MONETARY DETERMINANTS OF OUTPUT DYNAMICS
IN THE LIGHT OF THE STRUCTURAL VECTOR-AUTOREGRESSIVE SVAR MODEL:
A KEYNESIAN APPROACH

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ABSTRACT

PURPOSE: The purpose of the following paper is to analyze and empirically verify the monetary theory of business cycles as a mechanism for the interaction of the dynamics of production and money supply based on the example of the Polish economy. In order to identify and mitigate the risk of economic fluctuations as a function of the response of the central bank, it is necessary to conduct an extensive analysis of the indirect mechanism of transmission of monetary impulses on production in the economy.

DESIGN/METHOD: Empirical analysis was carried out by estimating a macroeconometric time series model taking into account the inductive information based on the Keynesian theory – the structural vector-autoregressive SVAR model. The stochastic process included in the study was based on statistical data of Poland, which were obtained from the cyclical reports: ‘Preliminary estimate of gross domestic product’ and ‘Quarterly accounts of gross domestic product in 2017-2021’, Poland’s Central Statistical Office and the National Bank of Poland’s databases for the time interval of 2007.Q1 -2022.Q2.

RESULTS/FINDINGS: The applied empirical analysis positively verified the existence of an indirect monetary impulse transmission mechanism in Poland’s economy. The obtained research has positively verified the compatibility of the monetary theory of the business cycle in terms of the Keynesian theory with the macroeconomic reality in Poland. The results of the research justify the measures to mitigate the risk of economic instability and impose a requirement for discretionary policy by the National Bank of Poland.

ORIGINALITY/VALUE: The following work addresses an important element of the macroeconomic analysis, specifically the monetary theory of the business cycle. The originality of the work stems from the empirical attempt to verify the monetary theory of the business cycle taking into account the indirect mechanism of transmission of monetary impulses on the grounds of the statistical data from the Polish economy.

KEYWORDS: endogenous money creation, macroeconometrics, monetary theory of the business cycle, time series decomposition, structural vector-autoregressive model.

JEL: C32, E12, E41, E51, E52.
1. INTRODUCTION

In a fractional reserve banking system, in which money creation is determined by liquidity preference, the investment contributions of individuals do not account for the full share of a bank's liquidity capital. Accordingly, the bank's assets on the balance sheet also include less liquid assets in the form of Central Bank money bills, treasury bonds or other securities. Such a monetary system, in which lending is determined by operations between the private sector and commercial banks and the Central Bank and commercial banks at a given percentage imposition, has been ably described by John Hicks (Boissieu, 1989) as an 'overdraft economy', or 'debit economy'. The money supply in such a system is created endogenously. The scheme of the mechanism of credit action in the fractional reserve system laconically visualizes the structuralist model of endogenous money creation, which is an extension of the accommodative model (Wray, 2007), in which the money supply curve is perfectly elastic (horizontal). The difference is important because, given the inductive information about the discretionary monetary policy of the monetary authorities, investment contributions do not represent the full share of liquidity capital, reserves and assets of commercial banks. Other assets are those less liquid for example in the form of bonds bought on the secondary market\textsuperscript{1}, in view of which the money supply curve has a positive slope, and an increase in liquidity preference must induce (at a constant level of the money supply function) an increase in the market interest rate since it is a decreasing function of liquidity capital. Thus, the monotonic relationship and harmony between the marginal efficiency of capital and liquidity preference indicate that the structuralist model is a model of endogenous structuralism (Palley, 1997) of the mechanism of credit action and thus the money supply in the circular flow of the economy.

Structural vector-autoregressive (SVAR) models are macroeconometric time series models used for empirical verification of the previously formulated economic theory by structuring the VAR model and identifying the impact of structural shocks resulting from the impact of structural parameters on a given endogenous variable. A particularly important tool for structural identification is the impulse response function (IRF), which allows to identify the effect of the dynamic impact of the impulse change of the structural parameter of the constructed vector-autoregressive model on the formation of the dependent variable after the construction of models based on a recursive linkage. Accordingly, structural vector-autoregression SVAR models become a useful analytical tool for analyzing and assessing the risks associated with the conduct of countercyclical monetary policy, among other things, in terms of the impact of monetary impulses through open market operations to inject liquidity into the banking sector and thereby affect the marginal efficiency of a private sector capital. In addition, SVAR models are a useful tool for risk analysis in the context of identifying and forecasting risks associated with changes in the phase of the economy affecting the health of the banking sector (financial acceleration effect), the threat of a run on banks and through this the risk of an equity gap. SVAR models are useful analytical tools for assessing the risk of conducting countercyclical monetary policy in terms of reaction functions and stress testing, as also pointed out by Antolin-Diaz, Petrella and Rubio-Ramírez (2017). The above segments of macroeconometric analysis, and especially in the empirical verification of the monetary theory of the business cycle, allow for the analysis in the context of stress testing, which is an indispensable tool in modern monetary policy for assessing the macroeconomic risk and shaping the loss function of the macroprudential supervisory authority of the monetary authorities. In empirical work, the application of the macroeconometric SVAR model finds application in the macroeconomic risk analysis and stress testing (Arias, Caldara, and Rubio-Ramírez, 2016a; Arias, Rubio-Ramírez, and Waggoner, 2016b) as well as in dynamic forecasting of the model's structural parameters (Waggoner, Zha, 1999; Andersson, Palmqvist, and Waggoner, 2010).
The theory of endogenous money and its creation mechanism in the economy is omitted by mainstream economics. The contemporary reference to the endogenous mechanism of money creation can be found in the work of representatives of the post-Keynesian school. However, the problem lies in the empirical work, as these representatives use mostly a normative approach (inductive-deductive) (Sawyer, 2013; Pilkington, 2014; Palley, 2017). The following work takes into account the recommendations of the trend of institutional economics with regard to the use of methodological positivism by analytical tools, which include time series model – the structural vector-autoregressive SVAR model used in the work. An economist who eschews empirical work rather than a doctrinaire description of the economic phenomena under the dogmatic a priori assumptions should be aware of doubts about the reliability of the analysis with an excessive concentration of purely empirical or theoretical approaches. Empirical studies based on the economic theory face a problem of traceability, which particularly characterizes studies based on the multi-equation DSGE structural models (Storm, 2021). This division is particularly defined by Hume’s Guillotine (Almeida, 2016), who divides the field of economic science into a positive approach (which treats economics as a science) and a normative approach (which treats economics as the art of making a policy). Common sense, as well as the demand for detailed research on economic fluctuations call for a synthesis of the approaches of the two fields. This approach should be particularly characteristic of the Keynesian and post-Keynesian schools (Davis, 1987; Parsons, 1996). John Maynard Keynes specifically, commemorating the contribution of his teacher Alfred Marshall (Keynes, 1924), pointed out that economics must use a wide methodological range since it is an interdisciplinary science.

The purpose of the following paper is to analyze and empirically verify the monetary theory of the business cycle as an endogenous process of interaction between real and monetary parameters on the basis of the statistical data of the Polish economy.

The methodology used in the paper employs analytical tools, the time series decomposition with the Hodrick-Prescott filter, the spectral analysis using a periodogram and the Bartlett spectrum, as well as the optimal construction and estimation of a structural vector-autoregressive SVAR model. The synthesis of tools in the methodology used finds its starting point in the approach of Wesley Mitchell (Sherman, 2001), who verifies the results of empirical work to study the nature of business cycles from a previously structured economic theory, which acts as a ‘setting’ to describe the results. The estimation was based on periodically published reports: ‘Preliminary estimate of gross domestic product’ and ‘Quarterly accounts of gross domestic product in 2017-2021’ of the Central Statistical Office in Poland and the National Bank of Poland’s database for the time interval 2007.Q1-2022.Q2 (62 observations).

The arbitrary decision to conduct the study based on the example of Poland is due to the shortage of empirical research work on the endogenous money creation in the fractional reserve system in Poland. A special contribution to the development of the monetary theory from a post-Keynesian perspective should be attributed to Prof. Izabela Bludnik (2018; 2019), but her monumental contribution needs to be supplemented by the empirical quantitative study with reference to Polish economic reality.

To accomplish the research purpose set, the work will be divided into the theoretical analysis – a literature review and the empirical analysis. The theoretical part will address the evolution and history of the theory of endogenous money derived from the thought of a German economist G.F. Knapp and the Keynesian school of economic thought. In addition, the theoretical part will review one of the first regression models for analyzing and verifying the monetary theory of the business cycle – the St. Louis equation. A part of the empirical analysis, upon the formulation of the preliminary methodological assumptions, will be related to the synchronization of the business cycle, the identification of seasonality and periodic oscillations using the spectral analysis, and the optimal construction of a structural vector-autoregressive SVAR model on the acquired secondary statistical data from the Polish economy.
2. Literature review

2.1. The theory of endogenous money in light of the State Theory of Money

The post-Keynesian trend points to the endogenous nature of money creation, the philosophy which was formed from the nominalist theory of money by G.F. Knapp’s ‘The State Theory of Money’ (Ehnts, 2019). The author of the revolutionary monetary thought identified money as a product of law, the source of which is the state. According to Knapp, money has a nominal value assigned by the state and it is a full-fledged measure of value. Such a philosophy of the existence of money as a unit of account (Knaikiewicz, 1984) is due to the fact that it is money based on trust ‘fiat money’, the fiat money, which is supposed to ensure the liquidity of money in the circular flow of transactions in the economic system and to be the only form of payment of taxes and other public tributes providing the state budget funds, which is also pointed out by L. Randall Wray (2006). The nominalist approach arose as a critique of the metallicist theory, where Knapp emphasizes that money is not a commodity with a specific marginal utility derived from its intrinsic substance, but a product of law, in which the physical value does not determine the nominal value and it occurs in the form of a record as it serves as a means of payment established by law. In addition, it serves as a link between the past and the future, in view of which bullion money would determine the economically harmful level of the share of profits in production as a ‘paradox of thrift’ mainly by increasing the importance of money’s hoarding function. As a result, there must be a certain level of a deflationary gap, which is not maintained in the sectoral equation as a deficit of the consolidated budget from the issued government treasury securities. An important problem of money in the gold standard or bullion money system is its satisfaction intensity (marginal utility), which, by virtue of the vertical structure of the supply curve of commodity goods, determines a much higher opportunity cost of storing money in liquid form, as it simultaneously serves as liquidity capital for commercial banks. The market interest rate, which is a decreasing function of liquidity capital, therefore determines risk aversion and thus a decrease in liquidity preference. Money in modern fractional reserve systems is physically created by a public institution – the central bank, which is determined by the put loss function as a result of a target rule, e.g. a direct inflation targeting strategy. Such a discretionary policy, however, requires operations to be carried out with specific instruments by virtue of indirect objectives, which are controlled by taking into account institutional barriers including, the stability of the financial system, the Forward Guidance policy, or other exogenous (usually political) conditions.

Knapp’s Nominalist (‘State’) Theory of Money, which formalizes the meaning of money as a unit of account, actually found support from John Maynard Keynes in ‘A Treatise on Money’ (where he directly shows approval of the philosophy of being settlement money derived from chartalism). The name of the trend is derived from the word charta (from Latin: ticket) and it is a trend that differed significantly from the philosophy of money known in Anglo-Saxon countries since in Germany at that time the state, specifically the authorities, played a significant role in the economic system (Gruszecki, 2015) with the help of discretionary policies using fiscal stimulus. Such an economic model of Germany found its name among economists in the form of the ‘Rhenish model’ (Swadźba, 2007). Knapp’s formulation of the nature of money in Chartalist terms came about as a result of the monetary reform of 1871, which was influenced by the establishment of the German Empire as a result of the German unification by Chancellor Otto von Bismarck. The standardization of the new monetary system through the monetary reform of the German Empire continued until about 1876. The monetary reform included the introduction of the ‘Reichsmünzgesetz’ law by the German Parliament on December 4, 1871, under which the German mark was based on the gold standard. That same year, the ‘Zentralbank für das Deutsche Reich’, the Central Bank of the German Reich, be-
gan operations in Berlin and served as the full-fledged issuer of the German mark in denominations of 5, 20 and 50 (Tomczyk, 2015). An implication that both differed from the Anglo-Saxon monetary system of the time and modern fractional reserve systems was the fact that, according to the law of April 30, 1874, the issuance of money in the German Reich was subject to the decision of the Chancellor with the approval of the Federal Council ‘Bundesrath.’ The regulation of the issuance policy did not take place until March 14, 1875, under the Banking Law ‘Bankgesetz’, which established the Reichsbank, which was given the ability to issue banknotes with higher denominations: 200, 500 and 1,000 marks. The Reichsbank, however, functioned as a joint-stock company whose circulating funds were contributions from the private sector and commercial banks, while it was fully subject to government authority. The gold standard performed the function of stabilizing both the lending and exchange rate, and consequently the terms of trade. Such a system, in which the gold standard served as a ‘golden anchor’ for the German mark survived until World War I, when most countries moved away from the gold parity to liquidate the financing of government expenditures on armaments, which in the case of the German Reich was done by the law of August 4, 1914. The monetary reform in the second half of the 19th century in Germany was not only an intellectual field for Knapp to formulate a state theory of money, but he pointed out the consistency of chartalism with high government participation in the circular flow of money in the economy.

The theory of creditism, which is based on the principle that money in circular flow is contingent on a contract between the parties, basically the incurring of the other party’s debt, is particularly consistent with the theory of endogenous money creation. Already at this stage, there is a contingency between the philosophy of the existence of fiat money and the theory of chartalism since debt is also the result of trust. Of course, the implication of this trust often becomes a leverage, but as a rule, it is the result of an agreement between these parties. The source of money creation, which can be divided into inside money (which is foreign capital raised by the private sector in lending) and outside money (raised by the government in the deficit of the consolidated budget to finance spending in the fiscal stimulus and recovered from taxes) is important in this trend (Marimon, Nicolini, and Teles, 2003; Mills, 2007a; Sun, 2007). This division is important outside of the money division scheme (Figure 1) formulated in Keynes’ (1930) Treatise on Money as the ‘money family tree’.

Money is treated in terms of this theory as a paper promissory note, which was created as a result of the issuance by the Central Bank, and then indirectly made available in a circular flow, and thus was created from debt on a contract between the ‘real; private sector and the banking sector, which conducts open market operations with the Central Bank. Indeed, it should be emphasized that in this case money as a paper bill of exchange is a special case since it has no maturity and no interest rate. Contractism, therefore, not only is consistent with the mechanism of endogenous money creation and chartalism, but also the consideration of the source of external money shows the effect of the income effect of the fiscal stimulus on the increment of the marginal efficiency of capital of the transaction motive, which, according to the Keynesian system of implicit functions (Meade, 1937; Torre, 1977; Zhang, 1991) affects the increment of liquidity preference being in harmony with the marginal efficiency of capital, in view of which a change in the effective demand function in accordance with the acceleration mechanism must also induce a change with a certain adjustment parameter, in the increment of the money supply to finance private sector investment decisions — this mechanism can be implemented in the economic theory as the principle of the primacy of fiscal policy.
According to the above considerations, an obvious conclusion has arisen over the years that with the growing importance of financial institutions in the economic system, which is conditioned by financialization (Rudny, 2018), there is an increasing demand for the liquidity of money circulation, its availability and ease of conducting operations with it. The demand, which is conditioned by the process of the social and technological change, in fact, is itself a stimulant of research and innovation in the banking system, as particularly demonstrated by the COVID-19 period (Wiśniewski, Polasik, Kotkowski, and Moro, 2021). The pandemic period and the structural changes in banking over the years became another era of the evolution of money—demand deposit money, which today in Poland is about (80;97)%. From this point of view, Poland’s economy and the monetary system structurally take on the characteristics of a modern fractional reserve system with an endogenously created money supply. This is also confirmed by empirical research from a post-Keynesian perspective, which, through the use of a vector-autoregression VAR model and a Granger causality test, verified the hypothesis of an endogenous money creation mechanism in the Polish economy (Bachurewicz, 2019).
2.2. Empirical research in monetary fluctuation theory: the case of the St. Louis equation and structural vector-autoregression models

The estimated regression prepared by L.C. Andersen and J.L. Jordan (1968) was a special contribution to the economic literature on the study of the monetary fluctuation theory. It was one of the first macroeconometric time series models that included a research sample of the U.S. macroeconomic data in a quarterly sequence. The model very quickly gained recognition among the classical-neoclassical school (monetarists) and became a reliable model for this trend, as it showed significance for the monetary structural parameters, whereas for fiscal structural parameters it did not show significance between 1953.Q1 and 1969.Q4. It should be noted here that despite the empirical verification of the a priori assumptions of this school in the time series under study, the estimation of the model after replication by modifying the time series did not show the same results. Moreover, the empirical studies showed the significance of the fiscal structural parameters of the model (Seaks and Allen, 1980) or the parameters that were inductively influenced by fiscal policy, e.g. consumer spending, where the monetary parameter did not show statistical significance in the estimated model. The estimated regression model in principle follows the approach of Klein’s method and analyzes the model regressor (gross national product growth) through the least squares (OLS) estimation. The distribution of lags of the structural parameters was optimally selected by Almon’s method in the classical version of Andersen and Jordan’s model and its replication by other researchers (Carlson, 1975; Lukman and Kibria, 2021). The classical form (Equation 1) of the model is adopted (Belliveau, 2011):

\[
\Delta Y_t = \alpha_0 + \sum_{i=1}^{K} \beta_i \Delta M_{t-i} + \sum_{i=1}^{K} \gamma_i \Delta (R - E)_{t-i} + \sum_{i=1}^{K} \delta_i \Delta Z_{t-i} + \epsilon_t
\]

where:
\(\Delta M_t, \Delta (R-E)_t, \Delta Z_t\) – vector of observations of increments in the population values of the model’s n variables.

Although the St. Louis equation model of Andersen and Jordan was a reliable model at the time, as it was structurally a macroeconometric time series model, the poor reliability and optimality assumptions of the model did not allow it to serve as a representative model for the study of economic prosperity by the analytical background of the monetary authorities. Only the vector-autoregressive models originally developed by C.A. Sims (1980), the assumptions of which included atheoreticality in the estimation of the effect of structural parameters on the dependent variable under study, constitute (to this day), by virtue of the assumptions of optimal estimation, a representative model for the study of macroeconomic phenomena by the autoregressive effect of structural parameters on the dependent variable under study. Thus, the vector-autoregressive models were an alternative to the study of the macroeconomic phenomena to the multivariate the models presented by the Cowles Commission, which focused on the theory based on Walras’ General Equilibrium, in view of which the particular implication of the reliability of these models was the traceability of the structural parameters (Wójcik, 2014). The researchers, performing the inductive synthesis conditioned a posteriori on the phenomenon of the specific economic mechanisms and estimations of the vector-autoregression models, have constructed a structural vector-autoregression model that responds to structural shocks resulting from specific shocks (Krajewski, 2019). The procedure for estimating such a model first involves a robust verification of the VAR model by testing optimality assumptions, including, the stationarity of the time series, distribution of lags by information criterion tests, and autocorrelation of the time series. Then a test of the reaction impulse function is performed to analyze structural shocks and to structure the model. The St. Louis equation structurally meets the theoretical (from an economic perspective) assumptions of the structural vector-autoregressive SVAR model in...
studying the impact of short-term monetary shocks on the output dynamics, but the lack of sufficient analysis of the model’s reliability assumptions conditions the model’s strong sensitivity to replicability. The researchers, estimating the SVAR model, have generally noted the significant impact of monetary shocks on the output change for the U.S. economy (Bernanke, 1986; Blanchard and Watson, 1986; Bezemer and Grydaki, 2014; Halvorsen and Jacobsen, 2014), however, indicating, with an optimally chosen lag distribution, the more significant impact of short-term shocks. Barkelmans (2005) argues that for Australia’s economy, the discretionary monetary policy and monetary shocks have a significant effect on the formation of both the exchange rate and the market interest rate in the short term, while an even greater effect has a longer-term impact on the change in the output and prices. Credit channel shocks have also had a significant impact on the output dynamics in G7 countries (Magkonis and Tsopanakis, 2014).

3. RESEARCH METHODS

To estimate the structural vector-autoregression model, secondary data from the Central Statistical Office in Poland was used for the dynamics of the production in the compiled cyclical reports: ‘Preliminary estimate of gross domestic product’ and ‘Quarterly accounts of gross domestic product in 2017-2021’, as well as secondary data of the National Bank of Poland ‘M3 money supply and its creation factors’. The observations of the compiled sample are presented as an index of a percentage change with the money supply determining the supply of cash money in circulation, which is the largest share of the monetary base (M0), where the chain index of the relative value takes into account the base value of the previous quarter (previous quarter = 100), similarly the dynamics of production as a chain index of the relative value of GDP. The sample determines the time series in the quarterly sequence of the interval: 2007.Q1-2022.Q2 (62 observations), as the time series under study recorded economic fluctuations as a result of impulses caused by, among other things, the sheepish momentum in financial markets or the exogenous impact of the pandemic period on the decline in the marginal efficiency of capital. The phenomenon of outliers, which in the studied period could have caused, through the base effect phenomenon, the intensity of the dispersion of outliers in the tails of the distribution is an important implication for the studied time series (Kochański, 2022). Accordingly, outliers are an integral part of the time series under study and their omission can negatively affect the reliability of the study (Faldzinski, 2014). In particular, this issue is emphasized by the theory of extreme values, which has gained importance with the development of the GARCH class models, which take into account the conditional heteroskedasticity (Bollerslev, 1986).

The first step will be to decompose the time series of the structural parameters of production dynamics and circular money supply dynamics. Filters are a useful method of synchronizing business cycles in the time series under study, so they are used in the initial estimation of the macroeconometric time series models (Baxter and King, 1999). Such a procedural approach to the estimation of a structural vector-autoregressive SVAR model is presented by Mazzi, Mitchell and Moauro (2016) for the Monetary Union country zone data using the Hodrick-Prescott filter. Decomposing the time series with the Hodrick-Prescott (Hodrick and Prescott, 1997) high-pass filter isolates the trend and cyclical component, as shown in Equation 2:

\[ y_t = g_t + c_t \]  

where:
- \( y_t \) – time series,
- \( g_t \) – trend component,
- \( c_t \) – stationary random component.
In order to optimize the trend component, function minimization should be performed (Equation 3):

$$\min \left[ \sum_{t=1}^{T} \frac{1}{\lambda} (y_t - g_t)^2 + \sum_{t=1}^{T} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 \right]$$

(3)

According to the recommendation of the authors of the filter, the trend smoothing parameter, \( \lambda \), takes a value of 1600 for the quarterly sequence of the time series, thus cutting off the length of 39.7 quarters (10 years). However, the choice of the parameter can remain at the discretion of the researcher, who intends to choose the trend smoothing parameter optimally. Finding the lambda parameter in Equation 3 requires its first estimation according to the formula in Equation 4 (Kufel, Osińska, Błażejowski and Kufel, 2014):

$$\lambda = \frac{1}{4(1-\cos(\omega_0)^2)}$$

(4)

where:
\( \omega_0 \) – cutoff moment of the gain function.

Whereby (Equation 5):

$$\omega_0 = \frac{2\pi}{m}$$

(5)

where:
\( m \) – length of compensation period.

Or (Equation 6):

$$\lambda = \frac{\sigma_\xi^2}{\sigma_g^2}$$

(6)

where:
\( \sigma^2 \) – variance.

In addition to the Hodrick-Prescott filter, the Baxter-King (Zarnowitz and Ozyildirim, 2002) or Christiano-Fitzgerald (Christiano and Fitzgerald, 2003) filters are also used to synchronize the business cycles of time series; in the case of the former the inability to forecast and model the filtered series due to the loss of observations at the beginning and end of the sample is an important implication. The researchers (Kaiser and Maravall, 1999; Skrzypczyński, 2006) use the time series decomposition with the Hodrick-Prescott filter because, unlike the Christiano-Fitzgerald filter, it is a symmetric filter and it does not introduce a phase shift. According to Equations 2-6, the Hodrick-Prescott filter is the optimal filter for analyzing the time series of Poland’s macroeconomic variables, although, as Beck (2017) points out, the measures of the synchronization of Poland’s cycle with the rest of the EU and euro zone countries showed a slow increase in the degree of synchronization, except for the downturn, when it intensified.

To construct the structural vector-autoregression model, at first it is necessary to identify the classical VAR model which will be estimated according to the optimal model verification procedure set out in the paper. Application of the vector-autoregression VAR model in the analysis of unemployment hysteresis in the context of Okun’s Law’ (Kolbyko, 2022). The classical VAR model takes the following form (Equation 7):

$$X_t = \sum_{i=1}^{k} A_i X_{t-i} + \varepsilon_t$$

(7)

where:
\( k \) – row of the VAR model,
\( X_t \) – vector of observations of population values of n variables of the model,
\( A_i \) – matrix of autoregressive operators of individual processes, in which it is a priori assumed that there are no zero elements,
\( \varepsilon_t \) – vector of the residual component, in which the given components are correlated with each.
In contrast, the structural vector-autoregressive SVAR model takes the form (Equation 8):

\[ AX_t = \alpha_0 + \sum_{i=1}^{h} A_i X_{t-i} + B \eta_t \]  

where:

\[ \eta_t \] – random components of the model accounting for structural shocks.

A direct estimation of the structural vector-autoregressive model is impossible due to the parameter (A and B) responsible for the endogenous dependencies in the model and the problem of model traceability, which will be solved by a recursive model linkage for both variables and the VAR model structuring. The relationships are conditional as shown in Equation 9 and Equation 10:

\[ \varepsilon_t = A^{-1} B \eta_t \]  
\[ \Sigma = A^{-1} BB' (A^{-1})' \]  

The first step is to convert the analytical form of the model to a reduced form (Equation 11):

\[ \Phi = A_0^{-1} \Omega_t A_0^{-1} \]  

In order to structure the model, it is necessary to perform the identification of shocks by Cholesky decomposition, in which a matrix (which has dimensions of 2x2 in the case of the study conducted here) of covariance-variance defines a lower triangular matrix, \( P \), by a square matrix, \( \Sigma \), of dimensions \( N \times N \) (Equation 12):

\[ \Sigma = PP' \]  

The number of identifying conditions then constitutes (Equation 13):

\[ K = K_{SVAR} - K_{VAR} = (\left( P + 2 \right) N^2 + N) - (PN^2 + N + \frac{N(N+1)}{2}) = N^2 + \frac{N(N-1)}{2} \]  

After structuring the model to identify the impact of shocks on the variable under study, it is necessary to carry out an impulse response function. The IRF takes the following form (Equation 14):

\[ IRF_k = \frac{\partial X_{t+k}}{\partial \eta_t} \]  

The described methodology aims to study the interaction in the studied time series and the optimal choice of interaction lags between the studied structural parameters, the time of interaction between variables, the impact of endogenous and exogenous shocks and the properties of cyclical oscillations of the time series of the dynamics of the production and money supply in the Polish economy. The results of the estimation will allow to verify the monetary theory of fluctuations and the significance of the impact of the monetary parameter on the change in production in Poland.

The use of the methodological procedure according to Equations 2-6 and Equations 7-14 is consistent with the recommendations of the National Bank of Poland regarding the synchronization of business cycles, the spectral analysis and the identification of structural shocks in the constructed vector-autoregression models (Skrzypczyński, 2008).

The confidence interval adopted to verify the research hypotheses was chosen arbitrarily and is \( \alpha = 0.05 \).

4. RESULTS

According to the established estimation procedure, a ‘smoothed’ trend component was generated first using the HP filter. The time series decomposition charts in Figure 2 were generated using the package “mFilter” in Rstudio.
Monetary determinants of output dynamics in the light of the structural...

Figure 2. Trend component of GDP growth and money supply dynamics
Source: Own elaboration.

After the testing of the monotonic relationship between the time series for the variables of the dynamics of production and the money supply through the cross-correlogram analysis as in Figure 3, the relationship between the variables showed a negative cross-correlation ($r \approx -0.55^{***}$), which was the largest value (the statistical significance of the monotonic dependence was also shown by the correlation at one lag unit with a positive estimate, $r \approx 0.27^{**}$ for the arbitrarily chosen confidence interval $\alpha = 0.05$), so that the impulse between the variables under study was immediate.

Figure 3. Correlationogram between Money Supply dynamics and GDP dynamics
Source: Own elaboration.
The spectral analysis (Iacobucci, 2003) through the use of a periodogram and a spectrum using the Bartlett’s method showed a high variability of the process, as shown in Figure 4, which indicates intense fluctuations in the time series of the variables studied. Indeed, it is important to note the intensity of changes in the short, medium and long periods, as well as the clear phenomenon of seasonality. The periodogram indicates seasonality for the variable of the production dynamics for the period of change; 15.5, 6.9, 4.13 and 2.58 quarters, where for the frequency parameter $\omega = 3.14159$ the length of the period of change is 2 quarters. The periodogram for the money supply dynamics the variable showed seasonality especially for a period of change of 3.88 quarters, as shown by both the periodogram and the spectrum using the Bartlett’s method.

![Figure 4. Periodogram and Bartlett’s spectrum in the spectral analysis of GDP growth and money supply dynamics](image)

Source: Own elaboration.

After extracting the cyclical component, as shown in Figure 5, periodic oscillations of the random components of the studied structural parameters, which are negatively correlated, can be observed. It is possible to assume the duration interval of a full cycle lasting from 4 to 6 quarters when the upper and lower local extremes of the function of the cyclical component of production dynamics are observed. A clear outlier is indicated by the observation in Q2 of 2020 for both variables under study. Despite the higher intensity of the outlier of the cyclical component of production dynamics, both cyclical components of the studied variables in the examined time series showed similar intensity. The longest upward phase of production and the downward phase of money supply changes lasted about 8 quarters for the observations of 2016.Q2-2018.Q1. However, it is worth noting Hamilton’s (2017) accurate criticism of the use of the HP filter. He points out the sensitivity of the use of this filter due to the spurious relationship, biasing the filtered time series of the cyclic component, the tendency to generate spurious cycles, and the high sensitivity to a sample change. This is due to the fact that the HP filter is a band-pass filter, so the selection of the band passing fluctuations is arbitrary by the researcher. In accordance with the above, for the
analysis of business cycles and their synchronization in practice, an expansion is applied through the use of the time series models, among others. The work of Milo and co-authors (2013) of the Polish economic data, who extended the study to include the use of a vector-autoregressive VAR model, is an example of such an expansion of the analysis.

![GDP dynamics cyclic component](image1)

**Figure 5. Cyclic component of GDP growth and money supply dynamics**

Source: Own elaboration.

According to the methodology adopted for the optimal estimation of the vector-autoregressive VAR model, which is a necessary step in the reliable estimation of the structural SVAR model, the lag distribution of the model was be selected by using the AIC, BIC, HQC information criteria and the LR reliability quotient (Bernanke and Mihov, 1998; Hamilton and Herrera, 2004). An optimal selection of the lag distribution for the variable that is a dependent variable of the estimated VAR structural model showed the optimal selection of the second-order lag for the Akaike information criterion AIC, BIC and HQC and the statistical significance for the LR credibility quotient, so a lag distribution of 2 was selected for further estimation of the VAR and SVAR models.

The undesirable phenomenon of the time series autocorrelation for the vector-autoregression model does not show a significant partial autocorrelation PACF at the second lag (for production dynamics = 0.0011; for the money supply dynamics = 0.0814). The Ljung-Box test conducted for the production dynamics showed statistical significance (9.1933***) for the adopted confidence interval, which is an implication for the estimated VAR and SVAR models. In the case of the money supply dynamics, the Ljung-Box test showed no statistical significance (2.3002), indicating the absence of the time series autocorrelation.

A unit root test conducted by estimating the ADF test for the variables under study showed the statistical significance in both cases, which indicates the stationarity of the time series and therefore the absence of a unit root.
The estimated vector-autoregressive VAR(1) model takes the following form (Equation 15):

$$ GDP_t = \alpha_1 GDP_{t-1} + \alpha_2 GDP_{t-2} + \beta_1 M_{t-1} + \beta_2 M_{t-2} + \varepsilon_t $$  \hspace{1cm} (15)

where:

- $GDP, M$ – vectors of observations of the model’s production and the money supply dynamics variables.

**Table 1. Results of the performed estimation of the vector-autoregressive model VAR(1)**

<table>
<thead>
<tr>
<th>Explained variable: GDP$_t$</th>
<th>coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP$_{t-1}$</td>
<td>-0.24</td>
</tr>
<tr>
<td>GDP$_{t-2}$</td>
<td>0.26**</td>
</tr>
<tr>
<td>M$_{t-1}$</td>
<td>0.09</td>
</tr>
<tr>
<td>M$_{t-2}$</td>
<td>0.16**</td>
</tr>
</tbody>
</table>

*** - $p$-value < 0.01, ** - $p$-value < 0.05, * - $p$-value < 0.1  

Source: Own calculations.

For the VAR(2) model (Equation 16), in which the dependent variable under study is the dynamics of the money supply during the period under study:

$$ M_t = \alpha_1 GDP_{t-1} + \alpha_2 GDP_{t-2} + \beta_1 M_{t-1} + \beta_2 M_{t-2} + \varepsilon_t $$  \hspace{1cm} (16)

**Table 2. Results of the performed estimation of the vector-autoregressive model VAR(2)**

<table>
<thead>
<tr>
<th>Explained variable: M$_t$</th>
<th>coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP$_{t-1}$</td>
<td>0.60***</td>
</tr>
<tr>
<td>GDP$_{t-2}$</td>
<td>0.33**</td>
</tr>
<tr>
<td>M$_{t-1}$</td>
<td>0.42**</td>
</tr>
<tr>
<td>M$_{t-2}$</td>
<td>0.15</td>
</tr>
</tbody>
</table>

*** - $p$-value < 0.01, ** - $p$-value < 0.05, * - $p$-value < 0.1

Source: Own calculations.

The estimation results in Table 1 of the VAR(1) model using the Klein method showed statistical significance for the second lag degree of both parameters, and the coefficients which showed a positive effect on the formation of the production dynamics in the current period. In the case of the VAR(2) model, as shown in Table 2, the estimation results showed a significant effect for the 1st and 2nd lag degrees of the production dynamics and the money supply for the lag degree of 1.

The impulse response graphs (Figure 6) of the structural parameters produced showed that the monetary impulse in the form of the money supply dynamics has an impact on the formation of the production dynamics, where the most intense impact occurs in the short and medium term (up to about Q7), in the longer term the change in money supply has a much smaller impact. In the case of the impulse response of the money supply as a result of changes in the dynamics of production, the change in production causes only an intense change in the dynamics of the money supply in the short term. In the medium and longer term, a change in the production dynamics does not cause intensive changes in the monetary parameter. Indeed, it is important to emphasize the significant influence of the endogenous factor on the formation of the production dynamics, where, of course, in the long term it did not show a significant effect on the change in the structural parameter, but in the short and medium term, the endogenous influence significantly determined the intensity of changes in the production.
It should be noted that in accordance with the methodological procedure adopted for identifying structural shocks using the SVAR model, the National Bank of Poland survey for 1999-2009 showed a significant effect of monetary impulses on economic activity (Postek, 2011). The same conclusion was empirically verified by replicating the survey on quarterly data for 2007-2022 using a structural vector autoregression model. Indeed, it should be pointed out that, both in the above study and the study on the National Bank of Poland, the results obtained from the estimation of the reaction impulse function verified the indirect mechanism of monetary impulse transmission in the Polish economy. However, it should be indicated that the impulse had a strong effect in the short term, which is consistent with the economic theory, in which the monetary impulse does not affect the change in GDP.

5. CONCLUSION

The carried out empirical analysis, taking into account the established model of optimization assumptions for the reliable estimation of the HP time series decomposition and the structural vector-autoregression model, positively verified the hypothesis of the existence of the monetary theory of fluctuations in the Polish economy. First of all, the results obtained using the cross-correlogram to test the cross-correlation between the time series showed the statistical existence of the interaction between the variables under study, but with a negative test statistic for the absence of lags. The negative correlation in the current period is due to the time horizon of the transmission of the monetary impulse, specifically the
external lag of the monetary policy. Such a conclusion is confirmed by further examination of the similarity of the formation of the cyclical component in the examined time series, but it is shaped ‘divergently’. In doing so, the similarity is sufficiently evident in the ‘reversal points’ of the course of the cyclical components of the studied variables. However, taking into account the main research tool in the work – the SVAR model, the reaction impulses showed a particularly significant impact of the monetary parameter on the dynamics of production in the short and, to a lesser extent, the medium term. Accordingly, the obtained results of the empirical study positively verify the indirect transmission mechanism of monetary impulses and the monetary theory of the business cycle, which meets the research objective of the work.

An important implication for an empirical quantitative study is the estimation method used. In the above study, the Klein method and the estimation of optimally constructed the VAR models using the least squares (OLS) method were suggested. However, in practice, the results of the study can change significantly when using the method of maximum likelihood (MLE) or the Bayesian inference. The former, by maximizing the credibility function, can significantly reduce the difference between the function derived from the data and the mathematical hope. In the case of the latter, with the initial assumptions established a priori, the estimation procedure will have greater consistency with the pre-determined economic theory.

It is recommended that researchers use quantitative methods to study business cycles and the transmission of monetary impulses through the use of macroeconometric time series models. Indeed, however, the application of macroeconometric time series models requires the selection of a representative research sample and a time sequence. In the above study, the selection of the research sample took into account the inductive information on the fluctuations of the Polish economic cycle and the time sequence conditioned on the carried out business cycle synchronization with the HP filter.

However, researchers should avoid using the multi-equation structural DSGE models based on the tautological doctrine of the neoclassical economics trend, which was used by the Cowles Commission. The methodological approach in this approach limits the identification of structural shocks, thus the stress testing macroeconomic reality.

It is necessary to refer to the teaching flowing at the outset - there are no ideal econometric models, just as there is no stationary state of the economy in which a change in the monetary parameter would only induce inflationary inertia as a result of the operation of monetary neutrality. A key implication of the theoretical interpretation of empirical results is to formulate it as a ‘setting’ derived from the positivist a posteriori assumption. This assumption, based on the inductive-deductive methodology, must be conditioned by complex information about the deterministic dynamic system of an economic system. The econometric models only aim to approximate a given problem and bring researchers closer to verifying the previously posed research problem.

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1 In Poland, PKO BP bank, which is the universal bank with the largest share in the issuance of treasury securities (as a single entity), is the market maker of treasury bonds on the primary market.

2 On a similar basis today, the Central Banks’ liquidity reserves are obtained through the reserve requirement rate from commercial banks.

3 In the case of the one-sided nature of the estimation as the end of the observation sample approaches, both the HP and CF filters show asymmetry.

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FUNDING I Not applicable.

CONFLICTS OF INTERESTS I The Author declares no conflict of interest.

DATA AVAILABILITY STATEMENT I Data available on request from the author.

AUTHOR CONTRIBUTIONS I Not applicable.
REFERENCES


Skrzypczyński, P. (2008). Wahania aktywności gospodarczej w Polsce i w strefie euro. Instytut Ekonomiczny NBP.


