S F W J F X   I P G I H F O F S B M I F Y Q P O F O U J B U J P O I B M H P S J U I N T I

B suvsiKbi vcti j-ISpcfsdQfsnj/ztlj!

Jotujuvuf!pgI0 pn qvufsiboel.Jogpsn bujpoITdjfodftl
0 {fttdch px blV ojwfstjuz!pgUfdi opmphz-IQpmboel
bsuvs/Kbi vctlJA jdjt/qdC/qmlspcfsv/qfsnjotlJA jdjt/qdC/qml

B ctusbdu/IB sjui n fqjdlpolnbshfljoufhfstljtlpgufolofdfttbszljoldszquphsbqiz z/IN bozldszquptzt.
ufn tifefqfoelpolui flqpttjcjmjuzlpgldpn qvujohlqpx fstl h ^]Fyqpofoujbujpo*/IJolui jtlqbqfslx fl
eftdsjcflx fmol opx olfyqpofoujbujpolbmhpsjui n tlx jui lui fjstdpn qmfyjuz!bobmztjt/IP vslbmhp.
sjui n ljouspevdftlbll olfx ljefblpgltpnwjohlui flfyqpofoujbujpolqspcmfn /IX fltipx ldbtft!joi
x ijdi lui fldpn qmfyjuz!pglpvslbmhpsjui n ljtlcfuufsuibolui fldpn qmfyjuz!pglbozlpui fslbmhp.
sjui n t/I

Jouspevdujpo

U i flqspcmfn lpglfggjdjfoulfyqpofoujbujpoljtlfyusfn fmzljn qpsuboulgpslui flefwfm
pqn foulpgln pefsoldszquphsbqiz z/IN bozldszquphsbqiz jdltztfm tlivtflui jtlpqfsbujpoljol
ui flnn pn bujpolfodszqujpolqspdfttlJfh/IS T B lpsiF ml bm bm*/I

U i jtlx psl ldpodfsotlui flqspcmfn lpglfggfdujwfmzldbmmvm bujoh h ^.lx i sfflhljtlboll
fm fn foulpglhspvq!H -lboelfjjtlblqptjujwfljoufhfslnvn cfs\2, 3^/IH fofsbmz.lui jtlqspc.
n fm ljtlpdptjefsfelx i fonljtlbllsfbmln n cfs/Jolui jtlqbqfslx flbobmtfliglfofsbmlfyqp.
ofoujbujpolbmlpsjui n tlboeldpn qbsfl fjsalfggfdujwfolttl/IB tlui fldsjufsjpolpgl fl fggfdl.
ujwfofttlpgluifl bmlpsjui n lx flbepqufelui flovn cfslpglhspvq!pqlpqfsbujpotlx i jdi lui f!
bmlpsjui n lqfsgpsn t/IX i fonljtlovn cfsljtln bmfslps!bltfupglwbmesfltlbepqufolczlf-l
tvdi lbolbmlpsjui n ljtldpoltjefsfelupjcfln psfligjdjfou/IX fbsflbx bsflpgl fjgbdttluibll
bobmztjtlpgl fl ovn cfslpglbmlpsjui n lbjul pqfsbujpotlx pvmlcfln psfldpodmstjwf/I
B x bsflpgl jtlgbdu-lx flbcboepolefjjdljptjtn qmgzlui flbobmztjtlboelvtlupjn blfl fl
qbqfsldmfbsfs/I

Jolui jtlqbqfslx flqsftfoutlpvslpx oldpouscjcvujpo-lboelx i flofsbmlfyqpofoujbujpolbm
hpsjui n /IX fldbndmelipjuli flbmlpsjui n lpgl{fsp.potjoltlsfvofodftlboel jujtldpoujofelpol
Tfdujpol3lpgluli jtlx psl/IX fldpn qbsfljutlfggfdujwofttlupi fslsbmlpsjui n tlpqsftfoufel
jolui fix psl/I

2 /IS fwjfx lpglhfofsbmlfyqpofoujbujpolbmlpsjui n tl

U i flqspcmfn lpglfyqpofoujbujpoldbolcfldpotjefsfelvfpjui fldi bohjohlwbmfttlpgl
hjpsfl/IX fldbolodpotjefsui flqspcmfn lpspchfldbtfligihlboeli bohjohlfyqpofoullipsl

jowfstfm-lgpsi gjyfel fl boel bscjusbsjnzi di ptfoi h /Ui flui jsai pqujpoljti x i foi cpui lui fl
wbmefflbsfltvckfdduipl di bohf /

   Ui fsflbsflui fsfgpsflui sefluzqfti pgi fyqpofoubipolbmpsjui m t;l

π   hfofsbmifyqpofoubipolbmpsjui m t-l

l   fyqpofoubipoi gpsi bscjusbsjmzi di ptfoi hi boel bscjusbsjmzi di ptfoi f-l tp.dbmefl
   hfofsbmqvspftfifyqpofoubipolbmpsjui m t;l

      À   tjm qmflcjobszlboell .bzlfyqpofoubipol

      À   tmjejoh.x joepx lfyqpofoubipol

π   gjyfe.fyqpofoufyqpofoubipol

π   gjyfe.cbtflfyqpofoubipo /

Jolui jtiqbpfslx fldpotjefslui flhfofsbmfyqpofoubipolqspcmfm -lx i fsflofjui flehloopsef
bsflqsefufufm jofe /Joipyvstuwez-lx flx jmtbobmztfluifl flwbmeftipgflijolbklcsbjolsbohf
jolposefsuipdpm qbsfui flfggfdujwfofttipgui flbmpsjui m t /

2 /2 /Fyqpofoubipol.mobjwflm fui pel

   Jutffm tluolbdtiui flfbtjftuwbzluplcbmdvmvuflhˆljtiuplqfsgpsm i f .2lpqfsbjpotijol
hspvql H /Cfmpx l jtibol fybm qmfi pgl boi bmpsjui m l qtfvepdpefl dbmefl ui flobjwfl
m fui pe /

Bmpsjui m l.lobjwflm fui pe

Joqvil h — H · f — 0̲

Pvuqvcl hˆ

2 /   B Æ h
3 /   G psijlgspm l2luplf . 2leplui flgompx joh;l
      3 /2 /   B Æ B /h
4 /   S fwsol B

   Ui jtltjm qmfm fui pelpqfsdbmdvmvbujohiqpx fsijtipgufoifijofggfdujwf-ltfqfdjbmzlx i fol
jtipgbikiqhi lwbmef /N osfowfslui fldpm pfyjuzipgui jtibmpsjui m lsfrvjsftldbmdvmvujohl
P lf^l m vmjqmjdbujpot / Gpsi qsbdjdbmi bqqmjdbujpot-lx i fsfl fi jtipgltfwfsbl i voesfe-l
bui pvuboe-lpsm psfcjujt-luifibqqmjdbujpolpgui jtim fuipiltim qbsdjdbml

2 /3 /Mfgu.up.sjhi ulbmpsjui m l

   Ui fiovm cfspgpqfsbujpotijolui flbmpsjui m lpgui flqsfwjpvtitfdujpoldboclfiltjhojg,
jdbouzlsfevdfe /Joi ui iboel ui fl fjyl tfdujpol x fl x jmlqsftfoul ui flbmpsjui m lti ef .
tdsjcfeljolinufsravrfltibtiui flcjobszlfyqpofoubipolbmpsjui m -ljtlsbefebmpsjui m lgpsl
sbjsjohiuplui fltrvbfspsuflfibmpsjui m lpgpgbtflfyqpofoubipol\2, 4^/l
Joi ui iblmpsjui m -lui flfiwbmflpgi B iiqbsjbmrfsfvml tipgl fyqpofoubipo*ijtisbjftfipi ui fl
trvbfsljolfbil ldpvrslpgm pql3-ljf /Au, l2luim ft /Joifbdi lfyfdvujpolpgi mppl3lui fl
bmpsjui m l di fdl ti ui fl ofyui cjui wbmfi-ljgi ui jti wbmfi jti frvbm lup l 2-i x flbttjohil
m vmjqmzjohiBiboeiliupisftvmiB /

B mhpsjun !pgfmfgu.up.sjhu!cjobszfyqpofoujbujpo

Joqvu! h — H -bklpptjujwfjoufhfs f >)f_n f_{n-2} ··· f_2 f_1 *_3

P vuqvu! h^f

2/ B Æ 2
3/ G psjjgspm !sdpx olup! 1 !eplu! f !gpmpx joh;!
    3/2/ B Æ B /B
    3/3/ Jg f_j > 2 !uibol B Æ B /h
4/ S fwsol B

E xfjoplu! fibqbctt butt fjn ptutjhojdboucjupgfyqpofouf!jtifrvbmupsipot-fx sfdbol pn jujuf fjqstdt fyfdvujpolpgfmpqpq3 !boejub vtibttjhohjupivbsjbcmf!B /X fsbl fijouplbd. dpvoujt jtifbodt x foflobobmztjoh!tfdpn qnfyjuzjpgdi jtibmpsjun /Joln psfifubjmfe bobmztjt-ul fjovn cfsipm vmjqnjdbupots!pqfsbujpoljolui fjhspvq *lpgui jtbmpsjun i jti !mhf f ⌐. w8f *. 2 !x ifsf!w8f *!jtli fjovn cfsjpgpofijolui fjcjobszisfqsftfoubujpo

pgtfj\5^/Gpsfi f!126 !ui mn fj!pelsfqv!sfttiyjn vmjqmdbujpot-fx sjmfif h^26 !dbolcfifef. ufsm jofe vtjoh!polmzjgwjn vmjqndbujpot/Ui fibmpsjun lepttopux psl jpquin bml- qbsujdvbsjmy!gpsicjobszfyqpofoutu! bu!dpoubjolmpoh!tfrvfodftipgpoft/Jolcpodlv. tjpo-ui fibtzm qupjdpm qnfyjuzipgfifmfgu.up.sjhu!tbmpsjun ji ti P)mghf *A

2/3/2/F ybn qmf!

C fmpx x fjqsftfoui fimfgu.up.sjhu!tbmpsjun /Ubcml 2!eftdsjcft!ui fipcubjofe! wbmft-!ubl folcz!B !jolt!vddft!jojmhppl !3ldpvsftt-gpsfyqpofout!> !394/

P qfsbujohipgmfgu.up.sjhu!tbmpsjun lgpsfl > !384!

j!	f_j	f_{)3+}	B
9!	2!	2!	h
8!	1!	21!	h^3
7!	1!	211!	h^5
6!	1!	2111!	h^9
5!	2!	21112!	h^28
4!	2!	211122!	h^46
3!	1!	2111221!	h^81
2!	2!	211122121!	h^252
1!	2!	2111221221!	h^394

2/4/IS jhI u. up.-nfgubnbpsjun !

    UI jtibnbpsjun -Iui pvhI IjuteIjggfesfigspno Iui fIqsfwjpvtiopofe Ix psI tIwfeszItjn jnbsnzI\2-I 4^/I UI fIm bjoI eIjggfesofodfI npItIjoI uI fIvtI fI pgI cjobszI fyqpofouI f /AU fI fI fyqpofouI bjtI frvbnbppl2IjoI uI jtIcdbtI fbesfIdI fdI fe IjoIntftpql3/2Ipg Iui fIbnbpsjun *IgspmI uI fIn fbtuItjh. ojgdbouI cjtuptoI uI fIm ptI tjhogjgdbou ponf -Itp -IgspmI rjhI uI pInfgw/IMppp I3I fyfdvuft Ibt I npohI btI f ' I -Iui bujt-IefigbduoI -IbtInpohIbtIx fI Ibwf Icjut Ipgf I fI fyqpofouI Joi fbd I fyfdvujpoIpgImppqI3-ITIjtIsbjtfeI upI uI fIsrvbsf /AUI vt -Iui fItfdipgvbmwI fbsbppqufeIbzITIjtI I h^2-h^3-h^6 -...-h^{2^{m-I}} ~/I S ftvmf BI jtI pcubjofe Iczim vmqjqmzjohI uI ftfI qpxf rstI h x I jcI IdpsrfspodItupIuwbnf fi2IjoI uI fIcjobszIgspm Ipgf IfyqpofouI/I Igspm Iui fImfbtutjhoj. gjdbouI cjt */AKvtu bt Ijoi uI fInfgu up. sjhI ubnbpsjun -Iui fIbtzm qutjdpn qnfyjuzIpgfI jtI bnbpsjun IjtI P)nphI f *./

B nbpsjun IpgfsjhI u. up.-nfgubcjobszI fyqpofoubujpoI

Jnqvuı h — H -IbIjoufhfsI f — 0 .

P vuqvuı h^f

2/   B Æ 2 -ITÆ h

3/   X jnjfI f ' I Ieplui fIgpmpwjoh:I

     3/2/ Joef IjtIpoduI foI B Æ B /T

     3/3/ f Æ ⌊ f 03 ⌋

     3/4/ Joif ' I tuif oIT Æ T /T

4/   S fuvsoI B

2/4/2/IFybmqnfI

    UI fIfybmqnfIpgf IfsjhI u. up.-nfgubnbpsjun IjtItI px oIjoIUbcmfI3/AWbsjbcnfIBItupsftI uI fIqbsujbnIsftvntIpgf IfIqpx frsIcpnqvufeIjoItvddffejohIdpvstftIpgmppqI3/AD pmmo oI 4IthI px tIuhfIcjobszIfyqpofouIgjfnbtbbihjwfoItbhftpgf IfIbnbpsjun /I

UI fIsjhI u. up.-nfgubnbpsjun IgpsIfI>I3848I

f_{(2)}*	f_{(3)}*	dpm qvuefIfyqpofouI	BI	TI
3948I	211122122I		2I	hI
2528I	21112212I	2	hI	h^3
818I	2111221I	22	h^4	h^5
468I	211122I	122	h^4	h^9
288I	21112I	2122	h^22	h^27
98I	2111I	22122	h^36	h^43
58I	211I	122122	h^36	h^75
38I	21I	1122122	h^36	h^238
28I	2I	11122122	h^38	h^367
18I	I	211122122	h^394	.I

2/5/N powhpn fszlMbeefsl

U fN powhpn fszlMbeefsl\6^ijtiboljouftustujohln pejgjdbujpolpgui flnfguup.sjhl ui bmbpsjun /Kvtubstijolul bubmbpsjun -lui fN powhpn fszlMbeefslvtiftlflbjobrylsfqsf. tfobujolpgli fyqpofoul fbstl x fm/tUi fl x pel jolnipgui fN powhpn fszlMbeefslboelui fl nfguup.sjhl ubmbpsjun -ljtlsfubeltploltrebejohli fl tvctfsrvou bjtlpgli fl fyqpofou-l gspn lui fln ptulpli flmbstljhojgdboucjul/tUi fljgjfsfodfjolui fN powhpn fszlMbeefrsl jtlb ubjtlcfhjotlsfbejohgspn lui fltfdpolbcjul/Jolui fN powhpn fszlMbeefrs-l x fldbmv. nbufl x pl opptsjcmfl qbsjbmlsftvml joljolpoltutp-l x ljdi ljbrsl; lsbjtjohl upl fl trvbsfl boel sbjtjohl upl fl trvbrfl x jul n vmjqmzjohlczlh/E fqfoejohlpolui fl vmvflpgui flofyul fyqpofoucjul-x fldl pptfl ui fl qsppfrlpqbrjbmlsftvm/

Jolui jtlbmpsjun -lui fsfl bsfl wpl bouyjmjbsyl vbsjbcmftl.l y_2 lbnel y_3 /Ui flgjstulpofl upl fltsiuvddftsjw nyjDeuvsjohli fl fydvujpolpgui fl tppppql3^4lui flwbmrflpgl h $^{lf_u{}^lf_u{}_2\cdot{}^*{}_3}$ -lh $^{lf_u{}^lf_u{}_2{}^*{}_3}$ -l fud/-lboelgjobmmzlsfbcli slui fl tftjsfelwbmrflpgl h $^{lf_u{}^lf_u{}_2\cdots{}_2lf_1{}^*{}_3}$ /Ui flwbmrflpgl y_3 lijoltvc. tfrvoulpppl3^lfyfdvujoslbstbkwbmrflpgl h /y_2 /Vbsjbcmfsl y_2 lbnel y_3 lbsfltpn fl ti jol njlflu fl fldpm pqfm fouarysluvohtlpgulbmbeefsl/X l fol y_2 ljtlpolwbmrfl h $^{lf_u{}^lf_u{}_2\cdots{}_n{}^*{}_3}$ lboelti flofydbujolpgui fl fyqpofoul jtl flofydbfgsm *ljtl2-lui fol y_2 lubl fltuil flwbmrflpgl y_2 /y_3 -ltp-l y_2 /h /y_2 /Ui jtln fbosltbjtioli jtldbsf-lui flwbmrflpgl y_2 ljtl h $^{lf_u{}^lf_u{}_2\cdots{}_n{}^*{}_3{}^2*{}_3}$ /X l fol y_2 ljtl pglwbmrfl h $^{lf_u{}^lf_u{}_2\cdots{}_n{}^*{}_3}$ lboel fl ofydbujolpgui fl fyqpofouljtll-lui folx fbsshjol y_2 /y_2 lupl y_2 -lui vt-l x fl flbwflopn qvufsl h $^{lf_u{}^lf_u{}_2\cdots{}_n{}_1{}^*{}_3}$ /

Ui fsflbsfl x pln vmjqmjdbujotlpfsgorn fejoljlbdi l fyfdvujpolpgln ppql3-lboelui fl mppql svotl ⌊ lpgl $_3$ l⌋ l jn tt/A B mlpvhl l ui fN powhpn fszlMbeefsl srvjsfsl bl rfgrbfrl ovm cfslpglpqfsbujpotlu bolui flnfguup.sjhl ubmbpsjun -ljtl bbijoljouftustujohlqspqfsuz/ Ui fldbmrvmbujolpgl y_2 lboel y_3 ljoltuql3/2ldbolcflqfsgorm fejolqbsbmmfml/0 ptjefsjoh ul fbszn qupdjdpn pfyjzjpgl jtlbmpsjun -lui fsflbsflP Imgh l^*jn vmjqplnujotlpl qfsgorn -lui fltbn lbstlpgtul fl x plqsfwjpvtlbmpsjun tl/

N powhpn fszlMbeefsl

Jbqvulbolfmn fouh —H boelblqptjujwfljoufhfsslf > Jf_u lf_u{}_2 \cdots lf_2 lf_1 {}^*{}_3

P vuqvulh ^{1}

2/l y_2 Æl h -l y_3 Æl h ^{3}

3/l Gpsljlgspn lu. 2lepx olupll lepltll flgppml x joh:l

   3/2/lbgl f_j > l lui fol; l y_3 Æl y_2 /y_3 lbnel y_2 Æl y_2^3

      P ui fsx jtf:l y_2 Æl y_2 /y_3 lbnel y_3 Æl y_3^3

4/l S fwsol y_2

2/6/lTmjejoh. x joepx lbmbpsjun l

Ui fljefbllpgl fltmjejoh. x joepx l\4^ibmbpsjun ljtlcbtfelpolvtjohlui flgbdul bul dfsbjol qbstl pgl ui fl cjobsyl gprn l pgl ui fl fyqpofou bsfl pgufol sfqfbufe/l l bwjoh l efutrn joefl ui flwbmrflpgl bl dfsbjol qbsjbm lppx fs-l x fl dbol vtfl jul sfqfbufemzl upl

cbmlvmbufl ul fl dpssfddl qpx fs/l Gpsl ul jtl sfbtpo-l x fl ejtjuohvjti l ul fl qsfdpn qvubjpol tubhfljolul fltmjejoh.x joepx lbmbpsjui n /AU l fl qsfdpn qvubjpoljtlpletfufsm joflbmbpsel qpx fsllpghlgspn l2luplu 3^{l-2} . 2 ljodmrtjwf-lx l fsfll ljtlul fltj{flpglul fx joepx /A

U l fl hsfbufsl x joepx ltj{fll l fmbsh fsl l *lul fm pslel lopqfsbujpol ofdftbsyl gpsl dbmlvmbujohl ul fl qsfdpn qvubjpo-l x l jdl l ti pvml sfsvml jol bl gfx fsl ovm cfsl pgl pqfsbujpoljolul fsfmfwboulqbsupglul fmbmbpsjui n /Al px fwfs-ljgl ul fx joepx ltj{fljtluppl mbsh f-ul fldpstupgll fl qsfdpn qvubjpolx jmbopuldfdpn pfotbufl czlbln bmfslovm cfsl pgm vmjqmjdbujoploljolul fsfmfwboulqbsupglul fmbmbpsjui n /A

U l fltmjejoh-x joepx lbmbpsjui n l

Joqvul h -l f >)f_{u}f_{u-2} ... f_{2}f_{1}*_{3} lx l fsfll f_{u} > -2 -ljoufhfsl l μ 2

Pvuqvul h^{f}

2/A Qsfdpn qvubjpo:l

    2/2/A h_{2} Æ h -l h_{3} Æ h ^{3}

    2/3/A Gpsljlgspn l2luplu 3^{l-2} . 2 lephul fl goppx joh:l h_{3j-2} Æ h_{3j-2}/h_{3}

3/A B Æ 2 -lj Æ u

4/A X l jmfl jl μ 1 lephul fl goppx joh:l

    4/2/A Juf_{j} > 1 lul f o :l B Æ B^{3} lboel y_{2} Æ y_{2}^{3}

      P ul fsx jtf :l

        Gjoel fl mpohftuldjutujohl f_{j}f_{j-2} ... f_{m} ltvdi ltul bul

        j - m, 2 - l lboel f_{m} > 2 -lephul fl goppx joh:l

        B Æ B^{3j-m 2}/h)f_{j}f_{j-2} ... f_{m}*_{3} -lj Æ m- 2

5/A Sfwsol B

U l fl mbmbpsjui n l lx psl l tjtm jmsarzluplul fm fguloy-sjhi tlbmbpsjui n /AU l fledjggfsfodfljtlbu l jun lpvftlevfluplul fx joepx ltj{fl -ljotufbellpglm vjohljudczlcjud/AU l vt-lx feldboltbwel upll - 2 lm vmjqmjdbujoplpqfsbujposlbmm -cjutlpgl fl fyqpofou/A

2/8/2/AF ybn qmfl

    X l fx boululdpvoul flovm cfslpgl h^{2285:} /Aloluljtldbtf-lul fl cjobsyl gpsm lpglfyqp- ofoul f > 2285:_{)21+} > 212212222112 12_{)3+}/AB tljulutimohl jtlts-l2 5 -lwbmfl u > 24 /AU l ftj{fll pgll fx joepx lx fl ettll l > l4 /AU l fwbmfl pglwbsjbemftltvcdtcqtuhldpssftpoetluplull wbmfl pgljtlpx fs/Jolqsfdpn qvubjpolx fl efufsm joe:l
Å   Joltufql2/2:l
    h_{2} Æ h -l h_{3} Æ h /h
Å   Joltufql2/3:l
    h_{4} Æ h_{2}/h_{3} -l h_{6} Æ h_{4}/h_{3} -l h_{8} Æ h_{6}/h_{3} /A

    U l flybn qmflpgl fmbmbpsjui n lpgsl f > 2285: ljtl tl px lol jol Ubcmfl 4 /Aljsvotlpgsl2 5 l wbmftlx l jdll dpssftpoetluplul fmfohll pgll fl cjobsyl fyqpofoul)gspn l l lupl 24*/A

Jo!ui f!dbtf!pg!ui f!vtf!pg!b!x joepx -x f!sfevdf!wbmvf!j!pg!ui f!x joepx !tj{f /X f!sbjtf!!
qbsujbmsftvmd B !up!ui f!trvbsf!bt!n boz!bt!jt!ui f!x joepx !tj{f!boe!n vmjqmz!cz!ui f!!
wbmvf!pg!ui f!vtfe!qbuufso /Jg!ui f!x joepx !jt!opo/ bti fe-wbsjbcmf!j!jt!efdsfn foufe!!
)qbsujbmsftvmd B !jt!sbjtfe!up!ui f!trvbsf */

Ubcmf!4!
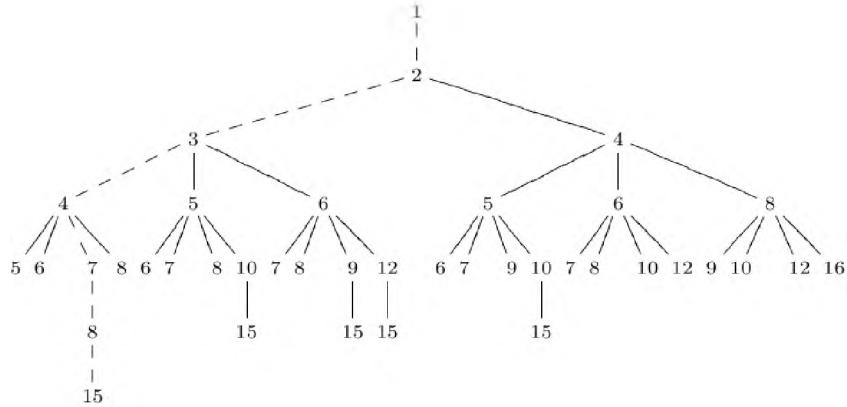
X psljoh!pg!tmjejoh.x joepx !bmhpsjui n -!gps!f!>!2285: !

j!	dpn qvufe!fyqpofou	B !	x joepx !
24!		2!	212!
21!	212!	h^6	212!
8!	212212!	$)h^6*9 h^6 > h^{56}$	222!
5!	21221222!	$)h^{56}*9 h^8 > h^{478}$	.!
4!	2122122221!	$)h^{478}*3 > h^{845}$	.!
3!	21221222211!	$)h^{845}*3 > h^{2579}$	212!
1!	21221222211212!	$)h^{2579}*9 h^6 > h^{2285:}$	.!

2/7 /!ÅQ px fslusff Ç!

U i f!bobmztjt!pg!bmhpsjui n t!gps!fyqpofoujbujpo!dbo!cf!qfsgpsn fe!cbtfe!po!ui f!
qpx fslusff!\5 ^/U i f!opeft!pg!ui jt!usff !bsf !wbmvft-!x i jdi !fyqpofou!dbo!bepqu/U i f!
gpmpx joh!mfwfmt!pg!ui f!usff !bsf !pcubjofe!bt!b!sftvmd!pg!n vmjqmjdbujpo!pg!opeft!bu!
ui f!mfwfm!bcpwf!ui f!mfwfm!bobmz {fe /U i f!sppu!pg!ui f!usff !sfqsftfou!ui f!ui f!wbmvf!!
pg!2/U i f!qbui !jo!ui f!usff !jt!ui f!bmhpsjui n !dpvstf!gps!b!gjyfe!wbmvf!pg!f/U i f!qbui !jt!
ui vt!b!tfrvfodf!pg!beejujwft-!gps!b!gjyfe!fyqpofou!\5 ^/U i f!qbui !!mfohu i !)ovn cfs!
pg!qbui !csbodi ft*!fyqsfttft!ui f!ovn cfs!pg!n vmjqmjdbujpot!x i jdi !ui f!bmhpsjui n !!
qfsgpsn t/

1
|
2
3 4
4 5 6 5 6 8
5 6 7 8 6 7 8 10 7 8 9 12 6 7 9 10 7 8 10 12 9 10 12 16
15 14 15 15 15
15

Gjh/12/!Mfgu.up.sjhi u!bmhpsjui n !.!qpx fslusff!

B obmztjoh!ui f!dpvstf!pg!ui f!nfgu.up.sjhi ubmpsjun !gps!fyqpofou!f!>!26!)Gjh ⁄2-! ebti fe!njof*!jo!ui f!qpx fslusff-!x f!dbo!tff!ui bui f!wbm f!pg!26!pddvst!bui f!tfwfou ! nfwfnU i fsfgpsf-!tjy!csbodi ft!dpoofdui ui jt!nfwfm x ju !ui f!lusff!spppu⁄U i jt!n fbot!bu ! ui f!bmbpsjun !sfrvjsft!jo!ui jt!dbtf!tjy!n vmjqmjdbujpot⁄U i f!wbm f!pg!26!jt!gps!ui f! gjstu!ujn f!bui f!tjyu !mfwfm!ui jt!n fbot!bui f!pqujn vm !ovm cfs!pgm n vmjqmjdbujpot! ofdfttbsz!up!dbmdvmbuf! h²⁶ !jt!6⁄U i f!bmbpsjui n !gspn !nfgu!up!sjhi ujt!opupqujn bm!

Gjh ⁄3 ⁄S jhi u.up..nfgubmbpsjun !.!qpx fslusff!

U i f!ebti fe!njof!jo!Gjh vsf!3!ti px t!ui f!bobmztjt!pg!ui f!sjhi u.up..nfgubmbpsjun !gps! fyqpofou!f!>!26⁄B t!jo!ui f!qsfwjpvt!qpx fslusff-!ui f!wbm f!pg!26!pddvst!bui f!tf. wfou!nfwfm⁄Gps!ui jt!fyqpofou-!ui f!bmbpsjui n !epft!opu!qfsgpsm !bo!pqujn vm !ovm cfs! pgm vmjqmjdbujpot⁄U i f!sjhi u.up..nfgubmbpsjui n !jt!opupqujn bm!ui fsf⁄

Gjh ⁄4 ⁄N pouhpn fsz!Mbeefs!.!qpx fslusff!

Mppl johlbmlGjhvsfl4-lx fldboltfflbmldvtjohlm flN ponlpn fszlMbaefsfgpsfryqp. ofoufl>l26-lm flbmbpsjdm n ljohlm flqpx feusff fmqbttff flm spvhl lm fltbm flopoftlbtlm fl sjhlm lp.mfmtlbmbpsjdm n AX fldponvefldm batfpstlm fltbm flsfbtpotlbtlm flbcpvflmhjwflm bmbpsjdm n t-lm jtlbmbpsjdm n ljtlopudpqm m bmfjm fs/l

3/AU lf lbmbpsjdm n lpglfsp.pofltfrvfodftl

U lfljebmlpgmfltjm futpomdpon ftfgpsm tlm flfbmduu batjtjtbffetlmpsfqspevdfm tlbmdjob. szlm qbmfsolmdpontjtujohlm pgfbjutlpgmwbm ff l2/AT vdm bmlqbmfsolm x fldbmtljtmt flmqbmfsolm pgl poft*ldbolcf lpctjofm mbfebczlm vmjqm johlm x plmdpontdvujwflmqbmfsotl)x m psfm lmdohmt lejgl gfstlczlpof*-ldpontjtjohlmpgmbjutl2lbbemllbmffsobmfnzl)x fldbmbmd fm m l(fsp.pofltfrvfo. dft*/AJolm fljtbmbpsjdm n -lx fm pvfm spvhl lm flfbjutjpogfyqponfoulfmpgmdomm flm nfohmlpgl m flmpohftutfrvfodflpgmbjutlpgmwbm fm l2l)m bsm fmdczle*/l

B lbpsjdm n lpglfsp.pofltfrvfodftl

Jnqvulm -lf >)f_nf_{n.2}\cdots f_2f_1^*_3)x mfsf lf_n > 2

Pvmqvulm ^t

2/A Qsfdpn qvubujpo:l
    2/2/A Gjoelm flmpohftutusjohlpdpon ftlboelbttjohljutlmefohm lm plwbsjbcm lm
    2/3/A h_2 Æ h -l0_1 Æ 1
    2/4/A Gpsljlgspn l2lmuplm)e . l2*l03leplm flgpmmpx joh:l

        2/4/2 l0_j Æ 5q_{j.2} , 2

        2/4/3 lh_{3*_j} Æ h_{*_j}^3 -lh_{5*_j} Æ h_{3*_j}^3 -lh_{6*_{j.2}} Æ h_{6*_j}/h

        2/4/4 lJgelfwf olm fo:l0_{n_{03}} Æ 5q_{n_{03.2}} , 2 -lh_{3*_{n_{03}}} Æ h_{*_{n_{03}}}^3

3/A B Æ 2 -lj Æ u
4/A X jmfljl µ l lepl m flgpmmpx joh:l
    4/2/A Jol f_j > l lm fo:lB Æ B^3 lboel j Æ j . 2
        Pm fsx jtf:l
            Gjoelm flmpohftulcjutlsjohlf_jf_{j.2}\cdots f_nlm bm bm lm flf lcjtlqbmfso -l
            tvdm lm bmlj . m_3 - e boelepl m flgpmmpx joh:l
            B Æ B^{3j.n.2}/h_{)f_jf_{j.2}\cdots f_n^*_3} -lj Æ m .2
5/A Sfwvso lB

Jolm flqsfdpn qvubujpo-lx fldpotuvdulbcbtfldpotjtujohlpgmbm lf l(fsp.pofltfqbu tfstlboelqbutfstlpgmpofmpgmbmfohm lmfttlm boljpsfrvbmlm plwbm l)e/AUbcmf l5tl pxs tl m fldbmvmbujpolpgmtvdsftuvfm (fsp.pofmfn m foufgpsmbmfohm lopulfydffejoh:/AU fl qbmfsolmpofotlbslm bm m bzlm pcmbjofm l czlm vmjqmmjohlx pltvdfttuvfm qbmfsolpgl ejggfsoum mfohm t/AU flqbmfsolpgmpoftlmfohm l5lx jmmlcflpcubjofm lczlm vmjqmmjohl m flqbmfsotlpgm0 p/A5lboelm0 p/A6ljolUbcmf l5/A

Kvtudbtljolui flmfgu.to.sjhi ubmbpsjun -lx fln pwfll fsflui spvhi lui flcjutlpgl fyqpofoui f-lgspm lui fln pttl sjhojgldbou tl lui flmfbtu sjhojgldbou/l X l folul fl qbwfskol pddvstljoi fyqpofoui f-l x fl sfqspevdfl jul ln vmjqmzl czl ul fl cbtfl fmfm fou*/l X l fol x fll bwfl bltsfuvfodfldpm pptfelpglcjutlx jul lwbmvfll-lui folx flsbjtfll fl wbmvfll pglpvstm pprbszl sftvmtl pl ui fltrvbsf /l

[fsp.po ffsrv fodfs-lqsfdpn qvubljpo

0 p /A	f_{13*}	f_{121*}	dp tul
21	21	21	1
31	211	31	21
41	2111	51	31
51	2121	61	41
61	21211	211	51
71	212111	311	61
81	212121	321	71
81	2121211	531	81
:1	21212111	851	81
211	21212121	861	:1
221	212121211	2811	211
231	2121212111	4511	221
241	2121212121	4521	231

3/2 /F ybn qmf

Tvqqptflx fl x boutlpldpvou h^{3916}/ Jolui jtldbtf-lui flcjobszlgpsm lpglfyqpofoui f >-l212122221212$_{13*}$-/C fdbvtflijutlmfohul ljtl23-lwbmvel >l122/A U l flmpohftuttsjohlpgl po ftlpglui flfyqpofoui bttlbmfohul lpgl5/AUbcmfl6lqsftfoutlbmmlfifm foutlpglqsfdpn .l qvubljpo/

Q sfdpn qvubljpolgpslfyqpofouf l>-l3916l

f_{13*}	f_{121*}
21	21
211	31
2111	51
2121	61
21211	211
22221	261

U i flfybn qmflpgl flflbmbpsjun lgpsfl>-l3916ljtltl pw oljolUbcmfl7/AU lflqstddpmm no l tl px tl fl wbmf ttlvddfttjwfmzlbepqufelczlvbsjbcmfljl/AU l flwbmflijtlsfevdfelczlul fl

nfohu !pg!u f!n bu !fe qbsfeto!u bu !x f !bwf!jo!dpm o ol 5/ Wbsjbcm B ! pmt!u f!
tvddfttjwfm z!e fufsn joe qpx ftt!

Ubcm 7!

B mpsju n !pg!{fsp.poftfrvfodft!gps!fyqpofou!f!>!3916!

j!	dpn qvufejfyqpofou	B !	x joepx !
22!	!	2!	2121!
8!	2121!	h^{21}	2222!
4!	212122221!	$jh^{21+27} h^{26} > h^{286}$	.!
3!	212122221!	$jh^{286+3} > h^{461}$	212!
1!	2121222121 21!	$jh^{461+9} h^6 > h^{3916}$	.!

C fmp x !x f!qsftfoubu f!dpn qbsjtpo!pg!u f!{fsp.poftfrvfodft!bpsju n -!tm jejoh.
x joepx !bpsju n !boe!mfgu.up.sjhi u!bpsju n -!gps!fyqpofou!f!>!3916:!

À 26!n vm qm dbujpo!gps!bpsju n !pg!{fsp.poftfrvfodft!
À 27!n vm qm dbujpo!gps!tm jejoh.x joepx !bpsju n !
À 28!n vm qm dbujpo!gps!mfgu.up.sjhi u!bpsju n !boe!gps!sjhi u.up.mfgu!bpsju n /!

4/D pn qbsjtpo!pg!qsftfoufe!bpsju n t!fggfdujwfoftt

U f!dpn qbsjtpo!ejggfsfou!fyqpofoujbpo!bpsju n t!ibt!cffo!cbtfe!po!u f!
bwfsbhfov n cfs!pg!pqfsbujpot!n vm qm dbujpo*!gps!2111!sboepn !ufttfmfdufe!239.cju!
fyqpofou/U f!sftvm t!bsf!qsftfoufe!jo!Ubcm 8/U f!sftvm t!pg!u f!x pcftubmp.
sju n !bsf!tipx o!jo!cpm /!

Ubcm 8!

B wfsbhfov n cfspn vm qm dbujpot!qfspn felps!239.cjufyqpofout!

B mpsju n !	B wfsbhfov cfs!pg!pqfsb.ujpot!gps!2111!uftut!
mfgu.up.sjhi u.cjobsz!	299/.4:!
mfgu.up.sjhi u.bsz-!!>!4!	29:/.:6!
mfgu.up.sjhi u.bsz-!!>!5!	283/363!
mfgu.up.sjhi u.bsz-!!>!6!	279/786!
tm jejohx joepx -!!>!4!	283/388!
tm jejohx joepx -!!>!5!	275/562!
tm jejohx joepx -!!>!6!	277/947!
{fsp.poftfrvfodft!	275/4:7!

B tdbo!cftffo-!u f!cftu!sftvm t!sfbdi!jfwfe!cz!u f!tm jejoh.x joepx !bpsju n !
boe-!czu f!qbqfsbu posu!bpsju n -.!u f!{fsp.poftfrvfodft!bpsju n /U f!cftu!

qptjujpolpgui fl{fsp.poflfrvfodftlbmbpsjun lsftvmtlgspn lblhppeltmfdujpolpgiqsf.
dpn qvubujpol.lui flovn cfslpglqsf.eftjhobufelqpx festlboelui fjslsfmbujpotljqt/

D podmvtjpol

Jolqsabdujdf-lui flbmbpsjun lpgl{fsp.poflfrvfodftlpsmkshlfyqpofoutljtlnfttlfg.
gfdujvfluibouflfstjejoh.x joepx lbmbpsjun lpgsalbqspqfsmztmfdufelvbmvfpglAUijtljtl
cfdbvtfluiflovn cfslpgm buljohlboelsfqspevdfelqbuutsotlgpsufl ftmjejoh.x joepx l
bmbpsjun ljtlhsfbufsltbolgpslui fl{fsp.poflfrvfodftl bmbpsjun /AUijtljtlevflupl
bltn bmfstlovn cfslpglfmm foutlhfofsbufeljolui flqsfdpn qvubujpolpgluljtlbmp.
sjun /AI

Jolgvuvsf-lx fljoufeelupljm qspvflui flbmbpsjun lpgl{fsp.poflfrvfodftljoltvdi l
blx bzluil buljodpvmelvtfblbmbshfslovn cfslpgsablejmzlhfofsbujohlpbuufsot/

S fgfsfodftl

[2^ D pol foll /-lB lD pvstfljolD pm qvubujpobmAmfcsbjdl/Om fslUl f poz-lT qsjohfs.Vfsmbh-lC fsmjol2: : 4/

[3^ C bdi ! F /-lTl bmqjul K/P /-lB mbpsjun jdlO vm cfslUl fpsz-lvpmm fl.l:lF qgdjfoulB mbpsjun t-lUl f! N AU l
Q sftt-lD bm csjehf!2: : 7/

[4^ M foefftl B /-lwboel O ostci putlQ /-lVbotupoflT /B /-ll boelbppl lpglB qqmjfelD szquphsbqi z-lD S lQsftt-l
C pdbl S bupol2: : : /

[5^ L ovul E /-lUl f!B srlpgllD pm qvufslQ sphsbm m joh-lvpm m fl3/lT em jovm fsjdbmAmbpsjun t-lB ddjtpol
X ftmfzlM pohm bo-l2: : 8l

[6^ K pzflN /-lZ foiT /N /-lUl f! N potom fszlppx fejoh!l deefs-lD szquphsbqi jchl aax bsflboel N cfeefel
T zstum .D ! F T !3113-lt/l2.22!