PERCOLATION WITH A BARIER IN FINITE SYSTEMS

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Abstract. The site percolation, where the long-range connectivity is the result of the occupancy probability defined on a site, is studied on the $L \times L$ square lattice. Method of determining of the location of the percolation pseudo-threshold $p_c(L)$ is proposed and the influence of a barrier on the percolation pseudo-threshold is analysed.

Introduction

The basic mathematical model of connectivity is called percolation theory. It was proposed by Broadbent and Hammersley [1] to study the flow of fluid in a porous medium with randomly blocked channels. Since that time percolation techniques have become a corner stone of the theory of disordered media [2].

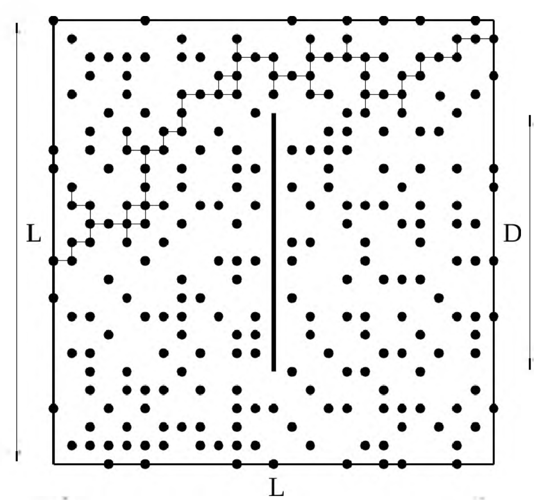

Fig. 1. The square grid on the $L \times L$ system. The vertical thick line denotes a barrier D, whereas dots are occupied sites. The thin lines indicate the spanning cluster created by randomly chosen sites

Let us take a grid and occupy sites on this grid with a probability p. For small values p one can see mostly isolated occupied sites with occasional small groups of them. As the occupancy probability increases, groups grow forming some clusters. Of course in the second limit, when $p = 1$, every site is occupied. Thus, there exists such a value of p for which one of the clusters starts to connect opposite edges of the square, which in practice means a maximal cluster size at a given square $L \times L$. This is called the spanning cluster (or the infinite cluster) as it spans the entire lattice (arbitrarily, we consider only the clusters connecting the left and right edges). This particular value of the occupancy probability, well defined for $L \to \infty$, is known as the percolation threshold p_c. So, the long-range connectivity is the result of the occupancy probability defined on a site. It can be described in terms of the probability of appearance of spanning cluster P as a function of p. The exact value of the threshold depends on the kind of grid considered and strongly on the dimensionality of the grid. In our case, for the two-dimensional square lattice, the percolation threshold has been designated numerically as $p_c = 0.59274621...$ [2].

Not all occupied sites are in the spanning cluster. That is why we can define another probability $P'(p)$ saying if an occupied site belongs to the spanning cluster. Below the percolation threshold that clearly must be zero. The function $P'(p)$ is particularly important when dealing with critical phenomena for percolative systems, but we do not treat this subject in the present study.

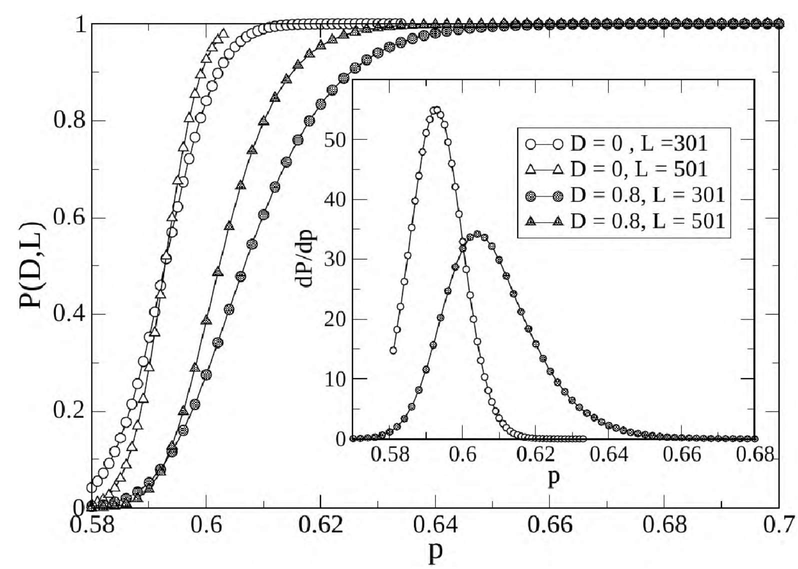

Fig. 2. The probability of appearance of spanning cluster P as a function of occupancy probability p for various system sizes L and barrier lengths D. The inset: the maxima of curves denote the positions of the percolation pseudo-threshold $p_c(L)$

The main aim of this paper is to answer the question of how the presence of a barrier in the finite system affects the behavior of the probability functions defined above. For simplicity, the barrier, which can be interpreted as a selected group of sites for which $p = 0$, has been chosen as symmetric with respect to the system edges (see Fig. 1). It is worth mentioning that the preliminary results has been presented in the reference [3].

1. Model

At finite systems there is no longer a sharp transition at p_c, it becomes smeared out to some extent of p around p_c. Therefore the percolation threshold defined by the singularity of $P(p)$ (or $P'(p)$) at p_c is replaced by the percolation pseudo-threshold $p_c(L)$. As one increases the system size, the smearing gets less (see in the main body of the figure 2 at $D = 0$). To determine the location of the percolation pseudo-threshold $p_c(L)$ we use the inflexion point of curve (or the equivalent position of the maximum of its first derivative). The same strategy for determining the $p'_c(L)$ can be used to the $P'(p)$ probability (see Fig. 3). Although both quantities $p_c(L)$ and $p'_c(L)$ vary for finite L, they tend to the same p_c when $L \to \infty$.

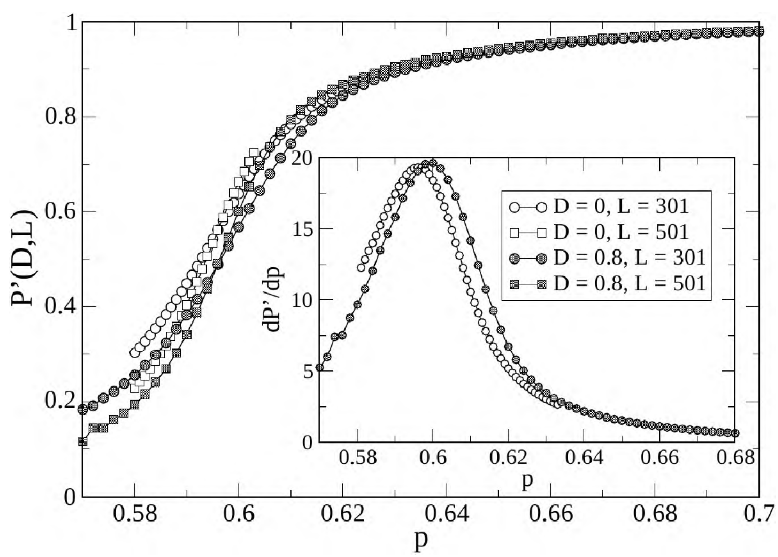

Fig. 3. The fraction of occupied sites belonging to the spanning cluster for various system sizes L and barrier lengths D. The inset: the maxima of curves denote the positions of the percolation pseudo-threshold $p'_c(L)$

The presence of a vertical barrier reduces the number of paths along which the cluster may extend between the edges. The only way to compensate the lack of some connections is to increase the probability p. Therefore we expect that for a non-vanishing D the percolation pseudo-threshold is shifted towards higher values of p. Our results presented in Figures 2 and 3, where the value of D has been chosen to be high enough to make significant a difference between the curves with zero and nonzero D, confirm this conjecture. It is worth noticing that this effect is much more pronounced for the $P(p)$ probability. In addition, in Figure 4, curves for different values of D at fixed $L = 101$ have been collected. It may be noted that for extremely large D shape of the function P and its derivative, changes dramatically.

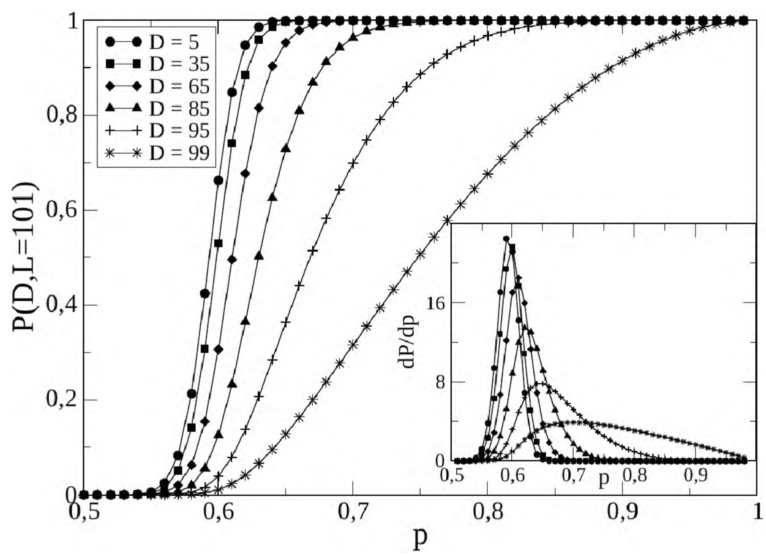

Fig. 4. The probability of appearance of spanning cluster P for $L = 101$ at various barrier lengths D. The inset presents their first derivative. Its maxima localize the percolation pseudo-threshold at finite systems

In order to combine the results for different values of L and D, it seems reasonable to confront the results obtained at a constant ratio of D/L. Both values are lengths characterizing the system, but one can expect that their ratio plays the role of an universal scaling parameter [4]. Thus, we are able to present a common plot for the percolation pseudo-threshold dependence on the ratio D/L.

The plots in Figure 5 show that the presence of a barrier affects the location of the both pseudo-thresholds in a different way. The deviation of the value of $p_c(L)$ for small D/L is relatively small, but it increases dramatically if the D/L is close to one. The position of $p'_c(L)$ varies almost linearly with the change of D/L, but is clearly lower in the whole range of D/L, compared with the previous case. It can be assumed that in the first case a very wide barrier makes practically impossible to create a cluster - only a few sites between the barrier and the edge are crucial. That is why so large p is necessary. In the second case, we consider only those situations, where the spanning cluster has already been formed and we inspect how large is the fraction of occupied sites belonging to it.

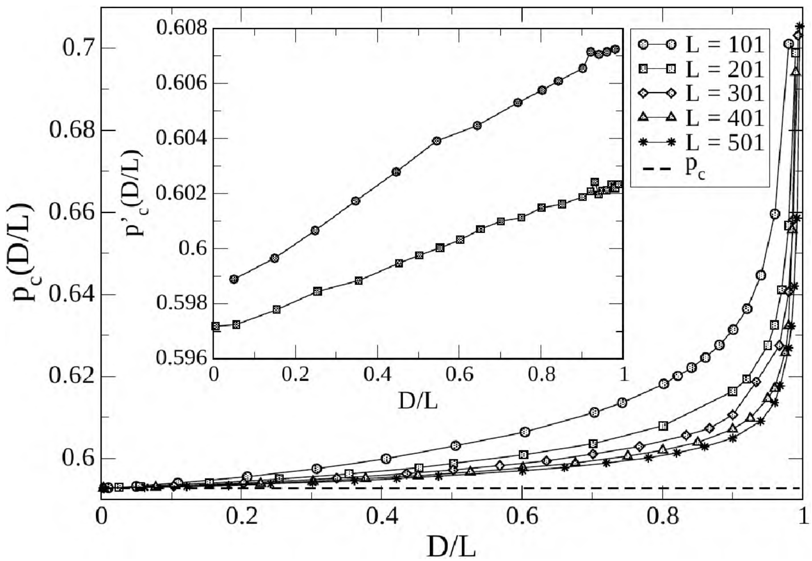

Fig. 5. The lines represent the percolation pseudo-thresholds for the square cluster *LxL* with the symmetrically located barrier with the height *D* parallel to one pair of edges

2. Discussion

We have confirmed the significant influence of a barrier on the localization of the percolation pseudo-threshold. Moreover, we showed that for different ways of determining the percolation pseudo-threshold in finite systems, the effect is diverse. To investigate thoroughly these behaviors further calculations are necessary for larger systems.

References

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