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COMPARATIVE ANALYSIS OF ELECTRIC ENERGY RISK CHANGES IN SELECTED EUROPEAN REGIONS

Summary: In the paper volatility of prices from selected European electric energy markets was described using multivariate autoregressive models VAR-GARCH. Quotations from Polish TGE, European EEX, Nordic Nord Pool and center Europe OTE were used from January 2014 to October 2016. We made risk analysis of change in average daily prices and proposed a portfolio of electric energy contract. The risk of single contracts was estimated by Value-at-Risk (VaR). The risk of portfolio was estimated by Conditional Value-at-Risk (CVaR).

Keywords: VAR, DCC, VaR, CVaR, portfolio analysis.

JEL Classification: C5.

Introduction

The aim of the paper is to assess the risk of volatility in the price of electricity on the Polish Power Exchange (TGE) compared to the risk of changes in prices on the neighboring electricity markets. Markets such as Nord Pool operating in Sweden, Norway, Denmark, Finland, Lithuania, Latvia, Estonia, as well as markets coordinated by the technical operator OTE in the Czech Republic, Slovakia, Hungary and Romania and the European Electric Exchange (EEX) trading of electricity in Germany, Austria, France and Switzerland were considered.

A comparative study was carried out on the basis of average daily electricity prices [EUR/MWh] in the period from 01.01.2014 to 10.30.2016. The average daily price on day-ahead electricity markets quoted in [EUR/MWh] were represented by the index POLPX spot base (POLPX) [www 1] on TGE and by spot index ELIX base (ELIX) [www 2] on EEX. For the Nord Pool spot market which covers very diverse countries in terms of level and volatility of prices all the publicly available average prices were taken into account. These prices include [www 3]:

- SYS – average system price for the whole Nord Pool,
- SE1 (Luleå), SE2 (Sundsvall), SE3 (Stockholm), SE4 (Malmö) – average daily prices for Swedish subregions,
- FI – average daily price for Finland,
- DK1 (Aarhus), DK2 (Copenhagen) – average daily prices for Danish subregions,
- Oslo, Kr.sand, Bergen, Molde, Tr.heim, Tromsø – average daily prices for Norwegian subregions,
- EE – average daily price for Estonia,
- LV – average daily price for Latvia,
- LT – average daily price for Lithuania.

Prices for markets using the services operator OTE are represented by symbols:

- CR– average daily price for Czech Republic,
- SR – average daily price for Slovakia,
- HU – average daily price for Hungary,
- RO – average daily price for Romania.

1. Introductory data analysis

Distributions of average electric energy prices from 01.01.2014 to 30.10.2016 for selected markets are represented by box-plots in Fig. 1.

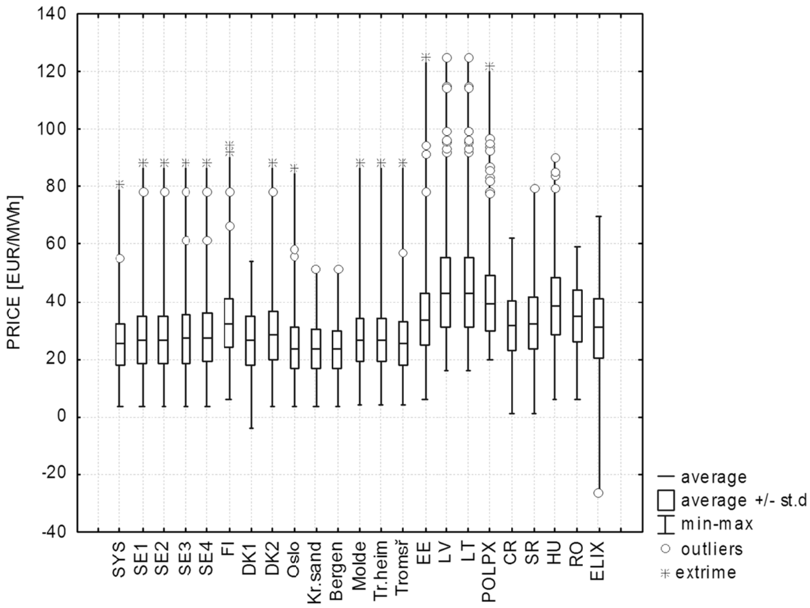


Fig. 1. Average electric energy prices noted form 01.01.2014 to 30.10.2016 on selected markets

Source: Own research.

Distributions of average electricity prices are characterized by high diversity. The highest average prices were recorded in Poland, Lithuania, Latvia and Estonia. In Denmark and on EEX negative prices were recorded. Prices distributions are further characterized by positive skewness. In the average daily prices outliers were observed. All five occurrences of negative average daily prices in the analyzed period were recorded on Sundays. The phenomenon of negative prices is associated with excess of electricity supply over demand, which occurs in markets with a large share of unconventional energy sources such as windmills. A negative average daily price was observed in Denmark e(DK1) on 16.03.2014 and four times in May 2015 on EEX. Due to the presence of negative values the assessment of volatility was carried out using dynamics indicators calculated as absolute price differences:

$$\Delta Y_t = Y_t - Y_{t-1}, \tag{1}$$

where:

Y_t – average price in day t,

Y_{t-1} – average price in day t–1.

Fig. 2 presents the empirical distribution of daily absolute price differences of the average electricity prices. The greatest volatility was observed for Polish and Eastern European markets of Lithuania, Latvia and Estonia. On Nord Pool the standard deviation is low which indicates that changes in the prices on this market are not as dynamic as on remaining markets. Meanwhile, unusual ups and downs of electricity prices are observed in markets in Northern and Central-Eastern Europe.

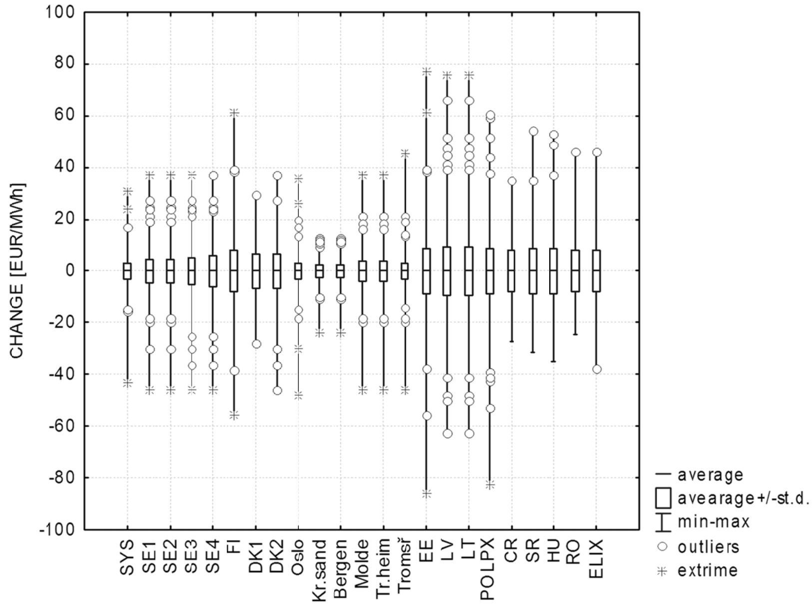


Fig. 2. Distributions of dynamics indicators observed in the period form 01.01.2014 to 30.10.2016 on selected market

Source: Own research.

To determine the relation between the average price of electricity, as well as the dynamics of change in prices on individual markets principal component analysis was carried out. Based on the scree plot (Fig. 3), three principal components were chosen to analyze average prices and two principal components were chosen to analyze dynamics indicators.

Table 1 lists the factor loadings for prices and dynamics indices using normalized varimax rotation.

Fig. 4 shows the average price of electricity in the system of three factors and dynamics indices in the absolute system of two factors.

In the analyzed period, average electricity prices in the markets of Sweden and Norway were strongly correlated. Another group of the prices in Denmark, Finland and Estonia, Latvia and Lithuania, and the Polish index POLPX are strongly correlated with the values of ELIX index. The price of electricity in the Czech Republic is correlated with the prices in Slovakia and Hungary, and Romania.

Generally, the average values of POLPX index are more correlated with the prices of the markets in the central and western Europe than the market in the north, on which TGE was modeled.

Given the correlations among dynamics indicators, markets may be divided into four groups: the first is Norway and Sweden, the second – Finland, Denmark, Lithuania, Latvia and Estonia, the third is Poland with index POLPX, the fourth is the Central-Western Europe.

It should be noted that dynamics indicators of electricity prices determined for POLPX is more strongly correlated with changes in prices on the markets of central-western than the northern markets.

Taking into account the results of PCA, for absolute differences time series of absolute increments for Sweden (SE4 – Malmö), Denmark (DK1 – Aarhus), Polish (POLPX), Hungary (HU) and ELIX were selected.

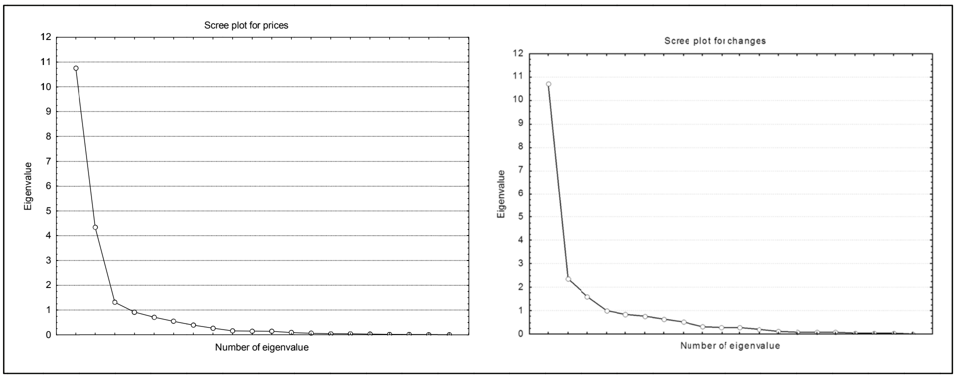


Fig. 3. Scree plots for prices and absolute changes of prices

Source: Own research.

Table 1. Loadings of factors in principal component analysis for prices and absolute changes

Market	PRICES			CHANGES	
	F1	F2	F3	F1	F2
1	*2*	*3*	*4*	*5*	*6*
SE2	**0,9348**	0,0880	0,1385	**0,8541**	0,2096
SE3	**0,9299**	0,1258	0,2101	**0,8762**	0,2362
SE4	**0,9103**	0,1663	0,2138	**0,8620**	0,2688
FI	0,5537	0,3220	0,5859	0,6262	0,4966
DK1	**0,7439**	0,3955	0,1301	0,5617	0,4407
DK2	**0,8007**	0,2976	0,2777	**0,7833**	0,3320
Oslo	**0,9317**	0,0614	0,0770	**0,8504**	0,1656

Table 1 cont.

1	*2*	*3*	*4*	*5*	*6*
Kr.sand	**0,9356**	0,0500	0,0071	**0,7378**	0,3441
Bergen	**0,9225**	0,0466	0,0136	**0,7335**	0,3423
Molde	**0,9545**	0,0760	0,0592	**0,8456**	0,1541
Tromsř	**0,9289**	0,0127	0,0539	**0,8039**	0,0374
EE	0,5368	0,2625	0,6211	0,6228	0,4557
LV	0,0349	0,3078	**0,8948**	0,4778	0,4197
LT	0,0369	0,2976	**0,8967**	0,4678	0,4195
POLPX	0,1238	0,4139	0,5017	0,1997	0,5632
CR	0,2507	**0,8726**	0,2612	0,3072	**0,8536**
SR	0,1916	**0,8810**	0,2899	0,2394	**0,8738**
HU	–0,0599	**0,8532**	0,2498	0,1445	**0,8065**
RO	–0,0326	**0,7735**	0,1840	0,0592	0,6905
ELIX	0,3573	**0,8055**	0,1806	0,3556	**0,8121**
Variance	8,9747	4,3505	3,0824	7,9058	5,1671
% of Variance	0,4487	0,2175	0,1541	0,3953	0,2584

Source: Own research.

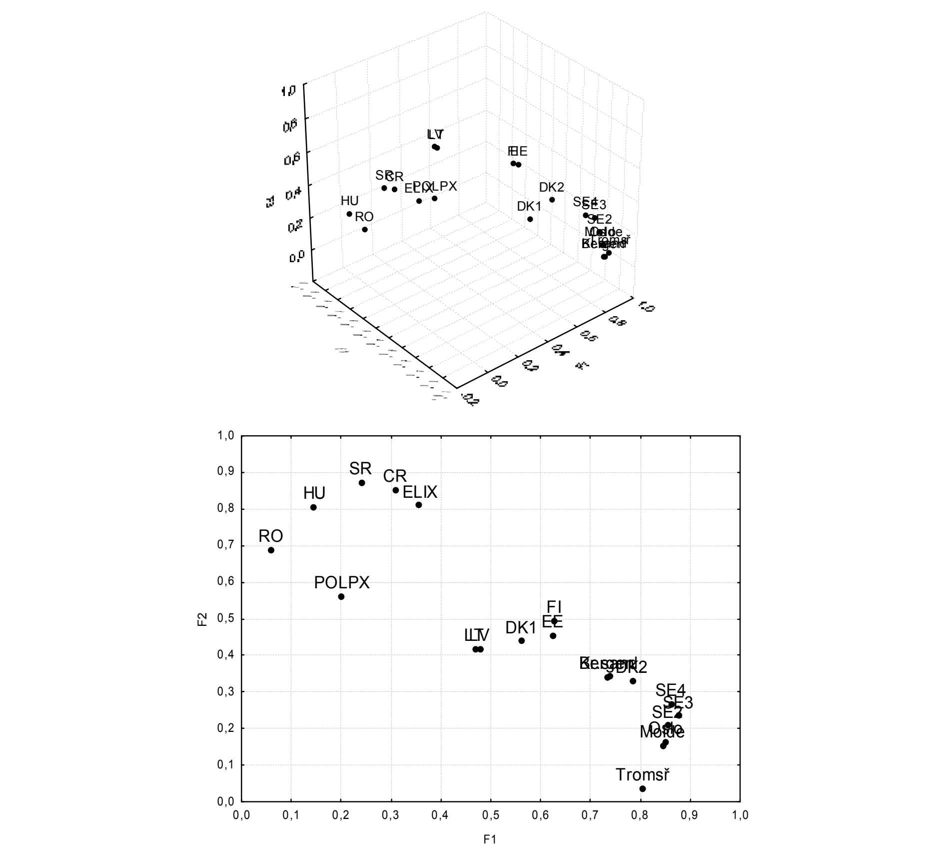


Fig. 4. Loadings for prices and changes of prices with normalized varimax rotation

Source: Own research.

2. Metodology

The following definition of Value-at-Risk (VaR) will be utilized in analysis of multivariate time series [Jajuga, 2000; Weron, Weron, 2000; Heilpern, 2011]. VaR is defined as such loss of value, which is not exceeded with the given probability α at the given time period Δt, and it is expressed by the formula:

$$\mathrm{P}(Y_{t+\Delta t} \leq Y_t - VaR_\alpha(Y)) = \alpha, \tag{2}$$

where:

Y_t – is a present value,

$Y_{t+\Delta t}$ – is a random variable, value of in.

VaR_α in the given time period Δt=1 (one day), may by written as a percentile of the order α of dynamics indicators:

$$\mathrm{P}(\Delta Y_t \leq -VaR_\alpha(\Delta Y_t)) = \alpha, \tag{3}$$

where ΔY_t is a dynamics indicators.

For a single contract $VaR_\alpha(\Delta Y_t)$ is α – percentile of dynamics indicators:

$$VaR_\alpha(\Delta Y_t) = -F_{\Delta Y}^{-1}(\alpha). \tag{4}$$

Using variance-covariance methods we obtain:

$$VaR_\alpha(\Delta Y_t) = -F^{-1}(\alpha) \cdot \sigma_{\Delta Y} + \mu_{\Delta Y}, \tag{5}$$

where:

$F^{-1}(\alpha)$ – is a α-quantile of standarized distribution,

$\sigma_{\Delta Y}$ – is a standard deviation of ΔY_t,

$\mu_{\Delta Y}$ – is an expected value of ΔY_t.

If we treat ΔY_t as a random realization of non-stationary process, then it may by expressed by the equation:

$$\Delta Y_t = \mu_{t\Delta Y} + \varepsilon_t, \tag{6}$$

$$\varepsilon_t = \sigma_{t\Delta Y}\xi_t, \tag{7}$$

where:

$\mu_{t\Delta Y}$ – a time-dependent expected value of dynamics indices described by SARIMA model,

$\sigma_{t\Delta Y}$ – a time-dependent standard deviation of dynamics indices described by GARCH model,

ε_t – the vector of heteroscedastic residuals,

ξ_t – white noise.

Then equation (5) can by written in the form:

$$VaR_{t\alpha}(\Delta Y_t) = -\ F^{-1}(\alpha) \cdot \sigma_{t\Delta Y} + \mu_{t\Delta Y}\,. \tag{8}$$

For m-dimensional matrix of absolute differences ΔY_{mt}, $VaR_{t\alpha}(\Delta Y_{mt})$ can written analogically:

$$VaR_{t\alpha}(\Delta Y_{mt}) = -F^{-1}(\alpha)\sqrt{\boldsymbol{x}^T \boldsymbol{H}_t x} + \boldsymbol{x}^T \boldsymbol{\mu}_t\,, \tag{9}$$

where:

$\boldsymbol{H}_t$ – variance-covariance matrix of multidimensional process of absolute differences,

$\boldsymbol{\mu}_t$ – vector of expected values multidimensional process of absolute differences,

$\boldsymbol{x}$ – vector of weights of individual contracts.

In portfolio analysis a coherent risk measure CVaR (Conditional Value-at--Risk), given by formula (10), is applied more often than VaR [Artzner et al., 1999; Rockafellar, Uryasev, 2000]:

$$\mathrm{C}VaR_{\alpha}(\Delta Y_{tm}) = E\{\Delta Y_{tmt} \mid \Delta Y_{tm} \le VaR_{t\alpha}(\Delta Y_{mt})_t\}\,. \tag{10}$$

Linear and non-linear models of single and multi-dimensional are discussed in many works [Osińska, 2006; Zivot, Wang, 2006]. Their applications to the financial market, the commodity market or the electricity market are described by [Weron, 2006; Fiszeder, 2009; Trzpiot, 2010; Krężołek, 2015].

SARIMA vector models incorporating autocorrelation and periodicity may be employed to describe multivariate time series. The m-dimensional processes of ΔY_t can be written as m-SARIMA model (Seasonal Auto-Regressive Integrated Moving Average), (p, d, q) × (P, D, Q) with period s [Brockwell, Davis, 1996]:

$$\Delta_{\mathrm{s}}^{\mathrm{d}} Y_{mt} = (1-B)^{\mathrm{d}}(1-B^{\mathrm{s}})^{\mathrm{D}} \Delta Y_{mt}\,, \tag{11}$$

where:

B – shift operator,

d – integration rank,

D – seasonal integration rank.

If $\boldsymbol{\varepsilon}_\mathbf{t}$ ~D($\mathbf{0}$, $\mathbf{H}_\mathbf{t}$), then we can write:

$$\boldsymbol{\varepsilon}_\mathbf{t} = \mathbf{H}_\mathbf{t}^{\mathbf{0,5}} \mathbf{u}_\mathbf{t}, \tag{12}$$

where:

$\mathbf{H}_\mathbf{t}$ – $m \times m$ dimensional conditional variance-covariance matrix,

$\mathbf{u}_\mathbf{t}$ – $m \times 1$-dimensional vector with zero expected value and unit variance-covariance matrix.

Multivariate GARCH family comprises models of conditional covariance matrix $\mathbf{H}_\mathbf{t}$. These include the generalization of a 1-dimensional model GARCH to form a multi-dimensional model Veche (*p*, *q*) [Kraft, Engle, 1983] and BEKK (*p*, *q*, *M*) [Engle, Kroner, 1995], factor models *K*-factor GARCH [Engle, 1987] and O-GARCH (1.1 m) [Alexander, Chibumba, 1996], as well as models of constant conditional correlation coefficients (CCC) [Bollerslev, 1990] and dynamic conditional correlation coefficients (DCC) [Engle, Sheppard, 2001; Engle, 2002; Tse, Tsui, 2002]. In this study, the following model of [Engle, 2002] is applied:

$$\mathbf{H}_\mathbf{t} = \mathbf{D}_\mathbf{t} \boldsymbol{\Gamma}_t \mathbf{D}_\mathbf{t}, \tag{13}$$

$$\boldsymbol{\Gamma}_\mathbf{t} = diag(q_{11,t}^{-0,5};...;q_{mm,t}^{-0,5}) \mathbf{Q}_t diag(q_{11,t}^{-0,5};...;q_{mm,t}^{-0,5}), \tag{14}$$

where:

$\mathbf{D}_\mathbf{t} = diag(\sigma_{1t}, \sigma_{2t}, ..., \sigma_{mt})$ – is a diagonal matrix with dimensions $m \times m$. Elements of this matrix are estimated using univariate GARCH models,

$\boldsymbol{\Gamma}_t$ – matrix of dynamic conditional correlation coefficient,

$\mathbf{Q}_t = (q_{ijt})$ – symmetric, positive definite matrix of dimension $m \times m$:

$$\mathbf{Q}_t = (1 - \alpha - \beta)\bar{\mathbf{Q}} + \alpha \mathbf{u}_{t-1} \mathbf{u}_{t-1}^{'} + \beta \mathbf{Q}_{t-1}, \quad u_{it} = \frac{\varepsilon_{it}}{\sigma_{it}},$$

$\bar{\mathbf{Q}}$ – unconditional variance matrix of u_t,

α, β – positive parameters, $\alpha + \beta < 1$,

m – dimension of time series.

The autocorrelation coefficient between two time series in Engle' DCC model is:

$$\rho_{ijt} = \frac{(1-\alpha-\beta)q_{ijt} + \alpha u_{it-1} u_{jt-1} + \beta q_{ijt-1}}{\sqrt{((1-\alpha-\beta)q_{iit} + \alpha u^2{}_{it-1} + \beta q_{iit-1})(((1-\alpha-\beta))q_{jjt} + \beta u^2{}_{jt-1} + \beta q_{jjt-1})}}. \tag{15}$$

Using a modification of the classical Markowitz [1959] model, the following optimization problem may be formulated:

$$\min \rightarrow \left| CVaR(\Delta Y_{mt}) \right|, \tag{16}$$

with restrictions:

$$x_i > 0, \quad i = 1, \ldots, 5,$$

$$\sum_{i=1}^{5} x_i = 1,$$

$$\boldsymbol{E(}\,\Delta Y_{mt}\,\boldsymbol{)} = \sum_{i=1}^{5} x_i \Delta Y_i \geq \mu_0,$$

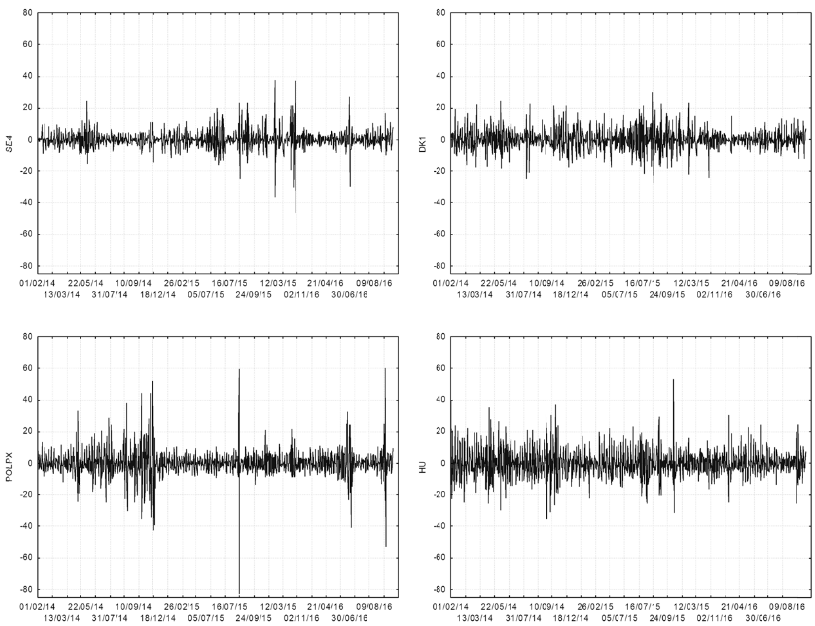
where:

μ_0 – the expected portfolio value before portfolio reconstruction (the expected value with equal share for every market),

x_i – the share of market in portfolio.

3. Empirical analysis

In the construction of the portfolio (16), contracts on absolute average daily gains of prices on five markets (Fig. 4): SE4, DK1, POLPX, HU, ELIX were used.



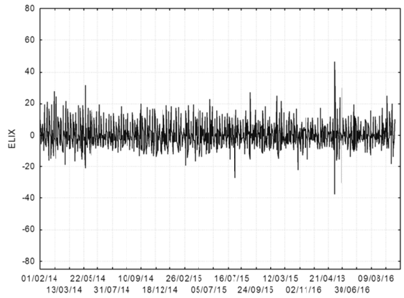


Fig. 5. Time series of dynamics indicators for selected market

Source: Own research.

Fig. 5 shows the time series of selected increments, Fig. 6 – the features of ACF and PACF series and a histogram graph and quantile-quantile distribution for Index POLPX. Considered differences characterized by high volatility, seasonality, autocorrelation. In series of differences we observed clustering of volatility effect. Distributions are leptokurtic and exhibit fat tails.

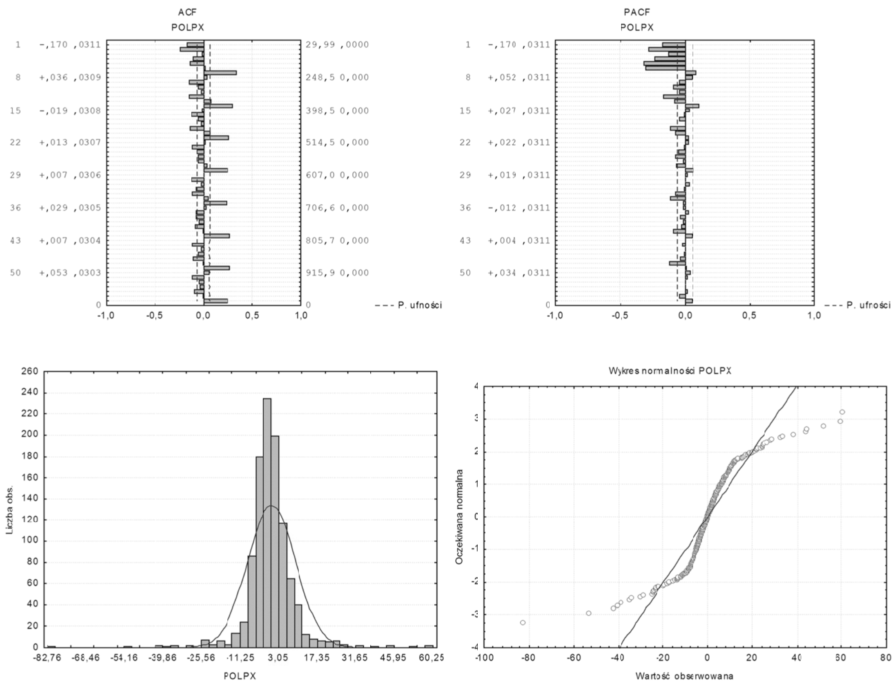


Fig. 6. Autocorrelation and distribution of POLPX dynamics indicators

Source: Own research.

To estimate a vector of expected values in multidimensional process of dynamics indicators $\boldsymbol{\mu}_t$, in $VaR_{t\alpha}(\Delta Y_{mt})$, given by formula (9), SARIMA models are used. In Table 2 results of vector-SARIMA model estimation are presented. Not all parameters differ significantly from zero. The best precision in SARIMA model estimation we obtained for time series from Sweden. In Table 3 the diagonal matrix $\mathbf{D_t}$ resulting from the first-step estimation of $\mathbf{H_t}$ by DCC model is presented. In Fig. 7 components of $\mathbf{D_t}$ matrix are presented. The highest estimated variances were obtained for POLPX index. The lowest variances were obtained for Danish dynamic indicator.

Table 2. Parameters of SARIMA(1,0,1)(1,1,1)$_7$ models

Markets	**SE4 - 1**		**DK1 - 2**		**POLPX - 3**		**HU - 4**		**ELIX - 5**	
Parameters	value	p-value	value	p-value	value	p-value	value	p-value	value	p-value
p(1)	0,5879	0,0000	0,4948	0,0000	0,4031	0,0000	0,1806	0,0006	0,6098	0,0000
q(1)	0,9335	0,0000	0,8869	0,0000	0,8433	0,0000	0,6719	0,0000	0,8204	0,0000
Ps(1)	0,1351	0,0001	-0,0411	0,2496	0,0833	0,0212	-0,0692	0,0816	0,0044	0,9125
Qs(1)	0,9604	0,0000	0,8993	0,0000	0,9009	0,0000	0,7377	0,0000	0,8270	0,0000
MS	22,3890		30,1750		49,3720		47,5990		25,6540	

Source: Own research.

Table 3. Parameters of first-step DCC model estimation– diagonal GARCH(1,1) models

Markets	**SE4 - 1**		**DK1 - 2**		**POLPX - 3**		**HU - 4**		**ELIX - 5**	
Parameters	value	p-value	value	p-value	value	p-value	value	p-value	value	p-value
ARCH(1)	0,1447	0,0351	0,0432	0,0147	0,4274	0,0003	0,2649	0,1075	0,1381	0,1503
GARCH(1)	0,8930	0,0000	0,9591	0,0000	0,7335	0,0000	0,8067	0,0000	0,8891	0,0000

Source: Own research.

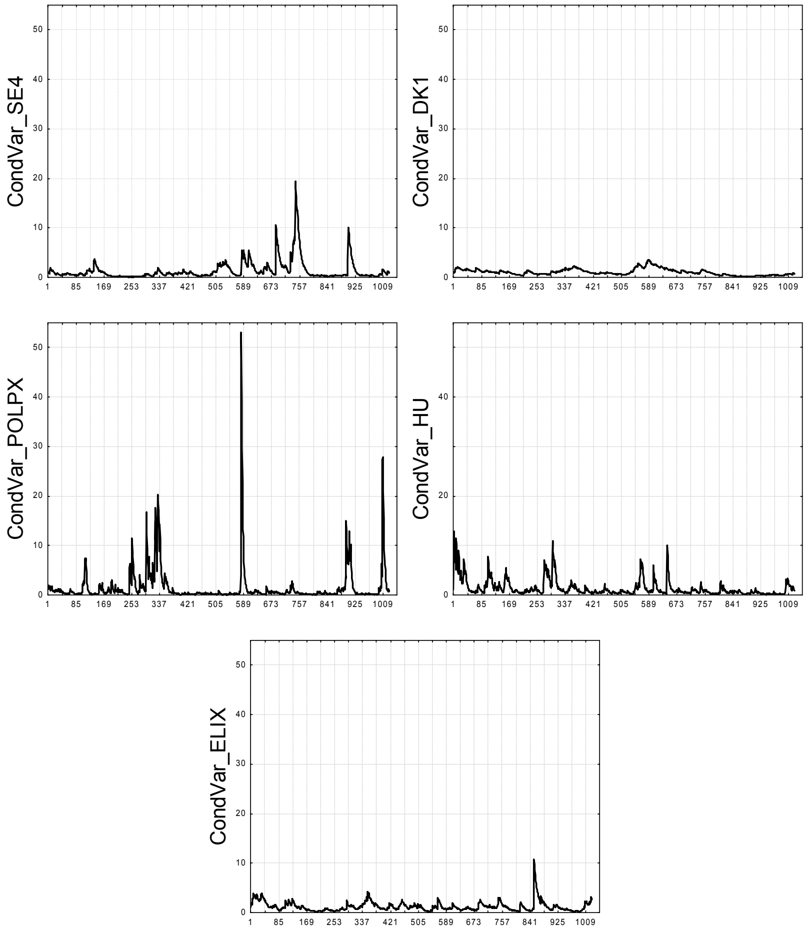


Fig. 7. Conditional variances of matrix D_t

Source: Own research.

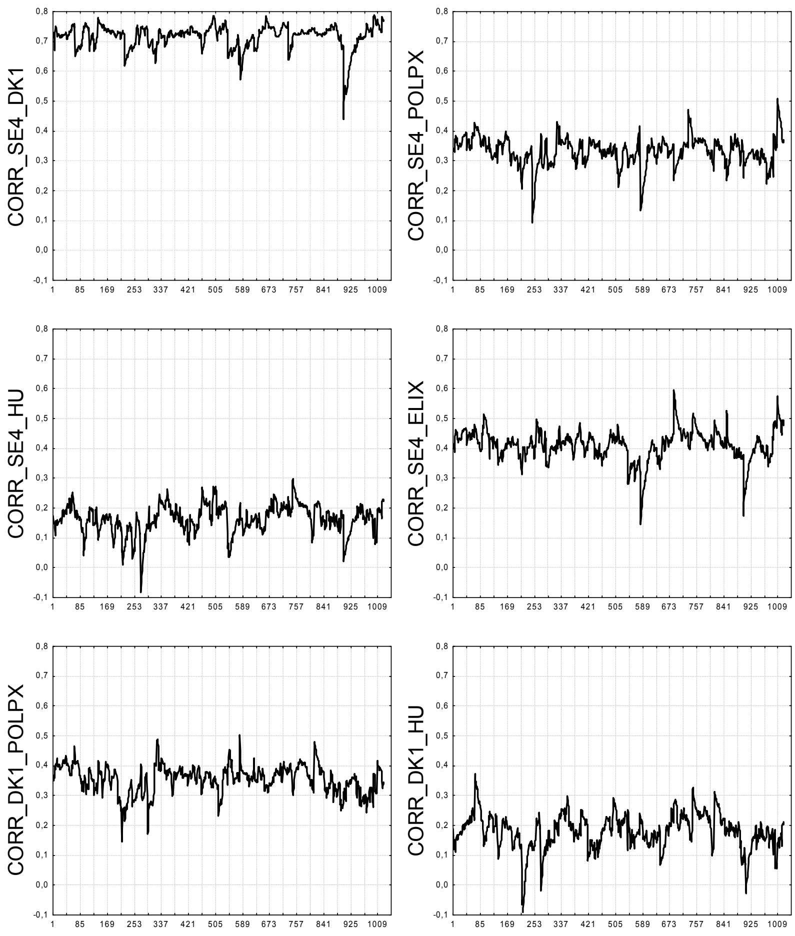
In Table 4 the matrix $\mathbf{\Gamma}_t$ resulting from the second-step estimation of $\mathbf{H_t}$ by DCC model is presented. In Fig. 8 components of $\mathbf{\Gamma}_t$ matrix are presented. Correlations are dynamic and positive but not strong. The strong dependence was observed only between dynamic indicators from Nord Pool (SE4 and DK1).

In Table 5 solutions of optimization task (16) are presented for different values of α. The greater level of risk we observed for short position (CVaR) than for long one. CVaR of portfolio means how much portfolio can average change next day bringing the lost. The most appropriate contract in every model it is SE4. But number *k* of exceed VaR in every portfolio is much greater than allowable. In every case share of *k* in number of observations – *w* is greater than α level. It means, that obtained portfolios' quintiles are underestimated.

Table 4. Parameters of second-step DCC model estimation – parameters of $\mathbf{\Gamma}_t$ matrix

Parameters	value	p-value
rho_21	0,7294	0,0000
rho_31	0,3409	0,0000
rho_41	0,1728	0,0000
rho_51	0,4179	0,0000
rho_32	0,3600	0,0000
rho_42	0,1880	0,0000
rho_52	0,5242	0,0000
rho_43	0,2138	0,0000
rho_53	0,3779	0,0000
rho_54	0,2885	0,0000
alpha	0,0216	0,0000
beta	0,9046	0,0000
df	5,2289	0,0000

Source: Own research.



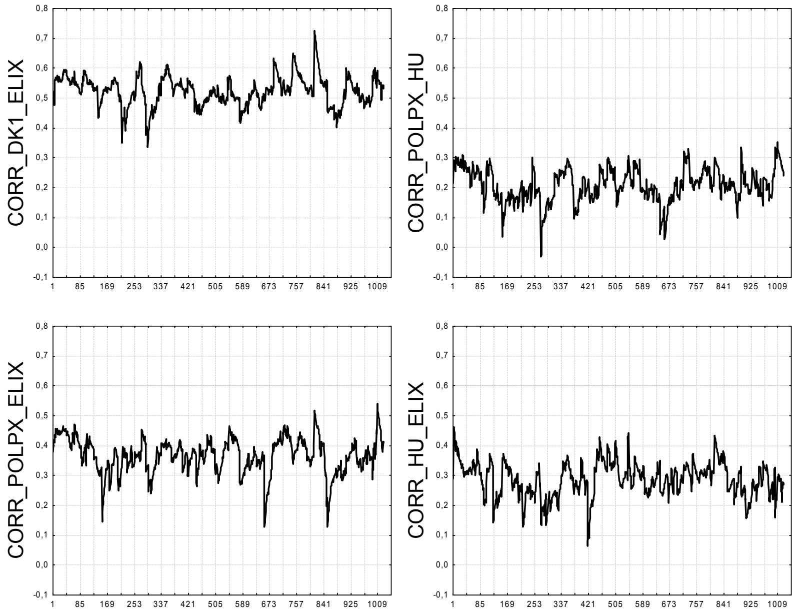


Fig. 8. Conditional correlations between daily indexes on selected markets

Source: Own research.

Table 5. Solutions of optimization task (16)

Posi-tion	Port-folios	α	Portfolios share x_i					Portfolios parameters			
			SE4	DK1	POL PX	HU	ELIX	C VaR	μ	k_i	w_i
short	P_1	0,001	**0,970**	**0,030**	0,000	0,000	0,000	–3,688	0,0143	115	0,112
	P_2	0,005	**1,000**	0,000	0,000	0,000	0,000	–3,437	0,0142	179	0,174
	P_3	0,01	**1,000**	0,000	0,000	0,000	0,000	–3,172	0,0143	207	0,201
	P_4	0,025	**0,965**	**0,035**	0,000	0,000	0,000	–2,837	0,0143	247	0,241
	P_5	0,05	**0,93**	**0,04**	**0,03**	0,000	0,000	–2,479	0,0139	279	0,272
long	P_6	0,05	**0,930**	**0,041**	**0,029**	0,000	0,000	1,426	0,0139	323	0,315
	P_7	0,025	**0,931**	**0,043**	**0,026**	0,000	0,000	1,306	0,0139	271	0,264
	P_8	0,01	**0,97**	**0,03**	0,000	0,000	0,000	1,35	0,0143	210	0,205
	P_9	0,005	**0,978**	0,000	0,000	**0,022**	0,000	1,580	0,0139	172	0,168
	P_{10}	0,001	**0,957**	0,000	0,000	**0,043**	0,000	1,711	0,0135	111	0,108

Source: Own research.

Fig. 9 presented value of $VaR_{0,001}$ for short and long position and value of P_1. If we compare dynamic time series of VaR and P_1 with means $CVaR_{0,001}$ = = –3,6875 for short position and $CVaR_{0,999}$ = 1,82 for long one.

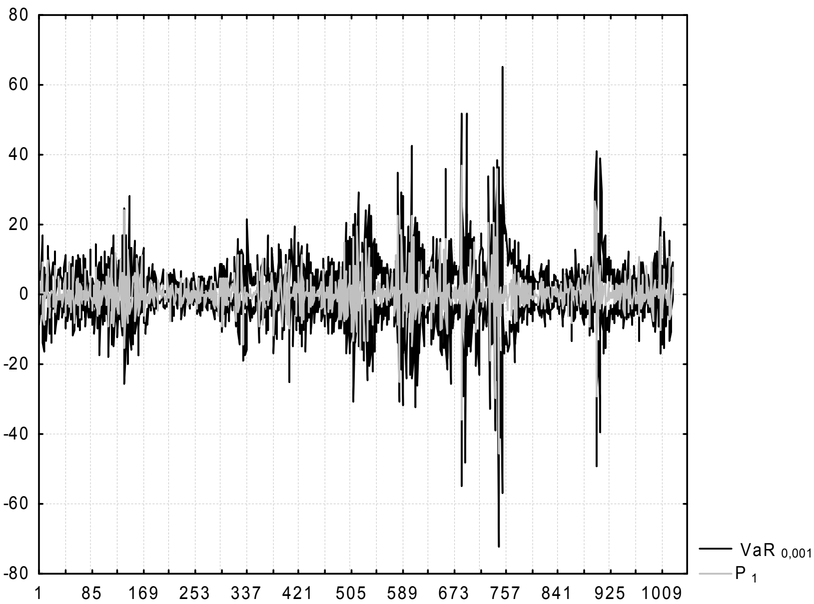


Fig. 9. $VaR_{0,001}$ and P_1 portfolio value

Source: Own research.

Conclusion

Empirical results show that in neighboring markets, both distributions of electricity prices and absolute growth rates are characterized by similar distributions. Time series of prices, as well as increments show stronger correlations in neighboring markets. Correlation among changing prices is also dynamic across all markets. For the optimization criteria of the task (16), the lowest risk with relatively higher profit can be achieved primarily on contracts issued in Sweden and Denmark. Nevertheless, taking into account absolute increases, due to the negative prices, worse fit was obtained in terms of the number of excesses of estimated quantiles than for the relative growth.

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ANALIZA PORÓWNAWCZA RYZYKA ZMIANY CENY ENERGII ELEKTRYCZNEJ W WYBRANYCH REGIONACH EUROPY

Streszczenie: W pracy opisano za pomocą wielowymiarowych modeli autoregresyjnych VAR-GARCH procesy zmienności cen energii elektrycznej na wybranych europejskich rynkach energii elektrycznej. Na bazie notowań z towarowych rynków natychmiastowych: polskiej Towarowej Giełdy Energii (TGE), European Energy Exchange (EEX), skandynawskiej Nord Pool oraz krajów Europy środkowej Czech, Słowacji, Węgier i Rumunii, korzystających z usług czeskiego operatora (OTE) w okresie od stycznia 2014 do października 2016, przeprowadzono analizę ryzyka zmiany średniej dziennej ceny energii elektrycznej oraz zaproponowano portfel kontraktów na energię elektryczną, minimalizując ryzyko straty w badanym okresie. Ryzyko straty pojedynczych kontraktów estymowano za pomocą wartości zagrożonej VaR. Do optymalizacji portfela kontraktów wykorzystano warunkową wartość zagrożoną CVaR.

Słowa kluczowe: VAR, DCC, VaR, CVaR, analiza portfelowa.