ENGINEERING ASPECTS OF STRUCTURAL RESPONSE OF MULTI-SPAN SANDWICH PANELS

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Abstract. The paper concerns a problem of static response of multi-span sandwich panels. The effects of transversal load and thermal actions are compared. The influence of material parameters on the sandwich behaviour is discussed. The examples illustrate practical approach to the problem of optimal design.

Introduction

Sandwich panels are commonly used in civil engineering as cladding elements. These panels are made of three layers: two external, thin and relatively rigid steel facings and thick, but light and flexible core (polyurethane, mineral wool, expanded and extruded polystyrene). The facings can be flat, micro-profiled or deep-profiled. The sandwich structures are very attractive for engineers because of a high load-bearing capacity at low self-weight, excellent thermal insulation, short time of erection and possibility of economical mass production. From the other point of view, such type of structure requires taking into account many apects of structural behaviour of sandwiches: various failure mechanisms, essential role of temperature actions, influence of creep, shear flexibility of the core and high susceptible to local instability of compressed faces.

The consistent theory decribing sandwich structure behaviour was originally published by Allen [1] and Plantema [2]. The approach was broadened and rearranged by Stamm in [3]. These publications gave the background to the current standard EN 14509 [11]. A wide variety of problems concerning sandwich panels with particular attention to engineering applications was presented by Zenkert [4] and Davies [5]. The importance of stress concentration and complex interactions between facings and core parts was underlined by Frostig [6]. The stress concentration leads to debonding and local instability of the sandwich. The paper [7] takes into account these phenomenon in the static analysis of continuous sandwich beams. Reliable analysis of any structure is connected with proper estimation of material parameters of the structure. The influence of material selection on the structural response was presented in [8]. Various failure mechanisms of sandwich structures and the possibility of mass production and market demands extort optimal design from engineers. Therefore, the relation between stress conditions and different failure modes is considered [9]. On the other hand, the producers are interested in structures which provide minimal cost of production and maximal range of applications [10].

In spite of the great importance of sandwich panels (taking into account universality of applications, costs of investments in the civil enigeenering industry, increase of production etc.), unfamiliarity with specific behaviour of the panels leads to misunderstandings or even mistakes in production, design and usage. The Authors make an attempt to present the most important solutions of static systems and the comparison of different actions' effects. The influence of variations in material and mechanical parameters on the structural response are discussed. The discussion about safety factors and practical hints for optimal design is presented.

1. Sandwich panel theory

This paper discuses multi-span panels with parallel facings and a soft core. The model of the three-span panel is shown in Figure 1.

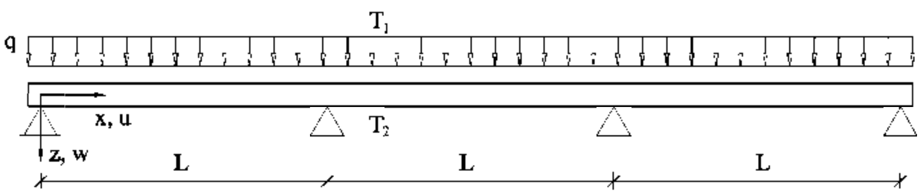

Fig. 1. Multi-span sandwich panel loaded mechanically (q) and thermally ($\Delta T = T_2 - T_1$)

In case of uniformly distributed loading and thermal actions, the Timoshenko beam theory generalized to sandwich sections is used [1-3]. In case of load or support conditions which demand 2-D description, the Reissner plate theory can be applied. It is assumed that the strains are small and materials are isotropic, homogeneous and linearly elastic. Because the Young modulus of the foam core is much lower than of the steel faces (about 50,000 times), the normal stress in the foam core is negligible ($\sigma_{xC} = \sigma_{yC} = 0$). Therefore, the shear stresses in the core are constant along transverse axis z ($\tau_{xzC} = \tau_{yzC} = \text{const.}$).

The cross sectional equilibrium condition for panels with thick or deep-profiled faces can be written in the form of two uncoupled differential equations (1). (2), for vertical displacement w and for shear strain γ [3]:

$$-\frac{B_{F1} + B_{F2}}{G_C A_C} \cdot w^{VI} + \frac{B}{B_S} \cdot w^{IV} = \frac{q}{B_S} - \frac{q''}{G_C A_C} - \theta'' \qquad (1)$$

$$-\frac{B_{F1}+B_{F2}}{G_C A_C}\cdot\gamma^{IV}+\frac{B}{B_S}\cdot\gamma''=-\frac{q'}{G_C A_C}-\frac{B_{F1}+B_{F2}}{G_C A_C}\cdot\theta''' \tag{2}$$

where w and γ are the functions of the position coordinate x. The G_C and A_C denote shear modulus and cross-sectional area of the core, q is the distributed transverse load and θ is an initial curvature induced by a temperature difference $\Delta T = T_2 - T_1$. Because the bending stiffness of the core is negligible, the total bending stiffness of panel B consists of three parts:

$$B = B_{F1} + B_{F2} + B_S \tag{3}$$

The term B_S represents the bending stiffness of the facings with respect to the global centre line of the sandwich panel, whereas B_{F1} and B_{F2} are the bending stiffness of the upper and lower facings with respect to their own centre lines.

In case of panels with flat and slightly profiled facings the B_{F1} and B_{F2} are negligible, $B = B_S$ and the equilibrium conditions (1), (2) change into (4), (5):

$$w^{IV}=\frac{q}{B_S}-\frac{q''}{G_C A_C}-\theta'' \tag{4}$$

$$\gamma''=-\frac{q'}{G_C A_C} \tag{5}$$

Integrating twice (4), (5) and using differential equations $M' = Q$, $Q' = -q$, the constitutive equations (6), (7) are obtained:

$$M = B_S\cdot(\gamma' - w'' - \theta) \tag{6}$$

$$Q = G_C A_C\cdot\gamma \tag{7}$$

The terms M and Q denote the bending moment and shear force, respectively. In order to solve the problem, the influence of temperature is usually analysed separately:

$$w_T'' = -\theta, \qquad \gamma_T = 0 \tag{8}$$

and the displacement w is divided into two parts $w = w_M + w_Q$ which refer to the bending and shear effect, respectively. Because $w_Q' = \gamma$ it follows to:

$$M = -B_S w_M'', \; Q = G_C A_C w_Q' \tag{9}$$

The bending and stresses in flat faces and shear stresses in the core are calculated using (cf. [11]):

$$\sigma_{F1} = -\frac{M}{eA_{F1}}, \quad \sigma_{F2} = \frac{M}{eA_{F2}} \tag{10}$$

$$\tau_C = \frac{Q}{A_C} \tag{11}$$

where e, A_{F1} and A_{F2} denote distance between centroids of faces, cross-sectional area of the external (upper) and interanal (lower) face, respectively.

In case of deep-profiled panels, the bending stiffness of faces must be taken into account and the bending moment and shear force are divided into parts which refer to each part of the panel (core, upper face, lower face). It results in the fact that even in the case of simply supported one-span panels, the structure is statically undetermined. The respective equations which allow static calculations for deep-profiled panels are given in detail in [3].

2. The effect of shear deformation

2.1. The example of one-span, hang-over beam

The shear deformation influences the structural response. The importance or even unpredictabilty of the effect can be observed in the example of the flat sandwich panel loaded by the concentrated force P (Fig. 2).

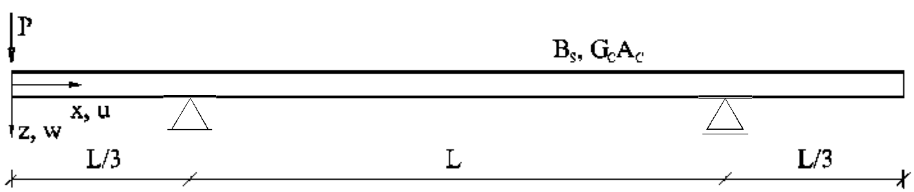

Fig. 2. The shear deformable structure loaded by concentrated force [4]

The displacement function w_M, which refers to the bending effect, has the typical form

$$B_S w_M(x) = \frac{4}{81}PL^3 - \frac{P}{6}L^2 x + \frac{P}{6}x^3 - \frac{2}{9}P\cdot\left(x-\frac{L}{3}\right)^3\cdot\Phi\left(x-\frac{L}{3}\right) + \frac{P}{18}\cdot\left(x-\frac{4}{3}L\right)^3\cdot\Phi\left(x-\frac{4}{3}L\right) \tag{12}$$

where the Φ is the Heaviside function and x is the position coordinate. The prediction of w_Q, which refers to shear deformation, is not so intuitive (Fig. 3):

$$G_C A_C w_Q(x) = \frac{4}{9}PL - \frac{4}{3}Px + \frac{4}{3}P \cdot \left(x - \frac{L}{3}\right) \cdot \Phi\left(x - \frac{L}{3}\right) - \frac{P}{3} \cdot \left(x - \frac{4}{3}L\right) \cdot \Phi\left(x - \frac{4}{3}L\right) \quad (13)$$

Both functions are presented in Figure 3. For typical parameters of the sandwich panel B_S = 332 kNm^2, $S = G_C A_C$ = 276.5 kN, the force magnitude P = 1.5 kN and the span L = 4.5 m the effects of shear and bending are comparable. Please note that if the bending stiffness increases, the form of shear deformation will dominate.

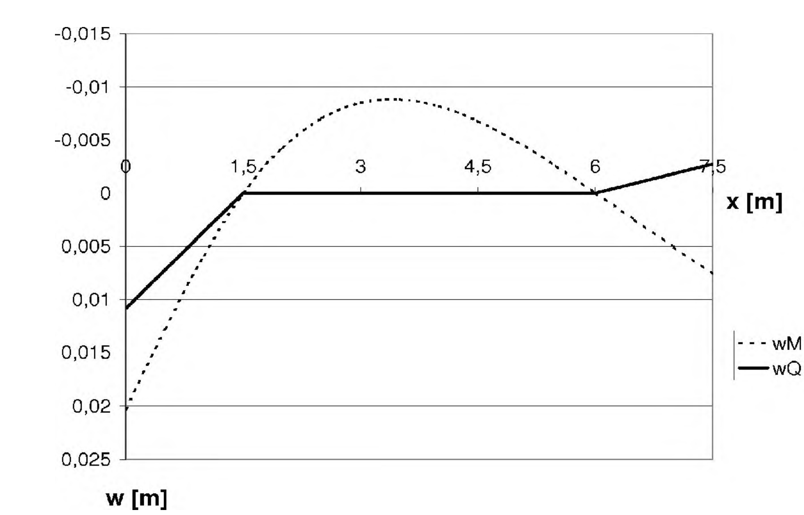

Fig. 3. The sandwich panel displacements with respect to bending (wM) and shear (wQ)

2.2. The multi-span panels

Consider the two- and three-span flat sandwich panels with equal spans L subjected to transverse uniform load q (cf. Fig. 1). It is well known that in case of Bernoulli beam ($G_C A_C = \infty$), the beam which has more spans is better because the extreme bending moments and deflections are lower. The question is whether it is also true in the case of sandwich beam structures. The respective values of bending moments are given in Table 1, where the parameter k is defined as:

$$k = \frac{3B_S}{L^2 G_C A_C} \quad (14)$$

The results presented in Table 1 show that the extreme bending moment for one-span beams is always higher than the others. Using the same values we can also find the conditions, when for the respective system the bending moment in span M_A is equal to the moment at the support M_B (to the absolute value). It happens when:

$$k = \frac{\sqrt{2}}{2} - \frac{1}{4} \approx 0.4571 \quad \text{for 2-span panel} \tag{15}$$

$$k = -1 + \sqrt{2} \approx 0.4142 \quad \text{for 3-span panel} \tag{16}$$

Table 1

Bending moments for one-, two- and three-span panels [11]

System	Bending moment in (end) span (M_A)	Bending moment at internal support (M_B)
Single span of L, uniform load q	$\frac{qL^2}{8}$	–
Two equal spans of L, uniform load q	$\frac{qL^2}{8}\left(1 - \frac{1}{4(1+k)}\right)^2$	$-\frac{qL^2}{8}\frac{1}{1+k}$
Three equal spans of L, uniform load q	$\frac{qL^2}{8}\left(1 - \frac{1}{5+2k}\right)^2$	$-\frac{qL^2}{10+4k}$

Please note that the bending moments attain the same values for two- and three-span systems if $k = 0.5$. This is presented in the last column of Table 2. The bending moment at the internal support is lower (to the absolute value) for 3-span systems than for 2-span ones when $k > 0.5$. The same relation is valid for thermal actions.

Table 2

Bending moments for one-, two- and three-span panels for different values of *k*

System	Bending moment	$k = 0.4142$	$k = 0.4571$	$k = 0.5$
Two equal spans of L, uniform load q	M_A	$+0.08471\, qL^2$	$+0.08579\, qL^2$	$+0.08680\, qL^2$
	M_B	$-0.08839\, qL^2$	$-0.08579\, qL^2$	$-0.08333\, qL^2$
Three equal spans of L, uniform load q	M_A	$+0.08579\, qL^2$	$+0.08630\, qL^2$	$+0.08680\, qL^2$
	M_B	$-0.08579\, qL^2$	$-0.08454\, qL^2$	$-0.08333\, qL^2$

It is worth noticing that extreme bending moment resulting from thermal action is attained at the internal support. Hence, the optimization of the structure is combined with the minimization of the moment M_B.

Figure 4 shows a graph of the values of k as a function of L for various depths of panel D. The following parameters were assumed: thickness of steel faces $t = 0.0005$ m, Young modulus of steel $E = 210$ GPa and shear modulus of the core $G_C = 3.5$ MPa.

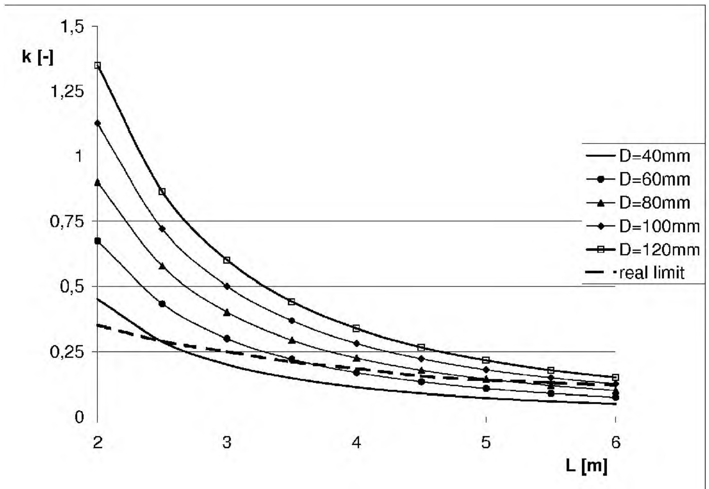

Fig. 4. The parameter *k* as a function of *L* for various panel depths *D*

Figure 4 shows that parameter *k* increases when the depth *D* increases and span *L* decreases. This is also clear by analysis of the equation (14) because the bending stiffness B_S increases with the growing of *D* faster than shear stiffness $G_C A_C$. The real range of sandwich panel application with respect to limit states is situated above the dotted line. The entire analysis proves that indication of a better structure is not automatic and 3-span systems can be worse than 2-span panels.

3. The influence of thermal action

As we previously mentioned, multi-span systems are considered by many people to be better than one-span structures. In fact, the bending moments presented in Table 1 are lower for multi-span systems. Nevertheless, we definitely can say that one-span sandwich systems are better. This is because of the influence of thermal actions. The temperature difference $\Delta T = T_2 - T_1$ between internal and external faces triggers the initial curvature θ:

$$\theta = \frac{\alpha_2 T_2 - \alpha_1 T_1}{e} \tag{17}$$

where α_1, α_2 are thermal expansion coefficients of respective faces, *e* is the distance between the centroids of faces. The curvature in one-span systems results in displacements (maximum deflection is $\theta L^2/8$), but it does not change the internal forces. In multi-span systems the thermal action plays a crucial role bringing on shear forces and bending moments. The significance of the effect is illustrated in Figures 5 and 6. Figure 5 presents the values of bending moments at the internal support of a two-span panel as a function of the span *L*. The curves on the graph refer to the effect of uniform load *q* (MB_q) and the temperature difference (MB_T). In calculations the following typical parameters were introduced: depth of the panel 0.08 m, thickness of the faces 0.0005 m, Young modulus of steel E = 210 GPa and shear modulus of the core G_C = 3.5 MPa. The load q = 0.50 kN/m and the temperature difference 40°C were taken. It corresponds to the temperature in winter: internal +20°C, external –20°C.

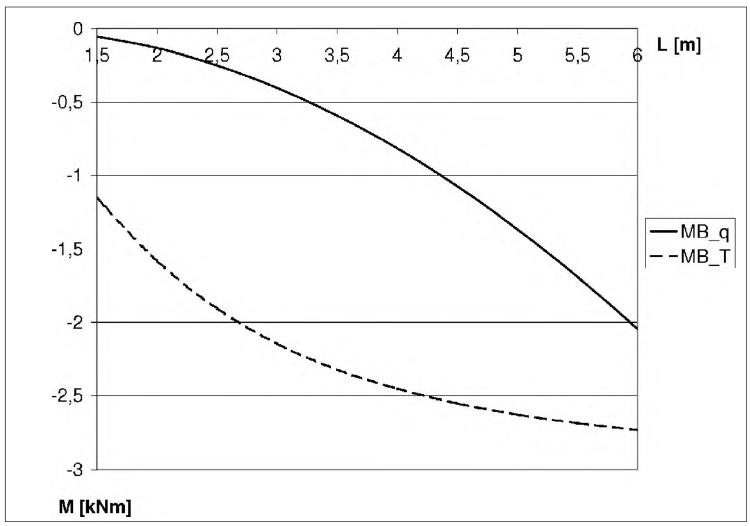

Fig. 5. The bending moment at the internal support of a two-span panel induced by load *q* (MB_q) and curvature *θ* (MB_T) as the function of the span *L*

Comparing the results on the graph it can be noticed that the bending moments caused by the temperature difference are much greater (to the absolute value) than the moments caused by the transverse load *q* for all values of the variable span *L*. Therefore, the effect of the combination of both actions (*q* and *θ*) gives much greater bending moments at internal supports for multi-span panels than the moment in span for one-span panels. It is also an interesting fact that function MB_T changes values nonlinearly, completely the opposite to MB_q, if span *L* increases the change in MB_T is smaller and smaller.

Figure 6 shows the intermediate support reaction as the function of span *L* for the same parameters of a sandwich panel. If the reaction is positive it causes compression at the support. If the reaction is negative it causes tension of the screws attaching the panel to the supporting structure.

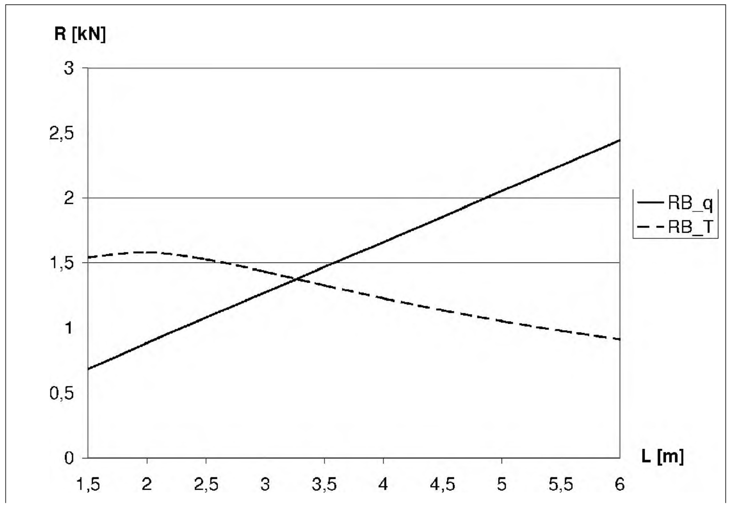

Fig. 6. The intermediate support reaction of a two-span panel induced by load q (RB_q) and curvature θ (RB_T) as the function of the span L

The curves in Figure 6 demonstrate that the temperature difference for small spans results in significant values of support reaction. In the case of a negative reaction the effect can lead to failure of the screws or the panel in the vicinity of the screws. It is interesting that if span L increases, *RB_T* decreases whereas the function of *RB_q* increases linearly.

4. Failure modes of deep-profiled panels

There are various failure modes of deep-profiled panels: shear failure of the core, shear failure of a profiled face layer, yielding of a face, wrinkling (local buckling) of a face, crushing of the core at a support, failure at the points of attachment to the supporting structure and the attainment of a specified deflection limit. In the case of flat multi-span panels it can be said that the local buckling is the most important. To present the importance of respective limit states the typical deep-profiled panel with the following parameters is analysed: total depth 0.14 m, profiling 0.04 m, thickness of the faces 0.0005 m, Young modulus E = 210 GPa, shear modulus G_C = 3.5 MPa, thermal expansion coefficient $\alpha = 12{\cdot}10^{-6}$ 1/°C, yield stress f_y = 280 MPa, bending stiffnesses B_{F1} = 20.6 kNm^2, B_S = 584 kNm^2 and shear stiffness $S = G_C A_C$ = 369 kN. The characteristic load q = 0.80 kN/m corresponds to snow load and the temperature differences +40°C (winter) and –40°C (summer) were taken into account. The results of the analysis of a deep-profiled three-span panel with equal spans is presented in Figure 7. The LS denotes the ratio of the effect of the action to the corresponding resistance. The graph presents the LS ratio as the function of the span *L*.

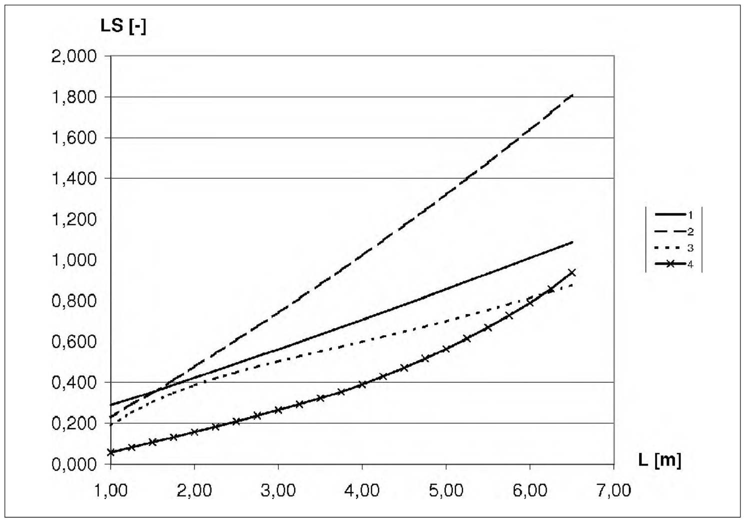

Fig. 7. The LS ratios for three-span panel: 1 - crushing of the core at the intermediate support, 2 - yielding of the face at the intermediate support, 3 - wrinkling of the face at the intermediate support, 4 - the attainment of a specified deflection limit

Surprisingly, the yielding of the face at the intermediate support appeared the most important. It limits the range of sandwich panel application. In our example the limit is equal to 3.92 m (maximum acceptable LS ratio is equal to 1.0). It is also an interesting fact that apart from the deflection, the other curves have almost linear form. The results show that the reducing of steel yield strength in the case of deep-profiled multi-span panels is absolutely uneconomical.

Concluding remarks

The presented analysis debunk many beliefs concerning sandwich panels. It appears that three- or more-span panels can be worse than two-span sandwich structure. Much more important is the fact that multi-span panels are worse than single-span systems. Of course roof panels should be multi-span systems to ensure water-tightness. If there are deep-profiled panels, the yield stress limits the range of practical applications. The examples prove that structural behaviour of panels can be at variance with engineering intuition. The problems which were taken up are important from the practical point of view because the producers press on the minimization of costs and maximization of permissible spans.

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