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BICRITERIAL ROBUST APPROACH IN PROJECT MANAGEMENT

1. ROBUST APPROACH – BASIC IDEA

In recent years a new approach to decision making in the situation of uncertainty and incompleteness of information has been used more and more often. It is a so called robust approach. Generally, in robust decision making a decision has to be taken when not all the parameters of the problem are known exactly yet. The question consists in taking such a decision which will be "good enough" when all the parameters become known and definitively fixed, even in case of their unfavorable perturbations. "Good" decision means in turn such one with which the decision maker will be sufficiently satisfied or which not require any significant corrections. The approaches proposed in the literature so far and the results gained seem promising, and at the same time this domain is in its initial development stage (if compared to the progress of the research concerning the fuzzy, interval and stochastic approaches). It seems natural to try to apply this approach to optimization and management decision making in the situation of uncertainty and incompleteness of information. It is also interesting that the notion of robust solution or robust decision is not unequivocal. Different authors understand them often in quite different ways.

The general philosophy of robust decision making, or at least a philosophy which the author claims to be such, is presented in [15]. The approach can be summarized through its 3 basic stages:

1) Elimination of uncertainty – to the extent it is possible.
2) Definition of the set of satisfying solutions.
3) Selecting from the satisfying solutions the one which is least sensitive to the uncertainty which we have not been able to eliminate.

However, this description does not cover the different understanding of robust approach that can be found in the literature. They will be discussed in the next section.

2. ROBUST APPROACH – EXISTING RESULTS AND APPLICATIONS

Robust approach in discrete optimization, together with numerous results, is presented in [10]. The basic notion in the robust approach discussed in this book is that of a scenario. Each scenario corresponds to a certain possible realization of the problem parameters. Two ways of defining scenarios are possible. The first one consists in enumerating a certain finite number of various scenarios. In the second one for each parameter an uncertainty interval is defined and the set of all the scenarios is the Cartesian product of these intervals. The essence of the discussed approach consists in determining a solution which minimizes the cost function value in the worst possible case (in the worst scenario).

A slightly different robust approach can be applied to the planning and scheduling of vehicle routes. One can search for solutions which will be robust in the sense that there will be enough "space" or reserve in them to protect them against the possible lengthening of the vehicle travel or unfavourable changes in the demand or supply [9].

In [6], for the colouring problem, yet another understanding of the robust solution for discrete optimization problems is proposed: it is a solution which will remain good in changed circumstances (e.g. in the situation when new edges are added to the graph or several existing edges disappear).

The notion of robust solution in the one and multicriteria linear programming problem is understood in many ways. The authors of [5] applied also the minimal regret criterion. However, the most interesting seems to be the worst scenario approach. The worst scenario can be defined here as the occurrence of the "biggest possible" number of "unstable" coefficients which have taken on other values than expected (such an approach has been used in [3], also there it is justified from the practical point of view: in practice usually not all the problem parameters attain the unfavorable values, and limiting the number of coefficients which may vary does not mean indicating which ones will it be). Another understanding of the worst scenario in linear programming refers to the highest possible magnitude of the deviation of all the parameters of the problem. In [11] the robust approach with the worst scenario understood in the former sense has been extended to goal programming.

The authors of [8] indicate, in the context of project management, another robust approach, used also in the present paper. In this approach the decision is taken in the moment when the coefficients are not fully known, yet is only an initial decision, allowing to undertake adequate preparations. However, it is the optimal solution (in the classic understanding of this notion) which will be implemented, determined only in the moment, when all the coefficients will be known exactly. The robust solution is the one determined in the conditions of incomplete information. It should differ as little as possible from the one which will be implemented in practice, and this not so much with respect to the objective function value, but rather with respect to the details, like the decision variables values – so that the preparations undertaken in the moment when the robust solution is determined make the most sense possible from the point of view of the yet unknown optimal solution.

Robust project management seems to be a domain in the initial stage of development, and at the same time very promising and important from the practical point of view, as a good project management is something needed by most companies of today. It is a very broad domain. It would comprise the same various aspects as project management *per se*, i.e. among other the estimation of project duration and cost, scheduling, progress tracking. The present state of knowledge in robust project scheduling is presented in [1; 4; 8; 16]. A robust schedule is usually defined as one which will not differ very much from the actual schedule, which will be actually put in practice. The open question is the meaning of the expression "not differ very much" with respect to project schedules. In [4; 8] the expression means either the difference in the total duration of the project or in the planned and actual starting times of individual activities. In [1] a robust schedule is defined as one which maximised the sum of free slacks – this sum, called "robustness of a schedule", is a second criteria, applied to the schedule evaluation and selection together with the "traditional" criteria of critical path minimisation. This approach has been modified in [13]. In [16] the authors use the worst case criteria and understand the robust schedule as one which also in the worst case will have a high chance of meeting the deadline. In [4] we find also other ways of understanding the notion of a "robust schedule". For example, a robust project schedule is defined as a schedule with a sufficient quantity of in-built reserves (of time, cost, resources), which assure that the schedule will be protected against unfavorable deviations and will be able to be put into practice whatever the actual values of the individual project parameters are. The best known example of this type of schedule is the schedule constructed according to the critical chain method [7; 12].

In the next section we will propose a new robust approach to constructing robust schedules in project management. It will be a bicriterial approach.

3. A NEW ROBUST APPROACH TO PROJECT MANAGEMENT

Let us consider the following critical path problem:

$$
\begin{aligned}
& x_n \to \min \\
& x_j - x_i \ge d_{ij}(t) \\
& x_i \ge 0 \ \ (I = 1,...,\mathrm{n})\ (j = 1,...,\mathrm{n})
\end{aligned}
\tag{1}
$$

where $d_{ij}(t) = d_{ij}^{'} + d_{ij}^{''}t$, $t \in S = [S_1, S_2]$ are duration times of a project activities, x_i are the occurrence times of events a project network, x_n is the final event.

$d_{ij}(t) = d_{ij}^{'} + d_{ij}^{''}t$ will depend on a parameter only in the initial stage. Later the duration times will take on crisp values. The final solution will be the optimal solution of (1) for the final value of the parameter, but while this value is not known, a robust solution is searched for. This robust solution should be as close as possible to all the possible optimal solutions (for various values of the parameter), because all the preparations to the project execution have to start now, while the final value of the parameter is still unknown. Once it becomes known, we do not want to be forced to undertake big changes while adapting the current, transitional solution to the final one.

As mentioned before, there are many possible ways of understanding the robustness of a schedule. Here, similarly as in [1] and [13], we propose a bicriterial approach, but using two different criteria. That is why we start with two definitions:

Definition 1

A robust solution of (1) is such a solution $\mathbf{x^0} = (x_1^0, ..., x_n^0)$ which minimizes the following objective function:

$$
\max_{\substack{j=1,...m; \\ t \in \langle T_{r-1}, T_r \rangle \\ r=1,...,l}} \lambda_r^j \left| x_j^0 - x_i^r(t) \right|
$$

where:

- $\mathbf{x}^{\mathbf{r}}(t) = (x_1^r(t), ..., x_n^r(t))$ ($t \in \langle T_{r-1}, T_r \rangle$, $T_0 = S_0; T_l = S_1; T_{r-1} \leq T_r$ for $r = (1, ..., l)$, $\langle\ \rangle$ stands for a one side or two side open interval) are all the optimal solutions of (1) for different crisp values of t (these solutions, in which the values of the decision variables are linear functions of t, can be found by means of the parametric version of simplex algorithm for the case of the parameter in the constraints right hand side coefficients). Each solution $\mathbf{x}^{\mathbf{r}}(t)$ is valid in an interval $\langle T_{r-1}, T_r \rangle$, whose exact form follows unequivocally from the algorithm.
- λ_r^j (j = 1,...n; r = 1,...,l) are weights chosen by the decision maker to control the importance of individual variances (several of those weights can be 0, in case the corresponding variance is if no importance) and to scale or to price them adequately.
- Signs $|\ |$ stand for the absolute value.

Thus, in the above definition we assume that a robust schedule is one in which the occurrence times of the events are as close as possible to the ones in the final solution, which is yet unknown.

Definition 2

A robust solution of (1) is such a solution $\mathbf{x}^{\mathbf{0}} = (x_1^0, ..., x_n^0)$ which minimizes the following objective function:

$$\max_{\substack{i,j=1,...m \\ t \in \langle T_{r-1}, T_r \rangle \\ r=1,...,l}} \eta_r^{ij} \left| (x_j^0 - x_i^0) - (x_j^r(t) - x_i^r(t)) \right|$$

where η_r^{ij} (i,j = 1,...,n; r = 1,...,l) are weights chosen by the decision maker to control the importance of individual variances and to scale or to price them adequately, the other notation is the same as in Definition 1.

The second definition wants the transitional, robust schedule to differ as little as possible from the final one with respect to the times between the individual events. These times are strongly linked to the actual duration times of activities (comprising free floats which can be used in the actual execution of the project without influencing the scheduled events times).

The proof of the following theorem is straightforward:

Theorem 1

A schedule satisfying to some degree Definition 1 and Definition 2 can be found by means of the following bicriterial parametric linear programming problem with n+2 decision variables:

$$\begin{aligned}
& y \to \min;\ z \to \min \\
& \lambda_r^j x_j - \lambda_r^j x_j^r(t) \le y \ \ (j = 1,...,n) \\
& \lambda_r^j x_j - \lambda_r^j x_j^r(t) \ge -y \ \ (j = 1,...,n) \\
& \eta_r^{ij} x_j - \eta_r^{ij} x_i - \eta_r^{ij}\left(x_j^r(t) - x_i^r(t)\right) \le z \ \ (i,j = 1,...,n) \\
& \eta_r^{ij} x_j - \eta_r^{ij} x_i - \eta_r^{ij}\left(x_j^r(t) - x_j^r(t)\right) \ge -z \ \ (i,j = 1,...,n) \\
& y \ge 0;\ z \ge 0;\ x_j \ge 0 \ \ (j = 1,...,n) \\
& t \in \langle T_{r-1}, T_r \rangle,\ r = 1,...l.
\end{aligned} \tag{2}$$

As $x_i^r(t)$ and $x_j^r(t) - x_i^r(t)$ (for $r = 1,...l$ and $i = 1,...,n$) are linear functions of t, the maximal and minimal values of $x_i^r(t)$ are attained in points T_{r-1} or T_r and are very easy to determine. Lets us thus denote by $x_i^{r,\min}$, $x_i^{r,\max}$ the minimum and maximum, respectively, of $x_i^r(t)$ in $\langle T_{r-1}, T_r \rangle$ and by $x_{ij}^{r,\min}$, $x_{ij}^{r,ma}$ the corresponding values for functions $x_j^r(t) - x_i^r(t)$. The proof of the following lemma is straightforward.

Lemma 1

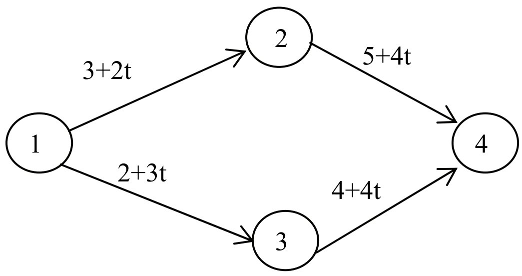
Solution of problem (3) can be found by solving the following linear programming problem:

$$\begin{aligned}
& y \to \min;\ z \to \min \\
& \lambda_r^j x_j - \lambda_r^j x_j^{r,\min} \le y \ \ (j = 1,...,n) \\
& \lambda_r^j x_j - \lambda_r^j x_j^{r,\max} \ge -y \ \ (j = 1,...,n) \\
& \eta_r^{ij} x_j - \eta_r^{ij} x_i - \eta_r^{ij} x_{ij}^{r,\min} \le z \ \ (i,j = 1,...,n) \\
& \eta_r^{ij} x_j - \eta_r^{ij} x_i - \eta_r^{ij} x_{ij}^{r,\max} \ge -z \ \ (i,j = 1,...,n) \\
& y \ge 0;\ z \ge 0;\ x_j \ge 0 \ \ (j = 1,...,n) \\
& r = 1,...l
\end{aligned} \tag{3}$$

Any multicriterial approach can be applied to the above problem. To adopt the easiest solution, we can to transform it to a one criterion problem, assigning weights to the objectives. We get then the objective function $\beta y + \chi z \to \min$ with the same constraints as above. Then, it will be enough to solve l one criteria linear programming problems (for each $r = 1,...l$) and to choose the best solution from the l solutions obtained.

4. EXAMPLE

Let us consider the following project network (t takes on values from the interval [0,5]):



If we solve problem (3) with the objective function $y + z \to \min$ (we assume both objectives to equally important), we get the following solution: $x_1 = 0,\ x_2 = 4,\ x_3 = 4,\ x_4 = 14,\ y = 6,\ z = 4.$

CONCLUSIONS

The robust approach to optimisation is very wide and promising. The robustness of a decision or solution, whatever definition of robustness has been selected, is an important feature and that is why the robustness should be used at least as one of the criteria in the decision making process. In this paper we propose one way of incorporating robustness into the search of project schedule. Other possibilities of using the robustness as a decision criterion are also mentioned in the paper and will be the object of further research.

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