A discrete set of alternatives and a discrete set of scenarios have been selected for the purpose of evaluating alternatives. For single-criterion problems we can apply decision rules, but here we consider the existence of multiple criteria.

First, we define the dominance relation which can be used for pre-selection of alternatives. Next, we discuss four decision aiding methods. We propose a hierarchy and quasi-hierarchy approach, for situations when DM is able to formulate his preferences in the form of order of criteria. For cases when DM can describe weights of criteria we propose the use of the distance function. Finally an interactive approach based on the idea of IMGP [17] is proposed.

Let:

n – number of alternatives,
m – number of scenarios,
K – number of criteria.

For simplicity let us assume that the values of all criteria are maximized.

Let the vector:

$$A_i^k = [a_{i1}^k, \ldots, a_{im}^k] \tag{1}$$

denote values of the k-th criterion of the i-th alternative. The matrix:

$$A^k = [a_{ij}^k]_{n \times m} \tag{2}$$

consists of n vectors A_i^k; it shows the values of the k-th criterion for all alternatives in each considered scenario.

2.2. The dominance relation

The proposed dominance relation is based on the "traditional" single--criterion max-min decision rule. Let:

$$\overline{a}_i^k = \min_{j=1,\ldots,m} a_{ij}^k \tag{3}$$

Thus $\overline{a}_i^k$ represents the worst value of the k-th criterion of the i-th alternative. A_i dominates A_j if:

$$A_i > A_j \Leftrightarrow \underset{k}{\forall} \overline{a}_i^k \geq \overline{a}_j^k \wedge \underset{k}{\exists} \overline{a}_i^k > \overline{a}_j^k \tag{4}$$

On the basis of the proposed dominance relation we can define the optimal alternative and efficient alternatives as follows:

optimal alternative $\overset{*}{A_i}$:

$$\overset{*}{A_i} \Leftrightarrow \underset{w \neq i}{\forall} A_i > A_w \tag{5}$$

efficient alternatives:

$$\widetilde{A}_i \Leftrightarrow \neg \underset{w \neq i}{\exists} A_w > A_i \tag{6}$$

The proposed dominance relation reflects the strong risk aversion approach to decision analysis. During the decision analysis we can search for efficient alternatives in the preselection of the universe of alternatives or – if we find an alternative suggested as a final decision – we can examine whether this alternative is efficient or not. Examples presented below show the use of the idea of proposed dominance relation at the first stage of the decision analysis (during the preselection of alternatives).

Example 1

Number of alternatives: $n = 4$
Number of scenarios: $m = 4$
Number of criteria: $K = 2$

Table 1

$\mathbf{A^1}$	S_1	S_2	S_3	S_4	MIN	
A_1	10	6	**4**	14	4	
$\mathbf{A_2}$	11	9	13	**8**	8	MAX
A_3	15	**5**	12	7	5	
$\mathbf{A_4}$	8	10	11	9	8	MAX

Table 2

$\mathbf{A^2}$	S_1	S_2	S_3	S_4	MIN	
A_1	110	**100**	120	130	100	
$\mathbf{A_2}$	130	**120**	140	150	120	MAX
A_3	130	**70**	80	110	70	
A_4	100	150	140	**30**	30	

It easy to see that alternative A_2 is optimal in the sense of the proposed dominance relation.

Example 2

Number of alternatives: n = 5
Number of scenarios: m = 4
Number of criteria: K = 3

Table 3

A^1	S_1	S_2	S_3	S_4	MIN
A_1	11	10	9	8	8
A_2	10	12	15	17	10
A_3	13	11	12	15	11
A_4	13	14	12	15	12
A_5	15	10	18	9	9

Table 4

A^2	S_1	S_2	S_3	S_4	MIN
A_1	140	160	190	180	140
A_2	160	150	175	190	150
A_3	130	160	120	200	120
A_4	200	150	145	130	130
A_5	150	110	140	130	110

Table 5

A^3	S_1	S_2	S_3	S_4	MIN
A_1	16	17	19	15	15
A_2	21	18	16	17	16
A_3	16	21	22	18	16
A_4	21	17	19	17	17
A_5	15	10	18	11	10

To simplify evaluation we show min values of each criterion for all alternatives ($\overline{a}_i^k$) in the next table:

Table 6

Criterion / Alternative	k = 1 $\overline{a}_i^1$	k = 2 $\overline{a}_i^2$	k = 3 $\overline{a}_i^3$
A_1	8	140	15
A_2	10	150	16
A_3	11	120	16
A_4	12	130	17
A_5	9	110	10

We can see that alternative A_2 dominates alternative A_1, alternative A_2 dominates alternative A_5, and alternative A_4 dominates alternative A_3. Finally we conclude that there is no optimal alternative in this case and that alternatives A_3 and A_4 are the efficient ones.

2.3. The hierarchy and quasi-hierarchy approach

Let us assume that DM is able to order criteria from the most to the least important. The lower the index of a criterion, the higher its importance:

$$k_1 \rangle k_2 \rangle \ldots\ldots \rangle k_m$$

2.3.1. The hierarchy approach

In the first step of this procedure we look for best alternatives with respect to the most important criterion (in the sense of traditional max-min rule). These alternatives are included in the first subset of alternatives $\widetilde{A}^{k_1}$. Next we consider the second criterion, obtain a subset of alternatives, and repeat the calculations.

Step 1

$$\widetilde{A}^{k_1} = \{A_i : \max_{i=1,\ldots m} \overline{a}_i^{k_1}\} \tag{7}$$

Step (t = 2,...K)

$$\widetilde{A}^{k_t} = \{\widetilde{A}^{k_{t-1}}{}_i : \max_{i=1,\ldots m} \overline{a}_i^{k_t}\} \tag{8}$$

Example 3

Example 3 shows the use of the hierarchy approach.

Number of alternatives: $n = 4$
Number of scenarios: $m = 4$
Number of criteria: $K = 3$

Table 7

A^1	S_1	S_2	S_3	S_4	Min
A_1	11	**10**	13	15	10
A_2	10	**8**	14	13	8
A_3	15	13	12	**10**	10
A_4	12	10	**8**	7	7

Table 8

A^2	S_1	S_2	S_3	S_4	Min
A_1	10	15	9	13	9
A_2	13	**11**	12	15	11
A_3	16	**13**	14	20	13
A_4	18	19	17	**16**	16

Table 9

A^3	S_1	S_2	S_3	S_4	Min
A_1	**10**	11	14	15	10
A_2	18	16	**12**	13	12
A_3	**14**	15	19	20	14
A_4	16	**15**	17	19	15

After the first step of procedure we select alternatives A_1 and A_3. In the second step we choose alternative A_3. Because the subset of alternatives consists of only one alternative, the procedure stops. We can see that criterion k_3 doesn't influence the final result. The alternative A_3 is suggested as the final decision.

2.3.2. Quasi-hierarchy approach

In this approach the decision maker describes the tolerance limit for each criterion. Let q_t denote the tolerance limit described for t^{th} criterion. Thus in the first step we should find the subset defined as follows:

Step 1

$$\widetilde{A}^{k_1} = \{A_i : \overline{a}_i^{k_1} \geq \max_{i=1,\dots m} \overline{a}_i^{k_1} - q_1\} \tag{9}$$

Step (t = 2,...K)

In steps $k_2,\dots,k_K$ we find the following subsets of the set of alternatives:

$$\widetilde{A}^{k_t} = \{\widetilde{A}^{k_{t-1}}{}_i : \overline{a}_i^{k_t} \geq \max_{i=1,\dots m} \overline{a}_i^{k_t} - q_t\} \tag{10}$$

Example 4

We consider data from Example 3 and tolerance limits described as follows: $q_1 = 2$, $q_2 = 3$, $q_3 = 1$. After the first step of procedure we select alternatives A_1, A_2 and A_3. In the second step we choose alternatives A_2 and A_3. Finally, taking into account the third criterion, we select the alternative A_3 which is suggested as the final decision.

2.4. The distance function

Let us assume that DM is able to describe the importance of the criteria using criteria weights $\mathbf{w_k}$, k = 1,…,K:

$$w_k \geq 0, \sum_{k=1}^{K} w_k = 1 \tag{11}$$

Let $\widehat{A}$ be the „ideal pessimistic point”:

$$\widehat{A} = [\widehat{a}_k : \widehat{a}_k = \max_{i=1,\dots,m} \overline{a}_i^{\,k} ; k = 1,\dots,K] \tag{12}$$

Let D be the distance function which measures the distance between the alternative considered and the ideal pessimistic point. The function D can be defined as follows:

$$D(A_i, \widehat{A}) = \sqrt{\sum_{k=1}^{K} w_k (\overline{a}_i^k - \widehat{a}_k)^2} \tag{13}$$

The lower the value of function D, the better the evaluation of the alternative; therefore, the alternative with the minimal value of function D should be suggested as the final decision.

Example 5

Number of alternatives: $n = 4$
Number of scenarios: $m = 4$
Number of criteria: $K = 3$
Criteria weights: $w_1 = 0.4$
$w_2 = 0.5$
$w_3 = 0.1$

Table 10

A^1	S_1	S_2	S_3	S_4	Min
A_1	**11**	**10**	**13**	**15**	**10**
A_2	10	**8**	14	13	8
A_3	**15**	**13**	**12**	**10**	**10**
A_4	12	10	8	7	7

Table 11

A^2	S_1	S_2	S_3	S_4	Min
A_1	10	15	**9**	13	9
A_2	13	**11**	12	15	11
A_3	16	**13**	14	20	13
A_4	**18**	**19**	**17**	**16**	**16**

Table 12

A^3	S_1	S_2	S_3	S_4	Min
A_1	**10**	11	14	15	10
A_2	18	16	**12**	13	12
A_3	**14**	15	19	20	14
A_4	**16**	**15**	**17**	**19**	**15**

The ideal pessimistic point is equal to $\widehat{A} = [10, 16, 15]$. The calculated distances from this point are presented in Tables 13 and 14.

Table 13

$$w_k(\overline{a}_i^k - \widehat{a}_k)^2$$

	k = 1	k = 2	k = 3
A_1	0.0	24.5	2.5
A_2	1.6	12.5	0.9
A_3	0.0	4.5	0.1
A_4	3.6	0.0	0.0

Table 14

Alternative	Distance: D
A_1	5.196152
A_2	3.872983
A_3	2.144761
A_4	**1.897367**

Looking at the distances from the ideal pessimistic point obtained above it is easy to see that A_4 has the lowest value of the distance function D. Therefore, alternative **A_4** should be suggested as the proposed decision.

2.5. An interactive procedure

We propose an interactive procedure based on the idea of Interactive Multiple Goal Programming (IMGP) suggested by Spronk [17]. Some important advantages are related to the IMGP approach.

First, the DM does not have to give his preference information on an a priori basis but has to consider all kinds of choices and trade-off issues which may be relevant (see [17, p. 104]). Another important advantage of IMGP is its relatively simple and easy to understand main idea. Finally, during an interactive procedure the DM has to answer the following simple questions:

1. Is the given solution acceptable or not?
2. Which goal value needs to be improved?
3. By how much (at least) should this goal value be improved?
4. Do you accept the consequences of the proposed improvement of the value of the indicated goal variable? (see [17, p. 250]).

The proposed interactive procedure also uses a potency matrix during decision aiding process but here the potency matrix consists of three vectors: ideal optimistic point, ideal pessimistic one, and current pessimistic solution which are defined below:

Let $\overset{***}{A}$ be the "ideal optimistic point" defined as follows:

$$\overset{***}{A} = [\overset{***}{a}_k : \overset{***}{a}_k = \max_{i=1,\dots,m} \max_{j=1,\dots,n} a_{ij}^k ; k = 1,\dots,K] \tag{14}$$

Let $\widehat{A}$ be the "ideal pessimistic point":

$$\widehat{A} = [\widehat{a}_k : \widehat{a}_k = \max_{i=1,\dots,m} \overline{a}_i^k ; k = 1,\dots,K] \tag{15}$$

Let $\breve{A}$ be the "current pessimistic solution" defined as follows:

$$\breve{A} = [\breve{a}_k : \breve{a}_k = \min_{i=1,\dots,m} \overline{a}_i^k ; k = 1,\dots,K] \tag{16}$$

The Potency Matrix P is described below (where "r" is the index showing the consecutive number of the iteration):

$$P^r = \begin{bmatrix} \overset{***}{A} \\ \widehat{A} \\ \breve{A} \end{bmatrix}$$

The decision aiding procedure can be written in form of three main steps.

Step 1

Calculate the first potency matrix P^1 presented to DM.

Step 2

After the analysis of potency matrix, DM chooses the criterion according to which the value for the current solution should be improved, and decides by how much this value will be improved. Thus DM chooses criterion **k** and describes the accepted value of that criterion: $\mathbf{d_k}$

$$\breve{a}_k < d_k \leq \hat{a}_k$$

Step 3

Alternatives which don't meet conditions set up by DM in the previous step are deleted from the set of alternatives and the new potency matrix P^r is calculated. DM compares values presented in the current potency matrix with values from the previous one. DM should decide whether he accepts the consequences of his last decision (he considers the local trade-offs between criteria).

a) If DM accepts the new solution we go back directly to Step 2.

b) If DM doesn't accept the new solution then the last condition put on criteria value is omitted and the previous set of alternatives is restored. Then go to Step 2.

Stop condition

The procedure stops when there is only one alternative in the set of current alternatives and DM accepts the last solution (potency matrix).

The flow chart of the procedure is presented on the next page.

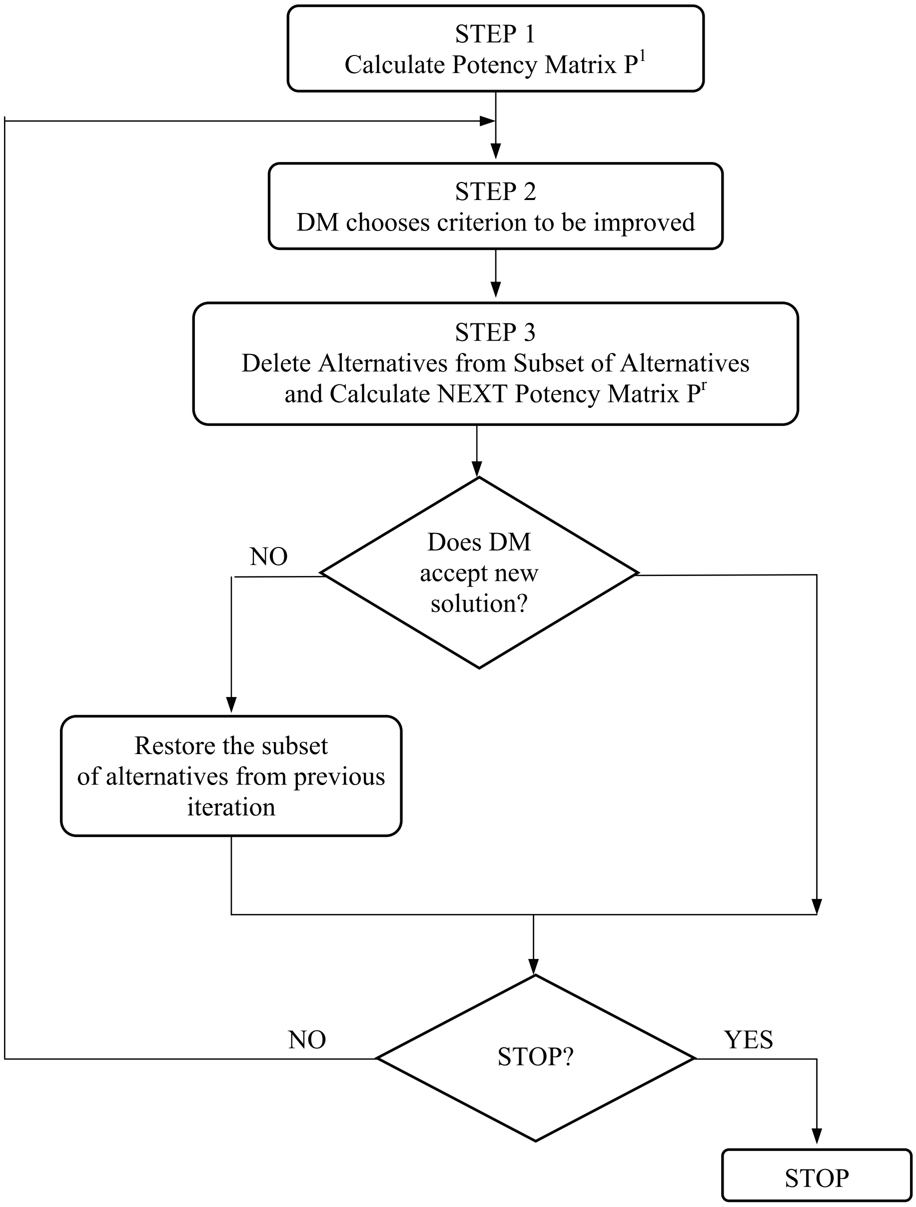

Fig. 1. The flow chart of the procedure

The next example presents the application of the proposed interactive decision aiding procedure.

Example 6

Number of alternatives: n = 4
Number of scenarios: m = 4
Number of criteria: K = 3

Table 15

A^1	S_1	S_2	S_3	S_4	min	max
A_1	7	8	10	**12**	7	12
A_2	14	**6**	11	13	6	14
A_3	11	**10**	11	**10**	**10**	11
A_4	18	15	10	9	9	**18**

Table 16

A^2	S_1	S_2	S_3	S_4	min	Max
A_1	**10**	13	14	16	**10**	**16**
A_2	**9**	11	13	**14**	9	14
A_3	12	**16**	**8**	10	**8**	**16**
A_4	**10**	11	13	**14**	**10**	14

Table 17

A^3	S_1	S_2	S_3	S_4	min	Max
A_1	21	23	4	9	**4**	**23**
A_2	20	15	16	18	**15**	20
A_3	15	13	10	11	10	15
A_4	14	15	13	14	13	15

The first calculated potency matrix is shown below:

Table 18

P^1	k = 1	k = 2	k = 3
Ideal optimistic point (max-max)	18	16	23
Ideal pessimistic point (max-min)	10	10	15
Current pessimistic solution (min-min)	6	8	4

Let us assume that the decision maker (DM) wants to improve the value of the third criterion and decides that the value of this criterion should be equal to at least 8. Then the set of alternatives is examined and as a result alternative A_1 is deleted from this set.

Table 19

A^1	S_1	S_2	S_3	S_4	min	Max
~~A_1~~	~~7~~	~~8~~	~~10~~	~~12~~	~~7~~	~~12~~
A_2	14	6	11	13	**6**	14
A_3	**11**	**10**	**11**	**10**	**10**	11
A_4	**18**	15	10	9	9	**18**

Table 20

A^2	S_1	S_2	S_3	S_4	min	max
~~A_1~~	~~10~~	~~13~~	~~14~~	~~16~~	~~10~~	~~16~~
A_2	**9**	11	13	**14**	9	14
A_3	12	**16**	**8**	10	**8**	**16**
A_4	**10**	11	13	**14**	**10**	14

Table 21

A^3	S_1	S_2	S_3	S_4	min	max
~~A_1~~	~~21~~	~~23~~	~~4~~	~~9~~	~~4~~	~~23~~
A_2	**20**	**15**	16	18	**15**	**20**
A_3	**15**	13	**10**	11	**10**	15
A_4	14	**15**	**13**	14	13	15

Thus the second potency matrix is:

Table 22

P^2	k = 1	k = 2	k = 3
Ideal optimistic point (max-max)	18	16	20
Ideal pessimistic point (max-min)	10	10	15
Current pessimistic solution (min-min)	6	8	10

Let us assume that the decision maker (DM) accepts the new solution and decides to increase the value of the second criterion to 9. Alternative A_3 is deleted from the set of alternatives. The third potency matrix is presented to DM.

Table 23

P^3	k = 1	k = 2	k = 3
Ideal optimistic point (max-max)	18	14	20
Ideal pessimistic point (max-min)	9	10	15
Current pessimistic solution (min-min)	6	9	13

DM accepts the results and decides to improve the first criterion which should be at least equal to 8 (A_2 is deleted).

Table 24

P^4	k = 1	k = 2	k = 3
Ideal optimistic point (max-max)	18	14	15
Ideal pessimistic point (max-min)	9	10	13
Current pessimistic solution (min-min)	9	10	13

Let us assume that DM does not accept the last solution. DM analyzes again the third potency matrix P^3 (A_2 is restored to set of alternatives) and decides to increase the value of the third criterion to 15. Alternative A_4 is deleted from further considerations and the potency matrix $P^{4'}$ obtained above is presented to DM.

Table 25

$P^{4'}$	**k = 1**	**k = 2**	**k = 3**
Ideal optimistic point (max-max)	14	14	20
Ideal pessimistic point (max-min)	6	9	15
Current pessimistic solution (min-min)	6	9	15

DM accepts this solution; the set of current alternatives consists of one element only – the procedure stops. Alternative A_2 is suggested as the final decision.

CONCLUSIONS

In this paper we discussed the problem of decision making under uncertainty. The main approaches to MCDA in uncertainty are shortly described. The scenario planning was applied as a useful technique to deal with uncertainty and the three procedures were proposed with respect to the way in which DM preferences are reflected. The hierarchy and quasi-hierarchy approach can be easily expanded to reflect the group hierarchy of the criteria.

The approach based on the distance function can be easily modified using different measures or taking into consideration the position of alternatives with respect to two reference points at the same time (e.g. ideal optimistic, ideal pessimistic).

The interactive procedure is the most flexible: it may be used without any a priori knowledge about DM's preferences and can also be applied when criteria are on the ordinal scale.

The proposed interactive procedure can be easily applied in real life problems; one of the main areas of such approach is the strategic management.

REFERENCES

1. Bazerman M.H.: Judgment in Managerial Decision Making. John Wiley & Sons, New York 2002.
2. Chang N.B., Wang S.: A Fuzzy Goal Programming Approach for the Optimal Planning of Metropolitan Solid Waste Management Systems. "European Journal of Operations Research" 1997, 99, pp. 303-321.
3. Cook W.D, Kress M.A.: Multiple Criteria Decision Model with Ordinal Preference Data. "European Journal of Operations Research" 1991, 54, pp. 191-198.

4. D'Avignon G.R.D, Vincke P.: An Outranking Method under Uncertainty. "European Journal of Operations Research" 1988, 36, pp. 311-321.
5. Fishburn P.C.: Foundations of Risk Measurement. I. Risk or Probable Loss. "Management Science" 1984, 30, pp. 396-406.
6. Goodwin P., Wright G.: Decision Analysis for Management Judgment. John Wiley & Sons, 1997.
7. Goicoechea A., Hansen D.R. Duckstein L.: Multiobjective Decision Analysis with Engineering and Business Applications. John Wiley & Sons, New York 1982.
8. Greco S., Matarazzo B., Slowinski R.: Rough Approximation of a Preference Relation by Dominance Relations. "European Journal of Operations Research" 1999, 117, pp. 63-83.
9. Greco S., Matarazzo B., Slowinski R.: Rough Sets Theory for Multicriteria Decision Analysis. "European Journal of Operations Research" 2001, 129, pp. 1-47.
10. Jia J., Dyer J.S.: A Standard Measure of Risk and Risk-Value Models. "Management Science" 1996, 42, pp. 1691-1705.
11. Kahnemann D., Tversky A.: Prospect Theory an Decision Analysis under Risk. "Econometrica" 1976, 47, pp. 263-291.
12. Klir G.J., Fogler T.A.: Fuzzy Sets. Uncertainty and Information. Prentice Hall, New Jersey 1988.
13. Klein G., Moskowitz H., Ravindran A.: Interactive Multiobjective Optimization under Uncertainty. "Management Science" 1990, 36, pp. 58-75.
14. Lahdhelma R., Salminen P.: Pseudo-Criteria versus Linear Utility Function in Stochastic Multi-Criteria Acceptability Analysis. "European Journal of Operations Research" 2002, 141, pp. 454-496.
15. Martel J.M., and Zaraś K.: Stochastic Dominance in Multicriterion Analysis under Risk. "Theory and Decision" 1995, 3, pp. 31-49.
16. Millet I., Wedley W.C.: Modelling Risk and Uncertainty with the AHP. "Journal of Multi-Criteria Decision Analysis" 2002, 11, pp. 97-107.
17. Nijkamp P., Spronk J.: Interactive Multiple Goal Programming: An Evaluation and Some Results. In: Multiple Decision Making Theory and Application. Eds. G. Fandel, T. Gal. Springer Verlag, Berlin 1980, pp. 278-293.
18. Pomerol J.C.: Scenario Development and Practical Decision Making under Uncertainty. "Decision Support Systems" 2001, 31, pp. 197-204.
19. Rios Insua D.: Sensitivity Analysis in Multi Objective Decision Making. Lecture Notes in Economic and Mathematical System, Vol. 347, Springer Verlag, Berlin 1990.
20. Rosquist T.: Simulation and Multi-Attribute Utility Modeling of Life Cycle Profit. "Journal of Multi-Criteria Decision Analysis" 2001, 10, pp. 205-218.
21. Sarin R.K. and Weber M.: Risk-Value Models. "European Journal of Operations Research" 1993, 70, pp. 135-149.
22. Stewart D.J.: Uncertainties in MCDA. In: Multi Criteria Decision Analysis. Ed. P. Greco. Springer Verlag, 2004.

23. Urli B., Nadeau R.: PROMISE/scenarios: An Interactive Method for Multiobjective Stochastic Linear Programming under Partial Uncertainty. "European Journal of Operations Research" 2004, 155, pp. 361-372.
24. Van der Heijden K.: Scenarios: The Art of Strategic Conversation. John Wiley & Sons, 1996.
25. Watkins D.W. Jr., McKinney D.C., Lasdon L.S., Nielsen S.S., Martin Q.W.: A Scenario-Based Stochastic Programming Model for Water Supplies from the Highland Lakes. "International Transactions in Operational Research" 2000, 7, pp. 211-230.
26. Yilmaz M.R.: An Information-Expectation Framework for Decision under Uncertainty. "Journal of Multi-Criteria Decision Analysis" 1992, 1, 65-80.
27. Zimmermann H.: An Application-Oriented View of Modeling Uncertainty. "European Journal of Operations Research" 2000, 122, pp. 190-198.